

*A. N. Bystriantseva, O. Yu. Khetselius, Yu. V. Dubrovskaya, L. A. Vitavetskaya,  
A. G. Berestenko*

Odessa National Polytechnical University, 1, Shevchenko pr., Ukraine  
Odessa State Environmental University, 15, Lvovskaya str., Odessa, Ukraine  
e-mail: quantsha@mail.ru

**RELATIVISTIC THEORY OF SPECTRA OF HEAVY PIONIC ATOMIC SYSTEMS WITH  
ACCOUNT OF STRONG PION-NUCLEAR INTERACTION EFFECTS:  
93Nb, 173Yb, 181Ta , 197Au**

It is presented a consistent relativistic theory of spectra of the pionic atoms on the basis of the Klein-Gordon-Fock with a generalized radiation and strong pion-nuclear potentials. There are presented data of calculation of the energy and spectral parameters for pionic atoms of the  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$ , with accounting for the radiation (vacuum polarization), nuclear (finite size of a nucleus ) and the strong pion-nuclear interaction corrections. The measured values of the Berkley, CERN and Virginia laboratories and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uhling-Serber radiation correction and optical atomic theory are listed too.

**1. Introduction**

In papers [1-3] we have developed a new relativistic method of the Klein-Gordon-Fock equation with an generalized pion-nuclear potential to determine transition energies in spectroscopy of light, middle and heavy pionic atoms with accounting for the strong interaction effects. In this paper, which goes on our studying on spectroscopy of pionic atoms, we firstly applied method [1-3] to calculating calculation of the energy and spectral parameters for pioninc atoms of the  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$ , with accounting for the the radiation (vacuum polarization), nuclear (finite size of a nucleus ) and the strong pion-nuclear interaction corrections..

Following [1-3], let us remind that spectroscopy of hadron atoms has been used as a tool for the study of particles and fundamental properties for a long time. Exotic atoms are also interesting objects as they enable to probe aspects of atomic and nuclear structure that are quantitatively different from what can be studied in electronic or "normal" atoms. At present time one of the most sensitive tests for the chiral symmetry breaking scenario in the modern hadron's physics is provided by studying the exotic hadron-atomic systems. Nowadays the transition energies in pionic

(kaonic, muonic etc.) atoms are measured with an unprecedented precision and from studying spectra of the hadronic atoms it is possible to investigate the strong interaction at low energies measuring the energy and natural width of the ground level with a precision of few meV [1-10]. The strong interaction is the reason for a shift in the energies of the low-lying levels from the purely electromagnetic values and the finite lifetime of the state corresponds to an increase in the observed level width. For a long time the similar experimental investigations have been carried out in the laboratories of Berkley, Virginia (USA), CERN (Switzerland). The most known theoretical models to treating the hadronic (pionic, kaonic, muonic, antiprotonic etc.) atomic systems are presented in refs. [1-5,7,8]. The most difficult aspects of the theoretical modeling are reduced to the correct description of pion-nuclear strong interaction [1-3] as the electromagnetic part of the problem is reasonably accounted for.

**2. Relativistic approach to pionic atoms spectra**

As the basis's of a new method has been published, here we present only the key topics of an

approach [1-3]. All available theoretical models to treating the hadronic (kaonic, pionic) atoms are naturally based on the using the Klein-Gordon-Fock equation [2,5], which can be written as follows :

$$+ \left( 1 + \frac{\pi}{2m_N} \right) \{ C_0 \rho^s(r) + C_1 \rho(r) \delta \rho(r) \}. \quad (1)$$

where  $c$  is a speed of the light,  $\hbar$  is the Planck constant, and  $\Psi_0(x)$  is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential  $[V_0(r), 0]$  approximation with the stationary solution:

$$r) \text{ scattering length in th} \quad (2)$$

where  $\varphi(x)$  is the solution of the stationary equation:

$$+ \text{amplitude } \pi \text{ N scattering} \\ (b_0 \rho(r) \rightarrow b_0 \rho(r) + b_1 \{ \rho_p(r) - \rho_n(r) \}, \quad (3)$$

Here  $E$  is the total energy of the system (sum of the mass energy  $mc^2$  and binding energy  $\varepsilon_0$ ). In principle, the central potential  $V_0$  naturally includes the central Coulomb potential, the vacuum-polarization potential, the strong interaction potential.

The most direct approach to treating the strong interaction is provided by the well known optical potential model (c.g. [2]). Practically in all papers the central potential  $V_0$  is the sum of the following potentials. The nuclear potential for the spherical-symmetric density  $\rho(r|R)$  is [6,13]:

Au	5g-4f	$532.5 \pm 0.5$	$533.16 \pm 0.20$
Nb	4f-3d	-	$140.3 \pm 0.1$

(4)

The most popular Fermi-model approximation the charge distribution in the nucleus  $\rho(r)$  (c.f.[11]) is as follows:

$$( \quad (5)$$

where the parameter  $a=0.523$  fm, the parameter  $c$  is chosen by such a way that it is true the following condition for average-squared radius:

$$\langle r^2 \rangle^{1/2} = (0.836 \times A^{1/3} + 0.5700) \text{fm.}$$

The effective algorithm for its definition is used in refs. [12] and reduced to solution of the following system of the differential equations:

### 3. Results and conclusions

$$\text{energies for pionic ato} \quad (6)$$

$$\text{181} \quad \text{197} \quad (7)$$

Virginia laboratories and alternative data (8)  
based on other versions of the Klein-Gordon

with the corresponding boundary conditions. Another, probably, more consistent approach is in using the relativistic mean-field (RMF) model, which been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [13]. To take into account the radiation corrections, namely, the effect of the vacuum polarization we have used the generalized Ueling-Serber potential with modification to take into account the high-order radiative corrections [5,12].

The most difficult aspect is an adequate account for the strong interaction. On order to describe the strong  $\pi$ N interaction we have used the optical potential model in which the generalized Ericson-Ericson potential is as follows:

-	-	-	-	$140.81$
-	-	-	-	$838.67$
-	-	-	$992.75$	$1008.80$

$$+ \left( 1 + \frac{\pi}{2m_N} \right) \{ C_0 \rho^s(r) + C_1 \rho(r) \delta \rho(r) \} \quad (10)$$

$$\alpha(r) = \left( 1 + \frac{m_\pi}{m_N} \right)^{-1} \{ c_0 \rho(r) + c_1 [ \rho_p(r) - \rho_n(r) ] \} + \quad (10)$$

$$+ \left( 1 + \frac{\pi}{2m_N} \right) \{ C_0 \rho^s(r) + C_1 \rho(r) \delta \rho(r) \}. \quad (11)$$

ere  $\rho_{p,n}(r)$  – distribution of a density of the protons and neutrons, respectively,  $\xi$  – parameter  $\xi=0$  corresponds to case of “no correlation”,  $\xi=1$ , if anticorrelations between nucleons); respectively isoscalar and isovector parameters  $b_0, c_0, B_0, b_1, c_1, C_0, B_1, C_1$  – are corresponding to the s-wave and p-wave (repulsive and attracting potential member) scattering length in the combined spin-isospin space with taking into account the absorption of pions (with different channels at p-p pair  $B_{0(p)}$  and p-n pair  $B_{0(p)}$ ), and isospin and spin dependence of an amplitude  $\pi$ N scattering

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$$(b_0\rho(r) \rightarrow b_0\rho(r) + b_1\{\rho_p(r) - \rho_n(r)\}),$$

the Lorentz-Lorentz effect in the p-wave interaction. For the pionic atom with remained electron shells the total wave-function is a product of the product Slater determinant of the electrons subsystem (Dirac equation) and the pionic wave function. In whole the energy of the hadronic atom is represented as the sum:

$$E \approx E_{KG} + E_{FS} + E_{VP} + E_N; \quad (12)$$

Here  $E_{KG}$  -is the energy of a pion in a nucleus ( $Z, A$ ) with the point-like charge (dominative contribution in (12)),  $E_{FS}$  is the contribution due to the nucleus finite size effect,  $E_{VP}$  is the radiation correction due to the vacuum-polarization effect,  $E_N$  is the energy shift due to the strong interaction  $V_N$ .

The strong pion-nucleus interaction contribution can be found from the solution of the Klein-Gordon-Fock equation with the corresponding pion-nucleon potential.

### 3. Results and conclusions

In table 1 our data on the  $4f-3d$ ,  $5g-4f$  transition energies for pionic atoms of the  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$  are presented. The measured values of the Berkley, CERN and Virginia laboratories and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uhling-Serber radiation correction [5, 15] and optical atomic theory [17,18] are listed too.

The analysis of the presented data indicate on the importance of the correct accounting for the radiation (vacuum polarization) and the strong pion-nuclear interaction corrections. Obviously, it is clear that that the contributions provided by the finite size effect should be accounted in a precise theory. Besides, taking into account the increasing accuracy of the X-ray pionic atom spectroscopy experiments, it can be noted that knowledge of the exact electromagnetic theory data will make more clear the true values for parameters of the pion-nuclear potentials and correct the disadvantage of widely used parameterization of the potentials (9)-(11).

Table 1. Transition energies (keV) in the spectra of some heavy pionic atoms (see text)

$\pi$ -A	Trans.	Berkley $E_{\text{EXP}}$	CERN $E_{\text{EXP}}$	$E_{\text{KGF+EM}}$ [5, 15]	$E_{\text{KGF-EM}}$ [16, 17]	$E_N$ [5]	$E_N$ [14, 18]	$E_N$ , Our data
$^{93}\text{Nb}$	5g-4f	-	$307.79 \pm 0.02$	-	-	-	-	307.85
$^{173}\text{Yb}$	5g-4f	-	-	-	-	-	-	412.26
$^{181}\text{Ta}$	5g-4f	$453.1 \pm 0.4$	$453.90 \pm 0.20$	453.06	453.78	-	453.52 453.62	453.71
$^{197}\text{Au}$	5g-4f	$532.5 \pm 0.5$	$533.16 \pm 0.20$	528.95	-	532.87	531.88	533.08
$^{93}\text{Nb}$	4f-3d	-	$140.3 \pm 0.1$	-	-	-	-	140.81
$^{173}\text{Yb}$	4f-3d	-	-	-	-	-	-	838.67
$^{181}\text{Ta}$	4f-3d	-	$1008.4 \pm 1.3$	-	-	-	992.75	1008.80
$^{197}\text{Au}$	4f-3d	-	$1187.3 \pm 1.9$	-	-	-	1167.92	1186.35

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**Abstract**

It is presented a consistent relativistic theory of spectra of the pionic atoms on the basis of the Klein-Gordon-Fock with a generalized radiation and strong pion-nuclear potentials. There are presented data of calculation of the energy and spectral parameters for pionic atoms of the  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$ , with accounting for the radiation (vacuum polarization), nuclear (finite size of a nucleus) and the strong pion-nuclear interaction corrections. The measured values of the Berkley, CERN and Virginia laboratories and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uhling-Serber radiation correction and optical atomic theory are listed too

**Key words:** strong interaction, pionic atom, relativistic theory

*А. Н. Быстрянцева, О. Ю. Хецелиус, Ю. В. Дубровская, Л. А. Витавецкая, А. Г. Берестенко*

**РЕЛЯТИВИСТСКАЯ ТЕОРИЯ СПЕКТРОВ ПИОННЫХ АТОМНЫХ СИСТЕМ С УЧЕТОМ ЭФФЕКТОВ СИЛЬНОГО ПИОН-ЯДЕРНОГО ВЗАИМОДЕЙСТВИЯ:  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$**

**Резюме**

Представлена последовательная релятивистская теория спектров пионных атомов на основе уравнения Клейна-Гордона-Фока с обобщенными радиационным и сильным пион-ядерным потенциалом. Выполнен расчет энергетических и спектральных параметров для пионных атомов  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$ , с учетом радиационных (поляризация вакуума), ядерных (конечный размер ядра) эффектов и поправки на сильное пион-нуклонное взаимодействие. Также для сравнения представлены данные измерений в лабораториях Berkley, ЦЕРН и Вирджиния и теоретические результаты, полученные на основе альтернативных теорий Клейна-Гордона-Фока с учетом конечного размера ядра в модели равномерно заряженной сферы и стандартной Юлинг-Сербер поправки.

**Ключевые слова:** сильное взаимодействие, пионный атом, релятивистская теория

*А. М. Бистрянцева, О. Ю. Хецеліус, Ю. В. Дубровська, Л. А. Вітавецька, А. Г. Берестенко*

**РЕЛЯТИВІСТСЬКА ТЕОРІЯ СПЕКТРІВ ПІОННИХ АТОМНИХ СИСТЕМ З УРАХУВАННЯМ ЕФЕКТІВ СИЛЬНОЇ ПІОН-ЯДЕРНОЇ ВЗАЄМОДІЇ:  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$**

**Резюме**

Представлена послідовна релятивістська теорія спектрів піоній атомів на основі рівняння Клейна-Гордона-Фока з узагальненими радіаційним і сильним піонія-ядерним потенціалом. Виконано розрахунок енергетичних і спектральних параметрів для піоних атомів  $^{93}\text{Nb}$ ,  $^{173}\text{Yb}$ ,  $^{181}\text{Ta}$ ,  $^{197}\text{Au}$ , з урахуванням радіаційних (поляризація вакууму), ядерних (кінцевий розмір ядра) ефектів та поправки на сильну піон-нуклонну взаємодію. Також для порівняння представлені дані вимірювань в лабораторіях Berkley, ЦЕРН і Вірджинія і теоретичні результати, отримані на основі альтернативних теорій Клейна-Гордона-Фока з урахуванням кінцевого розміру ядра в моделі рівномірно зарядженої сфери і стандартної Юлінг-Сербер поправки..

**Ключові слова:** сильна взаємодія, піонний атом, релятивістська теорія