STATIONARY STATES IN A 1D SYSTEM OF INELASTIC PARTICLES

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The existence conditions of asymptotic quasistationary states are found for a one-dimensional open system of inelastic particles. The influence of initial and external conditions on the structure of such states and transitions between them is investigated. The theoretical and numerical calculations are compared to the data of the direct physical experiment. The possibilities to apply the methods of statistical physics to the study of open systems (in particular, granulated materials) close to the discovered quasistationary states are discussed.

1. Introduction

During approximately two recent decades, the scientific community shows a heightened interest in the investigation of complex open systems where the energy dissipation results in the initiation of essentially nonlinear dynamic processes. Among such systems, one undoubtedly rates the so-called granulated materials (GMs). GMs consist of a large number of particles (granules) with complex surface morphology and size dispersion. The interaction between granules takes place only due to collisions of inelastic character. That is, GMs represent the example of an open system that particularly does not obey the law of conservation of energy.

An interest in the investigation of GMs is caused by the prospects of their application in many branches of production. They are also distributed in quantity in the environment. As an example, it's enough to mention ordinary sand.

GMs manifest unusual properties that differ from those of typical liquids, gases, and solids. They result, for example, in such phenomena as inelastic collapse, thread-like clusterization, compactization, segregation, fluidization of the type of the avalanche-like draining of a thin GM layer, the Brasil nut and arc effects (the latter results in the pressure saturation under a GM column in vertical containers), anisotropic clusterization, formation of patterns (defects), etc. [1–15].

The complexity of processes taking place in GMs stimulates their preliminary investigations with the help of studying the dynamics of simple model systems, where particles interact inelastically with one another, whereas the energy dissipation is compensated due to its supply from outer boundaries. Direct physical experiments with GMs (see Section 5) testify to the possibility for asymptotic stationary states to exist in them, which opens, in turn, possibilities for the application of the methods of statistical mechanics to their study.

In the case of studying both the structure and the dynamics of GMs, the role played by contacts between particles-granules is the leading one. For example, the stresses in a static granulated medium or the deformation waves in the case of the dynamic behavior of granules mainly appear and develop exactly within interparticle contacts or contacts between particles and a substrate. It is worth adding that the dissipative energy losses that represent one of the basic attributes of GMs also occur almost solely within interfaces.

In a certain sense, one-dimensional models of dissipative systems can facilitate a better understanding of physical processes in such interface structures and should be interpreted as the first step to the investigation of realistic and undoubtedly more complicated three-dimensional systems.

In what follows, we consider a trivial but rather evident model of a 1D system of inelastic particles in a gravitational field. The chief aim of the study of such systems consists in the direct demonstration of the existence of asymptotic quasistationary states in them, as well as the examination of their existence criteria and properties by means of the comparison of theoretical results with data of numerical and direct physical experiments. The existence of asymptotic quasistationary states in excited GMs (we certainly mean stationary states with a more complicated structurization) is confirmed by some recent experiments [7,15]. That is why the investigation of separate peculiarities of their formation even using the simplest models (in particular, low-dimensional ones) attracts interest.

2. Postulated Stationary States in a Vertical 1D System of Inelastic Particles in a Gravitational Field (Theoretical Determination)

Let us consider a system of N structureless particles of equal mass located vertically in vacuum (in the absence of friction) in the field of gravity forces $\overrightarrow{g}$. The energy losses due to binary collisions between particles can be compensated at the expense of the reflection of the lower particle from the horizontal "hot" solid substrate which represents, thus, an energy source for the system. In the case where such a reflection is absolutely elastic, the system is practically closed. In contrast, in the case where the hot substrate is able to supply an arbitrary (but definite) energy to the system, the latter is open.

The proposed model is constructed in such a way that, at arbitrary velocities of collision of an incident particle with the substrate, the velocity at the time moment of reflection also has the same constant value (let it be ω_0). Generally speaking, ω is the quantity distributed with some weight $\Phi(\omega)$. In our model, the initial velocity after a collision with the substrate obeys the distribution in the form of the Dirac delta-function $\Phi(\omega) = \delta(\omega - \omega_0)$.

Due to the binary character of collisions, the velocities of the particles before a collision (ω_1, ω_2) and after it (ω_1', ω_2') satisfy the following relations:

$$\omega_1' = \omega_1 - \frac{1+\varepsilon}{2}\omega_{12}, \quad \omega_2' = \omega_2 + \frac{1+\varepsilon}{2}\omega_{12}, \tag{1}$$

where $\omega_{12} = \omega_1 - \omega_2$; ε denotes the inelastic loss coefficient (at $\varepsilon = 1$, collisions are absolutely elastic and the summary kinetic energy conserves; at $\varepsilon < 1$, dissipative energy losses take place).

In the postulated stationary state, the periods of motion of the particles between collisions are equal to some constant T_N.

Taking into account the uniformly accelerated character of motion of particles between collisions, we obtained the following expression for the oscillation period T_N [5]:

$$T_N = \frac{2\omega_0}{g}\left[N + \frac{(1-\varepsilon)}{3(1+\varepsilon)}(N-1)(2N-1)\right]^{-1}. \tag{2}$$

Thus, as follows from (2), the period of the oscillatory motion of the N-th particle in the stationary state in the constructed model system depends on all parameters of the model ω_0, g, and ε and reaches its maximal or minimal values in the limits of absolutely elastic ($\varepsilon = 1$) and inelastic ($\varepsilon = 0$) collisions, respectively.

The stationary motion in such a model system must have form of the vertical stratification (i.e. layering) of the system into a sequence of intervals, within which the corresponding particles participate in a simple periodic motion. In this case, collisions of each pair of particles occur at some corresponding fixed heights.

The dimensions of the system, where the described stationary motion takes place, can be found with the help of the formula

$$L = \frac{gT^2}{8}\left(1 + 4\sum_{i=1}^{N-1}\frac{A}{(1+A)^2}(1 + 2(N-i))\right), \tag{3}$$

where

$$A = \frac{(1+2\varepsilon) - (1-\varepsilon)(N-i)}{(2+\varepsilon) + (1-\varepsilon)(N-i)}$$

denotes the ratio of the time, during which the i-th particle moves upward, to the time, during which it moves in the opposite direction. Each separate term in (3) specifies the size of the corresponding region, where the i-th particle performs its periodic motion. The condition, where at least one of these terms becomes negative, determines the criterion of decay of the stationary state:

$$\varepsilon \geq \varepsilon_c = \frac{N-2}{N+1}. \tag{4}$$

It follows from relation (4) for the critical value of the inelastic loss coefficient ε_c that, for systems that include only one or two particles, the stationary states are formed for arbitrary values of ε. If the number of particles in a system exceeds two, then the stationary states exist under condition of the limitation of the coefficient of inelastic energy losses due to interparticle collisions.

Thus, the asymptotic states cannot exist in a model with strongly inelastic collisions. The specified stationary state does not exist at sufficiently large N, i.e.

in the case of systems with large dimensions or many-particle systems.

The obtained results testify to the fact that, in large-scale systems or systems with strong dissipation, the supply of energy to the system from outside is insufficient for the creation of the existence conditions for stationary states (the dissipative and external energy flows cannot compensate one another).

3. Stationary States in a 1D Horizontal System of N Inelastic Particles

The problem of existence of stationary states in the case of one-dimensional systems, where inelastic particles collide in the absence of gravitational forces will be investigated by the example of the system considered in Section 1, by locating it horizontally. As in the previous case, the further analysis will be performed by postulating the existence of stationary stats in such a system.

Let us assume that two arbitrary particles move after a collision in the opposite directions. The corresponding stationary mode will have form of a simple periodic (oscillatory) motion of the particles of the system with the same period T. In addition, particle i in the stationary state of the 1D system moves within the interval x_i that satisfies the relation

$$L = \sum_{i=1}^{N} x_i, \tag{5}$$

where L is the size of the whole system.

In the specified stationary state, the velocity of any particle in the case where it moves from left to right will be denoted by v_i^r and in the case where it moves in the opposite direction – by v_i^l. The absolute values of the velocities of the particles immediately before and after the chosen collision are equal to [see (1)]

$$v_i^l = v_1^r - (i-1)v_{12} + (i-2)\frac{1+\varepsilon}{2}v_{12}, \tag{6}$$

$$v_i^r = v_1^r - (i-1)\frac{\varepsilon-1}{2}v_{12}, \tag{7}$$

where $v_{12} = v_{i,i+1} = v_i^r - v_{i+1}^l$ is the relative velocity in the pair of particles which varies only during the collisions, for which the inelastic loss coefficient ε =const. For the postulated type of a horizontal stationary (periodic) motion (without a gravity), it is set constant.

The size of the region x_i of the system, within which the i-th particle performs periodic oscillations with the corresponding constant amplitude in the stationary state, can be found with the help of the relation

$$x_i = \frac{v_i^r v_i^l}{v_i^l - v_i^r}T. \tag{8}$$

The period T of the stationary motion that satisfies conditions (5) and (8) is equal to

$$T = \frac{L}{\sum\limits_{i=1}^{N} \frac{v_i^r v_i^l}{v_i^l - v_i^r}}. \tag{9}$$

Thus, it follows from (9), (6), and (7) that, in order to determine the period T, it is necessary to determine the relative velocity of the particles v_{12} and specify the conditions of their reflection at the boundaries of the system. Now let us consider several examples, where energy is supplied to the system from outside (which ensures the maintenance of a specified stationary state of the dissipative system).

In the case where the energy is supplied from both boundaries of the system (i.e. the particles are reflected from the left and the right boundaries of the 1D system with some definite constant velocities v_1^r and v_N^l), v_{12} can be derived with the help of (6) as

$$v_{12} = \frac{2v_{1N}}{(1-\varepsilon)N + 2\varepsilon}, \tag{10}$$

where $v_{1N} = v_1^r - v_N^l$.

If the energy is supplied to the system only from one boundary (for example, the left one), let us suppose the interaction of the N-th particle (the nearest to the right wall of the system) with its boundary to be absolutely elastic. That is, its reflections from the boundary are specular and do not result in energy losses: $v_N^r = -v_N^l$. With regard for (7), one can express v_N^l in terms of v_{12}. Namely, using (10), we obtain

$$v_{12} = \frac{4v_1^r}{2(1-\varepsilon)N + 3\varepsilon - 1}. \tag{11}$$

Thus, we have obtained the analytic solution of the constructed model.

It is worth noting that the period T of a specified stationary state depends on the size of the system L and the inelastic loss coefficient ε, as well as on the energy obtained by the system from outside.

Now let us consider such a kind of motion in a two-particle system, where one particle has no time to get to the nearest wall after a collision, while the other one reflects from the opposite boundary and overtakes the particle which is "late". At the following step, the particles exchange the roles. In this case, one collision of the particle with the wall corresponds to its two collisions with the other particle. Let us demonstrate that, in this case, the stationary mode is also possible in principle.

The velocity of the particle nearest to the "hot" wall, from which the energy is supplied, will be denoted by v, while the velocity of the other one – by w. Let the velocities of the particles after their first collision v_1 and w_1 be distributed in such a way that the first particle will be "late".

After an elastic reflection from the wall, particle *2* overtakes particle *1*, and they collide again. In this case, the kinetic energy is lost (this process is determined by the coefficient ε), and the velocities of the particles after the collision will satisfy relation (1). Let us denote the velocities obtained by the particles after the second collision by $v_{2,i}$ and $w_{2,i}$ (here, index i specifies the number of the corresponding two-collision mode).

At the following stage, the other particle is late. Moreover, particle *1* moves much faster and, reflecting from the "hot" wall with the velocity v_0, overtakes particle *2* before it has time to reflect from the "cold" wall. After the collision, the velocities obtained by the particles again satisfy the relation of type (1). Let us denote these velocities by $v_{1,i+1}$, $w_{1,i+1}$. At this step, the specified two-collision cycle comes to the end.

In the stationary state, the corresponding values of the velocities and coordinates for each particle must be periodically reconstructed during the time interval equal to one period. Under these conditions, we obtain the following relations with the help of (1):

$$w_1 = v_0\frac{1+\varepsilon}{2}, \quad v_1 = v_0\frac{1-\varepsilon}{2}, \quad v_2 = -\varepsilon v_0, \quad w_2 = 0. \tag{12}$$

The determination of the coordinates x_1 and x_2 that specify the points of collisions of the particles gives

$$x_1 = x_2 = L. \tag{13}$$

From relations (12) and (13), one can see that, in the postulated state of the system, particle *1* moves in the whole bulk of the system, while particle *2* looks as if being stuck to the "cold" wall. Thus, the latter becomes effectively inelastic in the sense of its interaction with particle *1*.

The period T of the specified kind of motion in the system can be expressed in the form

$$T = \left(1 + \frac{1}{\varepsilon}\right)\frac{L}{v_0}. \tag{14}$$

The substitution of $\varepsilon = 0$ into (14) results naturally in the motion with an infinite period. This result can be explained by the coalescence of the particles with each other due to an absolutely inelastic collision, after which particle *1* will never come back to the "hot" wall (except the case where it returns coalesced with particle *2*).

Relation (14) can be obtained with the use of formulas (5)–(11). For this purpose, one should consider a one-particle system under the assumption that the interaction of the particle with the "cold" boundary takes place with the loss of the kinetic energy in the same way as in the case of collisions of particles of equal masses.

4. Non-Stationary States in a Horizontal 1D System of Inelastic Particles

Now let us consider the problem of stability of a stationary state by the example of a one-dimensional system consisting of two inelastic particles. The hot boundary of the system is specified in such a way that the particle nearest to it (let it be the first one) always reflects from it with the same constant velocity v_0. The reflection of the second particle from the opposite side is performed absolutely elastically, i.e. without any energy losses.

Let us consider the motion of particles in the constructed model that occurs according to the following scenario:
– the first particle with the velocity v_0 collides with the second one that moves toward the first particle with the velocity w_2 which is much lower;
– after the collision, the first particle goes on moving in the same direction with the lower velocity v_1, whereas the second particle changes the direction of motion to the opposite one and starts to move faster than the first particle with the velocity w_1;
– when the second particle reaches the boundary of the system L, it reflects and moves toward the first particle with the velocity $-w_1$;
– after the following collision of the particles, their velocities are distributed in such a way that the second particle moves in the initial direction but with the lower velocity w_2, while the first particle changes the direction of its motion, and its velocity v_2 exceeds the velocity of the second particle;

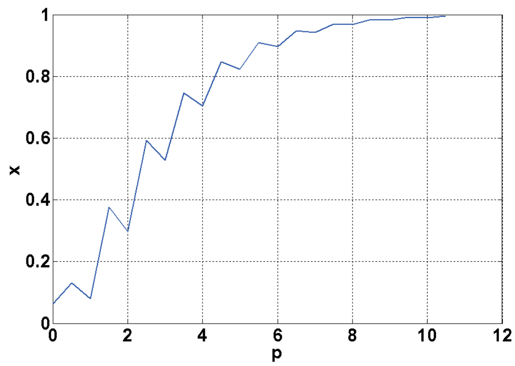

Fig. 1. Results of numerical calculations by formulas (21) and (22). Dependence of the collision coordinate of particles *1* and *2* on the number of their collisions at the following parameters (determined from the boundary and initial conditions): $L = 1$, $v_0 = 1$, $\varepsilon = 0.5$, $v_1 = 0.0250$, $w_2 = 0.675$, $x = 0.0632$

– after that the described scenario repeats.

In addition to the number of particles, we will also distinguish their velocities with the help of the number of the period of motion (p) described by the above-stated scheme.

Taking into account the laws of inelastic collisions, one can find the corresponding velocities of the particles:

$$w_{2,j+p} = \varepsilon^p w_{2,j}, \quad v_{2,j+p} = -\varepsilon v_0, \tag{15}$$

$$v_{1,j+p} = \frac{1-\varepsilon}{2} v_0 + \frac{1+\varepsilon}{2} w_{2,j} \varepsilon^{p-1}, \tag{16}$$

$$w_{1,j+p} = \frac{1+\varepsilon}{2} v_0 + \frac{1-\varepsilon}{2} w_{2,j} \varepsilon^{p-1}. \tag{17}$$

It follows from (15)–(17) that

$$\lim_{p\to\infty} w_{2,j+p} = 0, \ \ v_{2,j+p} = \text{const}, \tag{18}$$

$$\lim_{p\to\infty} v_{1,j+p} = \frac{1-\varepsilon}{2} v_0, \ \ \lim_{p\to\infty} w_{1,j+p} = \frac{1+\varepsilon}{2} v_0. \tag{19}$$

Thus, it turns out that the described system passes exactly to a stationary mode of motion after a series of collisions. Generalizing (15)–(19), we obtain the following recurrent relation for the velocity:

$$v_{k+p} = \varepsilon^p \left(v_k - v_\infty\right) + v_\infty, \tag{20}$$

where v_∞ is the velocity of a particle that corresponds to its motion in the stationary mode.

Let x_p be the coordinate of the collision of the particles in the above-stated type of motion. During the time interval equal to one period, there occur two collisions of the particles with one another at different points of space. The lower index p determines the number of the mode with periodic motion. Supposing that the particles move with constant velocities between collisions, x_p can be presented as

$$x_p = x_0 \prod_{k=1}^{p} B_{k-1} + L \sum_{l=1}^{p} A_{l-1} \prod_{m=1}^{p-l} B_{l+m-1}, \tag{21}$$

where x_0 is the coordinate of the first collision of particles *1* and *2*, L is the size of the system;

$$A_j = \frac{2v_{1,j}}{v_0 - w_{2,j}}, \quad B_j = \frac{v_{1,j} - w_{1,j}}{w_{2,j} - v_0}. \tag{22}$$

Using formulas (21) and (22), we can demonstrate that $\lim\limits_{p\to\infty} x_p = L$. On the basis of (18), (19), and (21), (22), we can conclude that the considered system asymptotically tends to the stationary state with the period determined by formula (14). Figure 1 presents the results of a numerical simulation of the behavior of the considered system of inelastic particles that obviously confirm the conclusions made above. Some kinetic processes taking place in GMs are determined with the help of the so-called order parameter [14]. Let us determine the order parameter $\varphi\left(p\right)$ of the considered system in the following way:

$$\varphi\left(p\right) = \frac{v_{k+p}/v_\infty - v_k/v_\infty}{1 - v_k/v_\infty}. \tag{23}$$

Using (20), we obtain the formula for $\varphi\left(p\right)$ which depends only on p and ε:

$$\varphi\left(p\right) = 1 - \varepsilon^p. \tag{24}$$

Under the conditions $p \gg 1$ and $\varepsilon \neq 1$, expression (24) can be presented in the exponential form

$$\varphi\left(p\right) = 1 - e^{-p(1-\varepsilon)}. \tag{25}$$

It is known [14] that, in some processes with GMs (for example, the radial size segregation of particles), the time dependence of the correspondingly determined order parameter φ that specifies the system is described exactly by the exponential law:

$$\varphi\left(t\right) = 1 - e^{-\frac{t}{\tau_0}}, \tag{26}$$

where τ_0 is the relaxation time of the order parameter.

Comparing (25) and (26) and taking $p \sim t$ into account, we can estimate the characteristic relaxation time τ_0 of the order parameter in the above-considered model:

$$\tau_0 = \frac{1}{1-\varepsilon}. \tag{27}$$

It follows from (27) that, with increase of the coefficient of inelastic energy losses ε, the time τ_0 of the relaxation of the system to the asymptotic quasistationary state increases. At $\varepsilon \to 1$, $\tau_0 \to \infty$, so that the system does not tend to a quasistationary state even asymptotically. The made conclusions completely agree with the data of numerical experiments performed with the help of the methods of molecular dynamics, as well as with the results of calculations by formula (21) obtained by means of analytical solutions. This dependence is also confirmed experimentally [15].

5. Motion of the Center of Mass of a Horizontal 1D System of N Inelastic Particles

Let us consider again a 1D system of N inelastic particles located between the "hot" and "cold" boundaries. We suppose that the velocities of all particles except the first one are equal to zero in the initial state. Obtaining a certain portion of energy from the "hot" boundary, the first particle moves with the velocity v_0 toward the rest of particles. After the first collision with the nearest particle, the velocity of the k-th particle amounts to $v_k = v_0 \left(\frac{1+\varepsilon}{2}\right)^{k-1}$, after the second collision – $v_k = v_0 \left(\frac{1+\varepsilon}{2}\right)^{k-1} \frac{1-\varepsilon}{2}$ (here, k is the number of the particle). In the case of the weak dissipation during collisions in the system (i.e. at $\varepsilon \approx 1$), one can consider that the velocity of the k-th particle after the second collision decreases almost up to zero $v_k \approx 0$. In this case, it is easy to imagine the character of motion of the particles after the reflection of the N-th particle from the absolutely elastic boundary of the system. Namely, after the first collision, we obtain: $u_k = -v_0 \left(\frac{1+\varepsilon}{2}\right)^{(N-1)+(N-k)}$. After the second one – $u_k = -v_0 \left(\frac{1+\varepsilon}{2}\right)^{(N-1)+(N-k)} \frac{1-\varepsilon}{2}$ (where u_k is the velocity of the k-th particle on its way from the N-th to the *1*-st particle). Similarly to the previous case at $\varepsilon \approx 1$, one can consider that $u_k \approx 0$ after the second collision of the k-th particle.

The velocity of the center of mass V_c can be obtained with regard for the fact that $V_c = V_{\text{in}} + V_{\text{out}}$, where $V_{\text{in}} = \frac{1}{N}\sum\limits_{k=1}^{N} v_k$, $V_{\text{out}} = \frac{1}{N}\sum\limits_{k=1}^{N} u_k$.

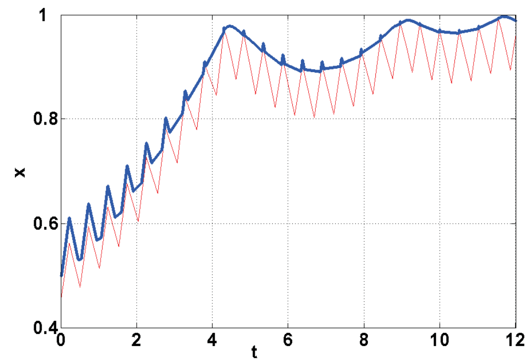

Fig. 2. Results of numerical calculations of the motion of the center of mass in a 1D horizontal system of 10 inelastic particles. Low-contrast solid line corresponds to the motion of the center of mass of the system as a whole; contrast solid line describes the motion of the center of mass of $N-1$ inelastic particles (except the motion of the first particle nearest to the "hot" boundary of the system); $\varepsilon = 0.95$, $v_0 = 5$, $dt = 10^{-5}$

Correspondingly, V_c has the form

$$V_c = \frac{2v_0}{N(1-\varepsilon)}\left[1 - 2\frac{(3+\varepsilon)}{\sqrt{8(1+\varepsilon)}} \times \right.$$

$$\left. \times \sqrt{\frac{1+\varepsilon}{2}}\left(\frac{1+\varepsilon}{2}\right)^{N-1} + \left(\frac{1+\varepsilon}{2}\right)^{2N-1}\right]. \tag{28}$$

For $0 \le \varepsilon \le 1$, we have $1 \le \frac{(3+\varepsilon)}{\sqrt{8(1+\varepsilon)}} \le 1.06$, and the expression for V_c can be presented in a simplified form

$$V_c \approx \frac{2v_0}{N(1-\varepsilon)}\left[1 - \left(\frac{1+\varepsilon}{2}\right)^{N-\frac{1}{2}}\right]^2 \approx$$

$$\approx \frac{2v_0}{N(1-\varepsilon)}\left[1 - \exp\left(-\frac{1-\varepsilon}{2}\left(N - \frac{1}{2}\right)\right)\right]^2. \tag{29}$$

It is worth noting that $V_c \ge 0$, i.e. the center of mass moves toward the "cold" boundary. In truth, the center of mass cannot permanently move in one direction because the system has limited dimensions. Thus, the obtained value of the drift velocity of the center of mass does not give the complete information on the behavior of the system. However, this quantity allows one to estimate the velocity of the center of mass in the limiting cases of large N and ε.

Near the boundary, where $\varepsilon \to 1$ (or $N \to \infty$), the velocity of the center of mass $V_c \to 0$. Thus, the velocity of the center of mass of the considered system is exponentially small in the case of large-scale systems

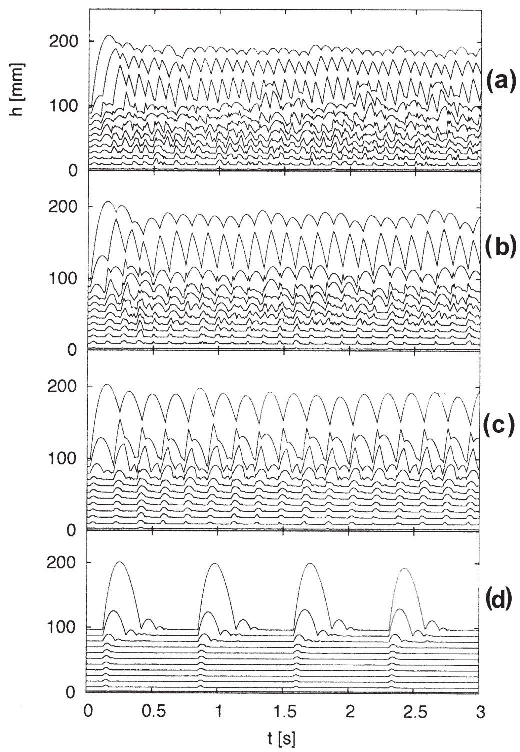

Fig. 3. Results of experimental measurements of trajectories in the vertical system consisting of 12 metallic balls obtained with the help of a high-speed digital camera. The excitation frequencies amount to: a – 13.8 Hz; b – 8.93 Hz; c – 5.61 Hz; d – 1.37 Hz

consisting of a large number of particles, as well as in the case of low inelastic energy losses during collisions.

Figure 2 presents the results of numerical calculations of the velocity of the center of mass that obviously confirm the theoretical conclusions made above.

6. Physical Experiment

The physical experiment aimed at the investigation of the dynamics of a model 1D granulated system consisted in the pulse excitation of a vertical column of metal spherical balls in the vertical direction.

With the help of a high-speed digital camera (500 fps), we determined the trajectories of the particles. The precision of measurements of the coordinates of the particles amounted to 0.5 mm. We used 12 metal balls 8.73 mm in diameter located in a glass tube 8.9 mm in diameter. The mass of each particle was equal to 2.74 g. The inelastic loss coefficient during central binary collisions is equal to 0.9. The excitation pulse amounts to 0.16 H $\cdot$ c.

Varying the excitation frequencies from 1 to 14 Hz, we managed to observe the layering of the system into subsystems with two qualitatively different patterns of motion. At the beginning, the particles rest in the state of direct contact one on another. Due to the excitation, the upper particle starts to oscillate almost periodically with respect to the rest of the particles increasing its amplitude with the frequency. In this case, all the other particles of the system move as a rather dense group (cluster). With the further increase of the excitation frequency, one observes a transition of the second from above particle to the state of simple periodic motion, after that – the third one (see Fig. 3).

Thus, we observe that, with increase in the energy supplied to the system from outside, the system passes to the asymptotically stationary state (in the form of a simple periodic motion). It is worth paying attention to the fact that the particles nearest to the substrate and almost static practically play a role of a conductor that transfers the energy to the highest particle of the column which passes to the stationary mode of motion.

Similarly to the data obtained here, the authors of [4] discovered that, in a one-dimensional system located between the "hot" and "cold" boundaries, the majority of particles represent an almost static cluster formed from the opposite side of the substrate (i.e. close to the cold boundary of the system), while the dynamic part of the system consists of one or several particles. In contrast to [4], the role of the cold wall in the problem considered here is played by the gravitational field that restricts the height, to which the upper mobile particle can rise, whereas the static cluster is formed near the hot wall. The difference is caused by different boundary conditions, while the observed phenomenon results from the nonlinear character of the dissipative system controlled by the corresponding parameter $N\varepsilon$.

The theoretical study of the motion of the center of mass of the investigated system confirms the experimentally observed behavior consisting in the layering of the system into an almost motionless cluster and a group of particles in the state of ballistic motion. According to the data obtained from the experimental observations, each upper particle that passes to the state of simple periodic motion reflects from the nearest particle (located below) belonging to the almost motionless part of the column (cluster) with a constant velocity. Just this criterion is inherent to the considered theoretical model constructed under the conditions of density restrictions, weak dissipation, and weak external excitations. Due to the vertical location of the particles in the column, as well as to the existence of own dimensions of particles and the really weak dissipation and excitation intensity, the physical model approximately satisfies the conditions of the theoretical model.

For a system consisting of particles in the stationary mode, we numerically calculated the dimensions of the regions, within which they perform their periodic motion. The comparison of these values with the data of the above-described experimental observations testifies to the fact that the size of the system in the stationary state of motion is in good agreement with the theoretically calculated value L [see (3)] if each separate interval of periodic motion is determined with regard for own dimensions of particles.

Thus, the asymptotic quasistationary limit of motion found in the physical experiment can be considered as an evidence of the existence of the theoretically postulated stationary states in 1D dissipative open systems.

7. Conclusions

Thus, the one-dimensional model of a system of inelastic particles in the case of the simplest modes of external energy supply (by means of the mirror reflection or a reflection with specified velocity distribution) allows the possibility for stationary states to exist in the form of simple periodic motion of each separate particle within the corresponding intervals of various lengths. The system passes to the specified state asymptotically under various ways of the energy supply through its boundaries. A criterion of the transition arising due to the condition of balance of the thermalization and dissipation processes is discovered. The established criterion [see (4)] depends multiplicatively on the initial size of the system and the absolute value of the coefficient of inelastic energy losses. The corresponding direct physical experiments devoted to the observation of stationary states in a vertical column of metallic balls (that represents a model realization of an open dissipative system excited due to a pulsed excitation of the substrate) testify to the existence of the theoretically predicted quasistationary states, to which a 1D open system of inelastic particles asymptotically tends.

1. H.M. Jaeger, S.R. Nagel, and R.P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).
2. P.G. de Gennes, Rev. Mod. Phys. **71**, 374 (1999).
3. J. Duran, *Sands, Powders and Grains* (Springer, New York, 2000).
4. Y. Du, H. Li, and L. Kadanoff, Phys. Rev. Lett. **74**, 1268 (1995).
5. O.I. Gerasymov, N.N. Khudyntsev, O.A. Klymenkov, and A.Ya. Spivak, Ukr. Fiz. Zh. **50**, 624 (2005).
6. O.I. Gerasymov, V.A. Idomskyy, and P.P.-J.M. Schram, Cond. Matt. Phys. **4**, 161 (2001).
7. S. Luding, E. Clement, A. Blumen, and J. Rajchenbach, Phys. Rev. E **49**, 1634 (1994).
8. J.M. Luck, A. Mehta, Phys. Rev. E **48**, 3988 (1993).
9. A. Mehta, J.M. Luck, Phys. Rev. Lett. **65**, 393 (1990).
10. N.B. Tufillaro and A.M. Albano, Amer. J. Phys. **54**, 939 (1986).
11. E. Falcon, C. Laroche, S. Fauve, and C. Caste, Eur. Phys. J. **B5**, 111 (1998).
12. J.-M. Hertzsch, Eur. Phys. J. **B18**, 459 (2000).
13. S.Bernard, T. Schwager, T. Poschel, and C. Saluena, Eur. Phys. J. **4**, 233 (2000).
14. O.I. Gerasimov, P.P.-J.M. Schram, and K. Kitahara, Ukr. Fiz. Zh. **48**, 885 (2003).
15. F. Cantelaube and D. Bideau, Europhys. Lett. **30**, 133 (1995).

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СТАЦІОНАРНІ СТАНИ У 1D СИСТЕМІ НЕПРУЖНИХ ЧАСТИНОК

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Р е з ю м е

В одновимірній відкритій системі непружних частинок знайдено умови існування асимптотичних квазістаціонарних станів. Досліджено вплив початкових та зовнішніх умов на структуру таких станів та переходи між ними. Теоретичні та чисельні розрахунки порівнюються з даними проведеного безпосередньо фізичного експерименту. Обговорено можливості застосування методів статистичної фізики до вивчення відкритих систем (зокрема, гранульованих матеріалів) поблизу виявлених квазістаціонарних станів.