NEW QUANTUM APPROACH TO DETERMINATION OF THE MOLECULAR SPECTRAL CONSTANTS AND PROBABILITIES FOR COOPERATIVE VIBRATION-ROTATION-NUCLEAR TRANSITIONS IN SPECTRA OF DIATOMICS AND THE HADRONIC MOLECULES

It is proposed a new approach to construction of the potential function of diatomic molecules as a sum of the known perturbed Morse oscillator function, the Simons-Parr-Finlan molecular potential in the middle of the potential curve, function of the \(-Cn/R^n\) type at the large internuclear distances. Within this approach it is presented a precise scheme for computing the molecular spectral parameters, namely, vibrational, rotational, centrifugal constants for the electronic states of diatomics. As application it was carried out calculation of the of molecular constants (cm\(^{-1}\)) for the \(X^1\Sigma^+\) \(B^1\Pi\) states of the KRb dimer and rubidium dimer and performed further comparison with experimental data. Within consistent approach to calculation of the electron-nuclear \(\gamma\) transition spectra (set of vibration-rotational satellites in molecule) of molecule there are obtained the estimates for vibration-rotation-nuclear transition probabilities in a case of the emission and absorption spectrum of nucleus \(^{127}\)I (\(E_{\gamma}^{(0)} = 203\) keV) in the molecule of H\(^{127}\)I for different approximations of the for potential curves: the harmonic oscillator, the Dunham model and presented approach.

From physical viewpint it is obvious that any alteration of the molecular state must be manifested in the quantum transitions, for example, in a spectrum of the g-radiation of a nucleus (see for example [1-9]). In result of the gamma nuclear transition in a nucleus of a molecule there is arised a set of the electron-vibration-rotation satellites, which are due to an alteration of the state of the molecular system interacting with photon. The known example is the Szilard-Chalmers effect which results to molecular dissociation because of the recoil during radiating gamma quantum with large energy.

In series of works [3-9] it has been carried out detailed studying the co-operative dynamical phenomena due the interaction between atoms, ions, molecule electron shells and nuclei nucleons. There have been developed a few advanced approaches to description of a new class of dynamical laser-electron-nuclear effects in molecular spectroscopy, in particular, a nuclear gamma-emission or absorption spectrum of a molecule. A consistent quantum-mechanical approach to calculation of the electron-nuclear \(\gamma\) transition spectra (set of vibration-rotational satellites in molecule) of a nucleus in the multiatomic molecules has been earlier proposed [5,7] and generalizes the well known approach by Letokhov-Minogin [4]. Earlier there were have been obtained estimates and calculations of the vibration-nuclear transition probabilities in a case of the emission and absorption spectrum of nucleus \(^{191}\)Ir (\(E_{\gamma}^{(0)} = 82\) keV) in the molecule of \(IrO_2\), \(^{188}\)Os (\(E_{\gamma}^{(0)} = 155\) keV in \(OsO_2\) and other molecules were listed. In Ref [8] there are firstly presented theoretical data on the vibration-nuclear transition
probabilities in a case of the nucleus of rhenium $^{186}$Re ($E^{(0)} \varepsilon = 186.7$ keV) in the molecule of ReO$_4$, estimated on the basis of the simplified version [5,7] of the consistent quantum-mechanical approach to cooperative electron-g-nuclear spectra (a set of the vibration-rotational satellites in a spectrum of molecule) of multiatomic molecules.

In this paper we present a generalization of the cited theory of cooperative electron-gamma-nuclear (vibrational, rotational) transitions in a case of the diatomic molecules using new principle of construction of the potential curves for diatomic, which is in some degree analogous to the Smirnov approach [10,11]. Moreover the proposed method allow to determine the molecular spectral parameters, that is checked on the example of the some alkali dimers. Besides, we will give a short generalization of the theory on a case of the exotic hadronic (pionic) molecules.

It should be noted that the diatomics potential function can be obtained on the decision of the electronic Schrödinger equation, however, due to significant computational difficulties in the present, this problem is reliably solving only for the case of the simplest diatomics having a small number of electrons [10-15]. In this regard, the first promising more used semi-empirical methods, where the potential curves are determined in the adiabetic approach using experimental vibrational and rotational spectroscopic constants. Some authors have studied solutions of the Schrödinger (or Klein-Gordon) equation with some known physical potential models, such as the Morse potential, Rosen-Morse potential, Manning-Rosen potential, Poschl-Teller potential, Deng-Fan potential, ring-shaped potential, and hyperbolic Scarf potential etc (look details for example in Ref. [10,16-18]).

Let us remind shortly a scheme for computing the cooperative on the vibration-nuclear transition probabilities in a case of the emission and absorption spectrum of the nucleus of diatomics as the corresponding method is earlier presented in details (look [5-8]). The aim is to compute parameters of the gamma transitions (a probability of transition) or spectrum of the gamma satellites because of changing the electron-vibration-rotational states of the molecule under gamma quantum radiation (absorption).

Our purpose is calculation of a structure of the gamma transitions (probability of transition) or spectrum of the gamma satellites because of the changing the electron-vibration-rotational states of diatomic molecules under the gamma quantum radiation (absorption). In adiabatic approximation a wave function of molecule is multiplying the electronic wave function and wave function of nuclei: $y(r_c)y(R,R,)$. Hamiltonian of interaction of the gamma radiation with system of nucleons for the first nucleus can be expressed through the co-ordinates of nucleons $r_n$ in a system of the mass centre of the first nucleus [4,7]:

$$H(r_n) = H(r_n) \exp(-ik_f R_1)$$

(1)

where $k_f$ is a wave vector of the gamma quantum. The matrix element for transition from initial state “a” to final state “b” is presented as usually:

$$<\Psi^*_b(r_n) | H(r_n) | \Psi_a(r_n) >$$

(2)

The first multiplier in (1) is defined by the gamma transition of nucleus and is not dependent upon an internal structure of molecule in a good approximation. The second multiplier is a matrix element of transition of the molecule from initial state “a” to final state “b”:

$$M_{ba} = <\Psi^*_b(r_n)| \Psi_a(r_n)>$$

(3)

The expression (7) gives a general formula for calculation of the probability of changing internal state of molecule under absorption or emitting gamma quantum by nucleus of the molecule and defines an amplitude of the corresponding gamma satellites. Their positions are fully determined by the energy and pulse conserving laws as follows [2]:

$$\pm E_\gamma + E_a + (1/2)M^2_0 = \pm E^{(0)}_\gamma + E_b + (1/2)M^2$$

(4)

$$M_0 \pm h k_f = M v$$

Here $M$ is the molecule mass, $v_0$ and $v$ are velocities of molecule before and after interaction of
nucleus with g quantum, E_a and E_b are the energies of molecule before and after interaction, E_g is an energy of nuclear transition. Then an energy of the g satellite is as follows:

\[ E_g = \frac{2ν_{g}}{M} + E_{g}^{(0)}(0,0) + \left( E_{b} - E_{a} \right) \]  

Here \( R_{om} \) is an energy of recoil: \( R_{om} = \frac{\left( ν_{g}^{2} \right)^{2}}{2M} \). It is well known (c.f.[4,7]) that the practical interest are presented only of the angle momentum operator, i.e.:

\[ \psi \equiv \left\langle \Psi_{b}(R_{1},R_{2}) | e^{i \frac{2ν_{g}}{M} R_{1}} | \Psi_{a}(R_{1},R_{2}) \right\rangle \]  

The values of energy, accepted by vibration and rotational degrees of freedom of the molecule are as follows:

\[ e_{vib} = \hbar \omega = R_{om}(m_{2}/m_{1}), \]
\[ e_{rot} = B^{2} = R_{om}(m_{2}/m_{1}). \]  

The simple adequate model for definition of the rotational motion is the rigid rotator approximation. In this approximation the wave functions with definite values of quantum numbers J,K are the eigen functions of the angle momentum operator, i.e.:

\[ \psi(R_{1},R_{2}) = Y_{j,k}(θ, φ). \]  

In a case of the vibration motion the wave functions with definite value of the vibration quantum number are numerically found by solving the corresponding Schrödinger equation with potential function, choice of which was discussed above. The simple approximation is surely the harmonic oscillator one. The harmonic oscillator wave functions were used for estimating matrix elements of the vibration-nuclear transitions in ref. [4]. In general the matrix element of the vibration-rotation-nuclear transition can be written as follows:

\[ M_{ba} = \langle \Psi_{b}(R_{1},R_{2}) | e^{i \frac{2ν_{g}}{M} R_{1}} | \Psi_{a}(R_{1},R_{2}) \rangle \]  

Here \( Q = (R-R_{o})^{2}/M \), \( m = m_{1}m_{2}/M \) is the reduced mass of the molecule, \( m_{1} \) and \( m_{2} \) are the masses of nuclei. The co-ordinate of mass centre of the first nucleus relatively the molecule mass centre is defined by expression:

\[ R_{i} = -(m_{i}/M)R_{om}Q/(m_{i}^{2}) = -(m_{i}/M)R_{om}Q/(m_{i}^{2}) \]  

The corresponding probability can be written in the following form:

\[ \gamma_{b}^{i} = \frac{2 J_{i} + 1}{\sum_{i} \left( \psi_{b}(\nu_{b}) \psi_{n}(\nu_{b}) \right)^{2} \psi_{a}(\nu_{a})^{2}} \]  

Our new approach in in construction of the final potential function as a sum of a few potentials. Each diatomic potential curve dimer is approximated by three functions corresponding to different portions of it. As in Ref. [10], the first portion is approximated by the known perturbed Morse oscillator function:

\[ y = 1 \exp \left(-\frac{\rho(R-R_{e})}{\rho} \right) \]  

where \( R \) and \( R_{e} \) is equilibrium internuclear coordinate; \( V_{c}, \rho, b_{n} \) - the parameters of the Morse oscillator function.

Often in the middle of the potential curve ( it is experimentally investigated range of vibrational quantum numbers) it is usually used the potential of the Rydberg-Klein-Rees (look for example [10-15,18]). This potential curve has not the analytical form, and it is built as a set of \( R_{min} \) and \( R_{max} \) classical turning points for experimental study of vibrational energy levels. Insit of it in our new scheme we apply the Simons-Parr-Finlan molecular potential which looks as follows [15]:

\[ U(r) = B_{b}(r-r_{o})/r^{2} \left[ 1 + \sum_{n} b_{n}(r-r_{o})/r^{2n} \right] \]  

or introducing \( x = r-r_{o} \):

\[ U(x) = B_{b}x^{2}/(x+r_{o})^{2} \left[ 1 + \sum_{n} b_{n}x^{2}/(x+r_{o})^{2n} \right] \]  

where the coefficients \( b_{n} \) are linked with corresponding molecular constants [14].
Finally, the plot of the potential curve for large values of the internuclear distance is approximated by the standard function:

\[ U(R) = D_e - \sum \frac{C_n}{R^n} - \Delta U_{\text{e.m.}}(R), \]  

(13)

where \( D_e \) - experimental value of dissociation energy; \( C_n \) – the function parameters (6); \( n = 3-8 \). Let us note that the model (11)-(13) is obviously more exact and consistent in comparison with a simple harmonic oscillator one. As an application it was carried out computing the rubidium dimers (Rb2, KRb) diatomics spectral parameters. The results of calculation of molecular constants (cm\(^{-1}\)) for the \( ^1\Sigma^+ \) state of the KRb dimer are presented in Table 1 toghether with experimental data \cite{10,19,20} and theoretical data \cite{10,11} obtained with using the Morse- Rydberg - Klein - Rees (M-RKR) method.

### Table 1
The molecular constants (cm\(^{-1}\)) for the \( ^1\Sigma^+ \) state of the KRb dimer: Experimental data – Exp; Theory: a- \cite{10}; b- our data

<table>
<thead>
<tr>
<th>KRb</th>
<th>( ^1\Sigma^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Th: a</strong></td>
<td><strong>Th: b</strong></td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>75,846</td>
</tr>
<tr>
<td>( \omega_x )</td>
<td>0,230</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>(-3,7(-4))</td>
</tr>
<tr>
<td>( \omega_z )</td>
<td>(-3,7(-6))</td>
</tr>
<tr>
<td>( B_e )</td>
<td>0,03815</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>1,21(-4)</td>
</tr>
<tr>
<td>( \gamma_e )</td>
<td>(-7,3(-7))</td>
</tr>
<tr>
<td>( D_e )</td>
<td>3,85(-8)</td>
</tr>
<tr>
<td>( H_e )</td>
<td>3,7(-14)</td>
</tr>
</tbody>
</table>

Table 2 contains the results of calculation of molecular constants (cm\(^{-1}\)) for the B'\( ^1\Pi \) (b) state of the KRb dimer.

### Table 2
The molecular constants (cm\(^{-1}\)) for for the B'\( ^1\Pi \) (b) state of the KRb dimer: Experimental data – Exp; Theory -our data

<table>
<thead>
<tr>
<th>KRb</th>
<th>B'( ^1\Pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exp</strong></td>
<td><strong>Theory: a</strong></td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>61,258</td>
</tr>
<tr>
<td>( \omega_x )</td>
<td>0,2095</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>2,88(-3)</td>
</tr>
<tr>
<td>( \omega_z )</td>
<td>(-1,034(-4))</td>
</tr>
<tr>
<td>( B_e )</td>
<td>0,03287</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>7,54(-5)</td>
</tr>
<tr>
<td>( \gamma_e )</td>
<td>(-1,12(-5))</td>
</tr>
<tr>
<td>( D_e )</td>
<td>3,75(-8)</td>
</tr>
<tr>
<td>( H_e )</td>
<td>5,5(-14)</td>
</tr>
</tbody>
</table>

Tables 3 and 4 contains the same data for states of the rubidium dimer for the \( ^1\Sigma^+ \) and \( ^1\Pi_u(B) \) states.

### Table 3
The molecular constants (cm\(^{-1}\)) for the \( ^1\Sigma^+ \) state of the Rb2 dimer: Experimental data – Exp; Theory: a- \cite{11}; b- our data

<table>
<thead>
<tr>
<th><strong>[16]</strong></th>
<th>Our data</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_e )</td>
<td>31,4883</td>
<td>31,4884</td>
</tr>
<tr>
<td>( \omega_x )</td>
<td>(-0,1140(-1))</td>
<td>(-0,1142(-1))</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>(-4,255(-4))</td>
<td>(-4,263(-4))</td>
</tr>
<tr>
<td>( \omega_z )</td>
<td>(7,20(-7))</td>
<td>7,31(-7)</td>
</tr>
<tr>
<td>( B_e )</td>
<td>0,13433(-1)</td>
<td>0,13435(-1)</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>(-1,449(-6))</td>
<td>(-1,468(-6))</td>
</tr>
<tr>
<td>( \gamma_e )</td>
<td>(-4,136(-7))</td>
<td>(-4,132(-7))</td>
</tr>
</tbody>
</table>
Table 4

The molecular constants (cm\(^{-1}\)) for the \((1)^1\Pi_u\) state of the Rb\(_2\) dimer: Experimental data – Exp; Theory: our data

<table>
<thead>
<tr>
<th>(Rb_x)</th>
<th>Theory</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_e)</td>
<td>47,471</td>
<td>47,470</td>
</tr>
<tr>
<td>(\omega_x e)</td>
<td>0,1431</td>
<td>0,1430</td>
</tr>
<tr>
<td>(\omega_y e)</td>
<td>(-8,351(-7))</td>
<td>-</td>
</tr>
<tr>
<td>(B_e)</td>
<td>0,19529(-1)</td>
<td>0,19523(-1)</td>
</tr>
<tr>
<td>(\alpha_e)</td>
<td>1,02(-4)</td>
<td>1,00(-4)</td>
</tr>
<tr>
<td>(\gamma_e)</td>
<td>1,564(-7)</td>
<td>1,561(-7)</td>
</tr>
</tbody>
</table>

Analysis of the listed data shows a physically reasonable agreement between the theoretical and experimental data. Further, we present the accurate data on the probabilities for vibration-rotation-nuclear transitions from state with \(v_a=0, J_a=0\) and state \(v_a=1, J_a=0\) in a case of the emission and absorption spectrum of nucleus \(^{127}\)I \((E(0)) = 203\) keV) linked with molecule \(H^{127}\)I in the ground electron state \(X^1S\) (molecular parameters: \(R_0=1,61\) Å, \(n_e=2309\) cm\(^{-1}\), \(B=6,55\) cm\(^{-1}\)). The recoil energy for this molecule is 0,172 eV. Parameters which define excitation of vibrations and rotations for this molecule because of the recoil, are as follows: \(a_\nu=1,30\) and \(e_\nu=5,29\times10^{-2}\). It should be noted also that a width of the gamma lines are corresponding to temperature \(T=300K\). In figure 1 we present the calculated spectrum of emission and adsorption of nucleus \(^{127}\)I in the \(H^{127}\)I.

![Fig. 1. Computed emission (solid curve) and absorption spectrum of nucleus \(^{127}\)I \((E(0)) = 203\) keV) in the molecule \(H^{127}\)I. Initial state of molecule: a). above \(n_a=0, J_a=0\) and b). below \(n_a=1, J_a=0\) (our data)](image)

We have made comparison of the corresponding vibration-rotation-nuclear transition parobabilities from state with \(v_a=0, J_a=0\) and state \(v_a=1, J_a=0\) in a case of the emission and absorption spectrum of nucleus \(^{127}\)I \((E(0)) = 203\) keV) in the \(H^{127}\)I for different approximations of the for potential curves: the harmonic oscillator \([4]\), the Dunham model \([5,7]\) and presented approach. The values for probabilities, calculated within the present approach and Dunham model for potential curve \([7,8]\), differ from the corresponding ones, calculated within the harmonic oscillator approximation \([1]\), in average on 5-20%. A direct experimental observation of the cooperative electron-gamma-nuclear effects represents a great fundamental interest. Finally let us note that the presented theory is related to usual molecular systems. At the same time in the last years a great attention is turn to the exotic (hadronic) atomic and molecular systems such pionic and kaonic atoms and molecules. The difference between the usual and exotic molecules at the theoretical level is obviously provided by using the Schrodinger equation in a case of usual molecules and the Klein-Gordon-Fock equation for the pionic and kaonic systems. Taking into account the results of the last two decades on successful solutions of the Klein-Gordon-Fock equation with the different (for example, Morse etc) \([18]\) potentials our theory is naturally generalized on a case of exotic diatomic molecules. All theoretical positions are remained the same. Simultaneously it is self-undrestood that the relatively quick radiative processes with
characteristic life less than the negative pion and kaon lifetime (~10^{-8}s) are of a direct theoretical and practical interest.

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Abstract

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Key words: electron-g-nuclear transition spectrum, molecules, spectral parameters
вращательных-ядерных вероятностей переходов в случае испускания и поглощения спектра ядра $^{127}\text{I} (E_g^{(0)} = 203 \text{ keV})$ в молекуле $\text{H}^{127}\text{I}$ для различных приближений для потенциальных кривых: модели гармонического осциллятора, модели на основе потенциала Данхем и предложенного в работе нового подхода.

**Ключевые слова:** спектр электрон- г -ядерных переходов, молекулы, спектральные параметры

УДК 539.183

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**НОВИЙ КВАНТОВИЙ ПІДХІД ДО ВИЗНАЧЕННЯ МОЛЕКУЛЯРНИХ СПЕКТРАЛЬНИХ КОНСТАНТ І ІМОВІРНОСТЕЙ КООПЕРАТИВНИХ КОЛИВАЛЬНО- ОБЕРТАЛЬНО-ЯДЕРНИХ ПЕРЕХОДІВ У СПЕКТРІ ДВОАТОМНИХ І АДРОННИХ МОЛЕКУЛ**

**Резюме**

Пропонується новий підхід до побудови потенційної функції двохатомних молекул у вигляді суми відомого обуреної функції осцилятора Морзе, молекулярного потенціалу Simons-Parra-Finlan в середній частині потенційної кривої, функції типу $-C_n / R^n$ при великих меж’ядерних відстанях. В рамках цього підходу розвинена прецисійна схема обчислення молекулярних спектральних параметрів, а саме коливальних, обертальних, відцентрових постійних для електронних станів двохатомних молекул. Як додаток проведено обчислення молекулярних констант (см$^{-1}$) для станів $X^1\Sigma^+$ В’П димера KRb і димера рубідію і виконано порівняння з експериментальними даними. В рамках послідовного підходу до розрахунку спектрів електронно- гамма-ядерних переходів (набір колебально-обертальних супутників в молекулі) в молекулі отримані оцінки для колебально-обертальних-ядерних ймовірностей переходів в разі випускання і поглинання спектра ядро $^{127}\text{I} (E_g^{(0)} = 203 \text{ keV})$ в молекулі $\text{H}^{127}\text{I}$ для різних наближень для потенційних кривих: моделі гармонійного осциллятора, моделі на основі потенціалу Данхем і запропонованого в роботі нового підходу.

**Ключові слова:** спектр електрон- г -ядерних переходів, молекули, спектральні параметри