THE MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
ODESA STATE ENVIRONMENTAL UNIVERSITY

Methodical instructions
for self-sufficient work of students and tests performance and distance learning in the discipline «Higher Mathematics» Part 2

for the 2nd year students of the extramural studies and distance learning (all directions of training)

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Methodical instructions for self-sufficient work of students and tests performance in the discipline «**Higher Mathematics**» (Part 2) for the 2nd year students of the extramural studies and distance learning (all directions of training)

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Foreword

Higher mathematics is one of the main disciplines of fundamental training cycle in the areas of hydrology, meteorology, ecology, computer science and more. It aims to study the basic provisions of the differential and integral calculus, multiple and curvilinear integrals, field theory, numerical and functional series, ordinary differential equations, the theory of functions of complex variable, mathematical physics, probability theory and mathematical statistics and generalization possibilities of practical use of studied methods for solving practical problems in a particular scientific practice.

The purpose of the discipline is to provide mastering of fundamental theoretical course of higher mathematics, to promote skills in the application of known methods of higher mathematics in different areas, creative research skills and mathematical modeling tasks. The total amount of the educational process in hours, level of knowledge and skills in the studied disciplines are defined by educational and professional programs.

Tasks of the "Higher Mathematics" course is to teach students to properly use the studied methods in solving problems and analyze the results of mathematical calculations. Study subjects "Higher Mathematics" are based on the principles of integration of theoretical and practical knowledge acquired by students in general-education schools.

After studying of the discipline a student must master the basic knowledge and skills; He should know - basic definitions, regulations and theorems of linear and vector algebra, differential and integral calculus of functions of one and several variables, theory of differential equations, multiple and curvilinear integrals, field theory, numerical and functional series, ordinary differential equations, the theory of complex variable , mathematical physics, probability theory and mathematical statistics; He should be able - to use theoretical knowledge and skills in solving problems of mathematical analysis, calculating derivatives and integrals, solving differential equations, both ordinary and partial derivatives apply a range of practical skills in the implementation techniques of higher mathematics applied to solving mathematical problems .

1 Syllabus of the Higher Mathematics course

1.1 Functions of several variables.

The functions of several variables. Region definition. Limits of functions. Continuity.
Partial derivatives. Full differential, its connection with partial derivatives. Full differential formula invariance. Geometric meaning of full differential. Tangent plane and normal to the surface.
Partial and full differentials of higher–order derivatives. Taylor Formula.
Implicit functions. Differentiation of implicit functions.
Extremes of functions of several variables. A necessary condition for extremum. Sufficient conditions. The method of least squares.
Conditional extreme. The method of Lagrange multipliers. Examples of applications in finding optimal solutions.

1.2 Ordinary differential equations and their systems.

Physical problems that lead to differential equations. Differential equations of the first order. Cauchy problem. Theorem of existence and cohesion of solution of the Cauchy problem. The main classes of equations which are integrated in quadratures. The use of first order differential equations in various fields of science.

Differential equations of higher orders. Cauchy problem. The concept of boundary value problems for differential equations. The equations that allow reduction procedure. Application to the solution of problems of escape velocity, physical pendulum motion, bending the rod.

Linear differential equations, homogeneous and heterogeneous. The concept of the overall solution.

Linear differential equations with constant coefficients. The equation of the right side of a special kind. Application to describe linear models.

The normal system of differential equations. Stand-alone system. Vector normal notation system. Geometric meaning of solution. The phase space (the plane), the phase curve. Application in the dynamics of material points, the automatic control theory in biology (model predator - prey), etc.

Cauchy problem for normal system of differential equations. Theorem of existence and cohesion of the solution of the Cauchy problem. Exclusion method to solve the normal system of differential equations. The simplest numerical methods.

Systems of linear differential equations. Properties of differential equations and their solutions. The simplest numerical methods.

Solving systems of linear differential equations with constant coefficients.

The concept of qualitative research methods of differential equations.

1.3 Numerical, functional, power series, Fourier series.

Numerical series. Sums and convergence of series. Convergence condition. Actions with the series. Methods of studying of convergence of series.

Functional series, convergence region, methods of determining power series.

Expansion of functions in power series. The use of power series in approximate calculations.

Fourier series. Decomposition of functions in trigonometric Fourier series. Terms of pointwise convergence and "average" convergence. The use of trigonometric Fourier approximate calculations.

1.4 Multiple integrals. Curvilinear and surface integrals.

The problems that lead to the concept of multiple integral. Double and triple integrals their basic properties. The concept of any integral ratio.

Calculation of double and triple integrals in Cartesian coordinates.

Replacement of variables in multiple integrals. The transition from Cartesian coordinates to polar, cylindrical and spherical coordinates. The use of multiple integrals to compute volumes and areas to solve problems of mechanics and physics.

The problems that lead to the line integral. Determining the line integral first and second kind, their basic properties and calculation. The geometrical and mechanical applications. Connection between the line integral of the first and second kind. Green formula.

Surface area. Definition of surface integrals, their properties and calculation.

1.5 Field Theory

Scalar field. Surfaces and lines of a scalar field. The direction derivative. The gradient of the scalar field and its coordinate and invariant definition.

Vector field. Vector lines and their equations. Unilateral and bilateral surface. The flow of the vector field through a surface. The physical meaning of the flow of fluid velocity field. Flow calculation. Ostrogradsky Theorem.

Divergence of a vector field and its invariant definition, physical meaning.

Line integral in a vector field. Force field work. The circulation of a vector field. Stokes' theorem. Rotor of the field it’s coordinate and invariant definition. The rotor physical meaning in the speeds field. Terms of independence of linear integral from integration way.

The potential field. Potentiality field terms. Calculation of linear integral in a potential field.

Hamilton Operator. Operations of the second order in vector analysis. Laplace Operator, its expression in cylindrical and spherical coordinates.

2 General guidelines for part-time students work over the course of Mathematics

The main learning form for part-time students is working on educational material, consisting of the following elements: learning material in textbooks, solving problems, self-testing, performance tests. To help external students University arranged lectures, practical classes and laboratory work. In addition, a student can contact the teacher with questions for the written or oral advice. Notes about the current work are also given in the process of test reviewing. However, the student must remember that only the systematic and persistent work of self-help university will be effective. The final stage of studying of individual parts of higher mathematics course is a compilation of tests and examinations in accordance with the curriculum.

3 General recommendations for part-time students on the implementation of control work

Preliminary results of students work under study are checked by control work. Work should be carried out independently and to some extent guarantee that the student learned this section.

According to a new topic of control work, an interview is held with the student to determine the independence of its implementation and identify skills in solving problems. Therefore, the performance should treated very seriously. Below is the table of numbers of tasks included in the test. The student must perform control tasks for the variant whose number matches the last digit of his training number (encryption).

variant	Numbers of tasks for execution																	
1	111	121	131	141	151	161	171	181	191	201	211	221	231	241	251	261	271	281
2	112	122	132	142	152	162	172	182	192	202	212	222	232	242	252	262	272	282
3	113	123	133	143	153	163	173	183	193	203	213	223	233	243	253	263	273	283
4	114	124	134	144	154	164	174	184	194	204	214	224	234	244	254	264	274	284
5	115	125	135	145	155	165	175	185	195	205	215	225	235	245	255	265	275	285
6	116	126	136	146	156	166	176	186	196	206	216	226	236	246	256	266	276	286
7	117	127	137	147	157	167	177	187	197	207	217	227	237	247	257	267	277	287
8	118	128	138	148	158	168	178	188	198	208	218	228	238	248	258	268	278	288
9	119	129	139	149	159	169	179	189	199	209	219	229	239	249	259	269	279	289
0	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	261	280	290

Let us have a look at the typical examples of control work №2 problems solution:

FUNCTION OF SEVERAL VARIABLES

Example №1. Find partial derivatives of the second order function

$$z = \cos^2(3x + 2y) .$$

Solution. Looking for partial derivatives of the first order, partial derivative by x is taken assuming that $y = const$, and partial derivative by y is taken with assumption that $x = const$.

$$\frac{\partial z}{\partial x} = 2\cos(3x + 2y)(-\sin(3x + 2y)) \cdot 3 = -3\sin(6x + 4y);$$
$$\frac{\partial z}{\partial x} = 2\cos(3x + 2y)(-\sin(3x + 2y)) \cdot 2 = -2\sin(6x + 4y);$$

For getting derivatives of higher orders, let us differentiate first equation by x, and the second one by y.

$$\frac{\partial^2 z}{\partial x^2} = -3\cos(6x + 4y) \cdot 6 = -18\cos(6x + 4y);$$
$$\frac{\partial^2 z}{\partial y^2} = -2\cos(6x + 4y) \cdot 4 = -8\cos(6x + 4y);$$

To get a mixed derivative, we need to differentiate by y partial derivative of x, or vice-versa, partial derivative of y should be differentiated by x. Results should be same according to theorem of independence of mixed partial derivatives from differentiating order.

$$\frac{\partial^2 z}{\partial x \partial y} = -3\sin(6x + 4y) \cdot 4 = -12\sin(6x + 4y);$$

Example №2. Find maximal and minimal value of function $z = x^2 y(2 - x - y)$ in triangle, limited by lines $x = 0;\ y = 0;\ x + y = 6.$

Solution. Let us find stationary points of the function,

$$\frac{\partial z}{\partial x} = xy(4 - 3x - 2y);\ \frac{\partial z}{\partial y} = x^2(2 - x - 2y),$$

Equating partial derivatives to zero. Inside the area $x>0,\ y>0$ we cut by x and y.

$$\begin{cases} 4-3x-2y=0 \\ 2-x-2y=0 \end{cases} \Rightarrow \quad \begin{cases} 3x+2y=4 \\ x+2y=2 \end{cases}$$

$x=1;\ y=1/2$ — solutions of the system, critical point coordinates P(1, 1/2).

Let us find values pf function in this point: $z(1,\ 1/2)=1\cdot 1/2\cdot(2-1-1/2)=1/4$. For finding maximal and minimal function values in an area, we need to study the function at area borders. In our case at borders $x=0$ and $y=0$ function equals to zero ($z=0$).

Let us find minimal and maximal value of the function at the side of triangle $x+y=6$. For this side $y=6-x$, $0\le x\le 6$ and

$$z=x^2(6-x)(2-x-6+x)=-4x^2(6-x)$$

At the ends of the interval, $x=0$ i $x=6$ function values are $z(0)=z(6)$. Let us find function's stationary points at this line:

$$z'=-48x+12x^2=0;\ 12x(x-4)=0;\ x_1=0,\ x_2=4.$$

The remaining options is $x=4$, as $x=0$ - border point and was already studied by us. $z(4)=-4\cdot 16\cdot(6-4)=-128$; with $x=4$; $y=2$, i.e. $z(4;\ 2)=-128$. Obviously minimal and maximal values of the function should be searched among it's values at the critical P, at the sides of triangle and on it's vertices, i.e.

$z(1;\ 1/2)=1/4$ – value at point P;

$z=0$ – at sides $x=0$ i $y=0$, and also at the triangle's vertices;

$z=-128$ – at point (4; 2) on side $x+y=6$.

This leads to conclusion, that maximal value $z=1/4$ function reaches inside the area at point P(1; $1/2$), and minimal value $z=-128$ function reaches at point P_1(4; 2), which lies at the line $x+y=6$.

ORDINARY DIFFERENTIAL EQUATIONS AND THEIR SYSTEMS

Example 1. Find a line, which normal length (i.e. segment from its normal line to a point on the horizontal axis) is constant a.

Solution. Let us make normal equation for needed curve, assuming x and y as coordinates of the point which lies and the curve, while X and Y as current normal coordinates.

$$Y - y = -\frac{1}{y'}(X - x).$$

At the point A normal is crossing horizontal axis $Y = 0$, and $X = x$. Let us write normal length using coordinates of points A and P:

$\sqrt{(X - x)^2 + y^2} = a$, so we can find $X - x$ and put it in the normal equation: $X - x = \sqrt{a^2 - y^2}$. On the other hand, as point A lies on the normal, it's coordinates must follow normal equation. Putting them: $-y = -\frac{1}{y'}(X - x)$ or

$y' = \frac{1}{y}(X - x) = \frac{1}{y}\sqrt{a^2 - y^2}$, we will have differential equationof the curve we are looking for

$$y' = \frac{\sqrt{a^2 - y^2}}{y}.$$

Integrate it:

$$\frac{dy}{dx} = \frac{\sqrt{a^2 - y^2}}{y}; \quad \frac{ydy}{\sqrt{a^2 - y^2}} = dx; \quad C - \sqrt{a^2 - y^2} = x,$$

$$-\sqrt{a^2 - y^2} = x - C; \quad a^2 - y^2 = (x - C)^2$$

We got $(x - C)^2 + y^2 = a^2$. We got family of circles with radius a with centres at the axis OX.

Example 2. Find common solution of equation $y'' = \frac{y'}{x} + x$.

Solution. Equation does not contain y and belongs to type $y' = f(x, y')$. so let us change $y' = p$ and $y'' = p'$. now we got linear equation of the firth order

$$p' = -\frac{1}{x}p = x \; .$$

Let us solve it, using exchange $p = uv.$ $p' = u'v + v'u$. Equation is transformed to form $u'v + v'u - \frac{1}{x}uv = x.$ Using Bernoulli method we get two equations with two variables: $v' - \frac{1}{x}v = 0$ and $u'v = x$. From the first one $\frac{dv}{dx} = \frac{v}{x}$; $\frac{dv}{v} = \frac{dx}{x}$;

$\ln y = \ln x;$ and $y = x.$ Let us pup value v in the second equation and integrate it: $u'x = x$ or $u' = 1$, so $u = x + c.$

This way we have found $p = x(x+c) = x^2 + cx.$

Changing $p = \dfrac{dy}{dx}$ and integrating this equation we'll get final solution

$$\frac{dy}{dx} = x^2 + cx;\ dy = (x^2 + cx)dx;\ y = \frac{x^3}{3} + \frac{cx^2}{2} + c_1 .$$

Example 3. Find partial solution of the equation $y'' - 2y' = e^x(x^2 + x - 3),$ meeting conditions $y\big|_{x=0} = 2; y'\big|_{x=0} = 2.$

Solution. Let us study corresponding homogenous equation $y'' - 2y' = 0.$ Partial solution of the equation should be looked in the form $y = e^{kx}$. Putting this solution in the equation and cutting by e^{kx}, we got characteristic equation $k^2 - 2k = 0$, so $k_1 = 0,\ k_2 = 2$.

Now we get partial solutions $y_1 = e^{ox} = 1$ and $y_2 = e^{2x}$. This syatem is fundamental, as $\dfrac{y_1}{y_2} \neq const$. So we can base on them common solution of homogenous equation $\bar{y} = c_1 + c_2 e^{2x}$.

As right part of the equation looks like $e^{\alpha x} p_n(x)$, partial solution will be searched in similar form $(k_1 \neq \alpha;\ k_2 \neq \alpha)$:

$$y = l^x(ax^2 + bx + c),\ y' = e^x(ax^2 + bx + c) + c^x(2ax + b);$$
$$y'' = e^x(ax^2 + bx + c) + 2e^x(2ax + b) + 2ue^x.$$

Putting these values in the target equation and get the equality (which is preliminary cutted by $e^x \neq 0$): $-ax^2 - bx + 2a - c = x^2 + x - 3$. Equating coefficients of similar powers of x in the left and rights parts, we come to the system of equations for determining of *a*, *b* i *c:*
$-a = 1; -b = 1;\ 2a - c = -3,$ so $a = -1;\ b = -1;\ c = 1,$ and partial solution get form: $y = e^x(-x^2 - x + 1)$.

Making common solution of the heterogeneous equation:

$$y = c_1 + c_2 e^{2x} + e^x(-x^2 - x + 1) .$$

Determining c_1 and c_2 from the initial conditions:

$$y' = 2c^2 e^{2x} + e^x(-x^2 - x + 1) - e^x(x + 1)$$

$$\begin{array}{l} y|_{x=0} = 2 = c_1 + c_2 + 1 \\ y'|_{x=0} = 2 = 2c_2 + 1 - 1 \end{array} \text{ or } \begin{cases} c_1 + c_2 = 1 \\ 2c_2 = 2 \end{cases}.$$

From the last equation $c_2 = 1$. Then $c_1 = 0$ and partial solution heterogeneous equation, which meets initial conditions, will have form:

$$y = e^{2x} + e^x(-x^2 - x + 1) = e^x(e^x - x^2 - x + 1).$$

Example 4. Find common solution of system with help of characteristic equation. Notate in matrix form given system and its solution.

$$\begin{cases} \dfrac{dx_1}{dt} = 2x_1 + x_2 \\ \dfrac{dx^2}{dt} = 3x_1 + 4x_2 \end{cases}$$

Solution. Let us remind, that partial solutions of system are searched in the form $x_1 = \alpha_1 e^{kt}$ and $x_2 = \alpha_2 e^{kt}$. After putting them in the given system and cutting by, e^{kt} we will get algebraic system

$$\begin{cases} (2-k)\alpha_1 + \alpha_2 = 0 \\ 3\alpha_1 + (4-k)\alpha_2 = 0 \end{cases}$$

It has nontrivial (nonzero) solutions only if its determinant is zero. After recording this condition, we obtain the characteristic equation for the system of linear homogeneous differential equations:

$$\begin{vmatrix} 2-k & 1 \\ 3 & 4-k \end{vmatrix} = 0.$$

Reveal determinant and get $(2-k)(4-k)=0$ or $k^2 - 6k + 5 = 0$, so $k_1 = 1,\ k_2 = 5$. Putting each of values k to the system of algebraic equations, getting values α_1 and α_2, and so – partial solutions x_1 and x_2. Let us take $k_1 = i`$ and put it in the system. We will get

$$\begin{cases} \alpha_1^{(1)} + \alpha_2^{(1)} = 0 \\ 3a_1^{(1)} + 3a_2^{(1)} = 0 \end{cases}$$

(here unknown a_1 and a_2, which matched to values k_1 are marked as $a_1^{(1)}$ and $a_2^{(1)}$). Received equations are dependent. So let us determine (up to a constant factor $a_1^{(1)}$ and $a_2^{(1)}$) from the first equation $a_1^{(1)} = 1$, $a_2^{(1)} = -1$ and get the first partial solutions $x_1^{(1)} = e^t$ i $x_2^{(1)} = -e^t$. Let us take $k_1 = 1$ And put it in the system. We will get

$$\begin{cases} -3\alpha_1^{(2)} + \alpha_2^{(2)} = 0 \\ 3a_1^{(1)} - a_2^{(2)} = 0. \end{cases}$$

As in the previous case from the first equation, we will find $a_1^{(2)} = \frac{1}{3} a_2^{(2)}$ i,

assuming $a_2^{(2)} = 3$, getting $a_1^{(2)} = 1$.

Therefore, the second partial solutions will be $x_1^{(2)} = e^{5t}$; $x_2^{(2)} = 5e^{5t}$.
Multiplicating $x_i^{(1)}$ by C_1, and $x_i^{(2)}$ by C_2 and adding appropriate solutions, we will find common solution of the system

$$x_1 = c_1 x_1^{(1)} + c_2 x_1^{(2)} = c_1 e^t + c_2 e^{5t}$$
$$x_2 = c_1 x_2^{(1)} + c_2 x_2^{(2)} = -c_1 e^t + 3c_2 e^{5t}$$

Let us note it in the matrix form.
System:

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} c_1 & c_2 \\ -c_2 & 3c_2 \end{pmatrix} \cdot \begin{pmatrix} e^t \\ e^{5t} \end{pmatrix}.$$

SERIES

Example 1. Explore the convergence of numerical series:

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8} + ... + \frac{2n-1}{2n} + ... = \sum_{n=1}^{\infty} \frac{2n-1}{2n}.$$

Solution. Let us check is the necessary requirement of convergence met: $\lim\limits_{n\to\infty} U_n = \lim\limits_{n\to\infty} \frac{2n-1}{2} = 1 \neq 0 =>$ series diverges according to the necessary requirement.

Example 2. Explore the convergence of numerical series:

$$1+\frac{1}{11}+\frac{1}{21}+\frac{1}{31}+...+\frac{1}{1+10n}+...=\sum_{n=0}^{\infty}\frac{1}{1+10n}+...=\sum_{n=0}^{\infty}\frac{1}{1+10n.}$$

Solution. Comparing this series with harmonic one $\sum\limits_{n=1}^{\infty}\frac{1}{n}$, on the basis of comparison we will get $\lim\limits_{n->\infty}\frac{u_n}{\tau_n}=\lim\limits_{n->\infty}\frac{n}{1+10n}=\frac{1}{10}\neq 0.$

Example 3. Explore the convergence of numerical series:

$$1+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+...+\frac{1}{1\cdot 2\cdot 3\cdots n}+...=\sum_{n=1}^{\infty}\frac{1}{n!}.$$

Solution.Let us write $u_n=\frac{1}{n!}$ and $u_{n+1}=\frac{1}{(n+1)!}$, use Dalamber sign:

$\lim\limits_{n->\infty}\frac{u_{n+1}}{u_n}=\lim\limits_{n->\infty}\frac{n!}{(n+1)!}=\lim\limits_{n->\infty}\frac{1}{n+1}=0<1,\ =>$ series converges.

Example 4. Explore the convergence of numerical series: $\sum\limits_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^n.$

Solution. Let us note $u_n=\left(\frac{n+1}{2n-1}\right)^n$ and use Koshy radical sign

$\lim\limits_{n->\infty}\sqrt[n]{u_n}=\lim\limits_{n->\infty}\sqrt[n]{\left(\frac{n+1}{2n-1}\right)^n}=\lim\limits_{n->\infty}\frac{n+1}{2n-1}=1/2<1$, => series converges.

Example 5. Explore the convergence of numerical series: $\sum\limits_{n=0}^{\infty}\frac{1}{\sqrt{4n+1}}.$

Solution. Let us use Koshy integral sign.

$$\int_0^{\infty} f(n)dn=\int_0^{\infty}\frac{dn}{\sqrt{4n+1}}=1/4\lim_{N->\infty}\int_0^{N}(4n+1)^{-1/2}d(4n+1)=$$
$$=1/4\lim_{N->\infty}2\sqrt{4n+1}\Big|_0^N=1/2\lim_{N->\infty}\left(\sqrt{4N+1}-1\right)=\infty,=>$$

Improper integral diverges, so the series is also diverging.

Example 6. Explore the convergence of numerical series: $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{1}{2n-1}$.

Solution. Since the series has alternating sign let us use Leibniz sign.

$$\left.\begin{array}{l} a)\ 1>1/3>1/5>\ldots>\dfrac{1}{2n-1}>\ldots \\ b)\lim\limits_{n->\infty}u_n=\lim\limits_{n->\infty}\dfrac{1}{2n-1}=0 \end{array}\right] \text{=> series converges.}$$

Let us determine the type of the convergence, exploring $\sum\limits_{n=1}^{\infty}|u_n|=\sum\limits_{n=1}^{\infty}\dfrac{1}{2n-1}$, using sign of comparison with harmonic series $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ we will have $\lim\limits_{n->\infty}\dfrac{u_n}{v_n}=\lim\limits_{n->\infty}\dfrac{1}{2n-1}=1/2\neq 0$, =>, both series diverges. Therefore series $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{1}{2n-1}$ conditionally converging.

Example 7. Find the area of convergence of power series:

$$\text{a) } \sum_{n=1}^{\infty}2^{n-1}x^{n-1};\ \text{б) } \sum_{n=1}^{\infty}\frac{(x-2)^n}{n}$$

Solution. a) Let us note $a_n=2^{n-1}$, $a_{n+1}=2^n$, then $R=\lim\limits_{n->\infty}\left|\dfrac{a_n}{a_{n+1}}\right|=\lim\limits_{n->\infty}\dfrac{2^{n-1}}{2^n}=1/2$, therefore, $|x|<1/2$ or $-1/2<x<1/2$ – convergence interval. Let us explore convergence of the series at the ends of the found interval. As $x=1/2$ we will have series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}$, which diverges, and with $x=1/2$ we will have series. $\sum\limits_{n=1}^{\infty}1$, which also diverges. Therefore, convergence area for the given series is $-1/2<x<1/2$ or $x\in\left]-1/2;\,1/2\right[$.

b) Let us note $a_n=1/n$, $a_{n+1}=1/(n+1)$, then $R=\lim\limits_{n->\infty}\dfrac{n+1}{n}=1$, so, $-1<x-2<1$, or $1<x<3$ – convergence interval. Series convergence at the interval ends: with $x=1$ series is $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{1}{n}$, which converges conditionally (Leibniz sign); with $x=3$ series is $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$, which diverges. Therefore, convergence area of the explored series is $1\le x<3$ or $x\in\left[1;\,3\right[$.

If function $f(x)$ has derivatives of all orders in the neighborhood of $x=a$, then for it formally we can write the expansion in power series (Tailor series):

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+...+\frac{f^{(n)}(a)}{n!}(x-a)^n+...$$

However, in any case it is necessary to explore, with this values of x series converges and when it's sum will be equal to the function $f(x)$. If $a=0$, then we will get separate case of Teilor series which is called McLaren series:

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+...$$

Let us show decomposition of some important basic functions in power series indicating areas of convergence:

1. $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+... \quad (-\infty<x<+\infty)$
2. $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\ldots \quad (-\infty<x<+\infty)$
3. $\cos x=x-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots+(-1)^n\frac{x^{2n}}{(2n)!}+\ldots \quad (-\infty<x<+\infty)$
4. $(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\ldots+\frac{m(m-1)\ldots(m-n+1)}{n!}x^n+\ldots$
$(-1<x<1)$
5. $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{n+1}\frac{x^n}{n}+\ldots \quad (-1<x\le 1)$

Example 8. Approximately calculate integral $\int\limits_0^{\alpha}\frac{\sin x}{x}dx.$

Solution. Using decomposition for $\sin x$, we will get:

$$\frac{\sin x}{x}=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\ldots \quad .$$

Then integrating this equation in limits from zero to α:

$$\int\limits_0^{\alpha}\frac{\sin x}{x}dx=\alpha-\frac{\alpha^3}{3\cdot 3!}+\frac{\alpha^5}{5\cdot 5!}-\frac{\alpha^7}{7\cdot 7!}+\ldots$$

Sum of the series can be easily calculated with needed precision with given α.

Example 9. Find approximated solution of differential equation $y' = xy^2 + 1$ with $y(x=1) = 0$.

Solution. Let us note searched solution in form of Teilor series by power of $(x-1)$

$$y(x) = y(1) + \frac{y'(1)}{1!}(x-1) + \frac{y''(1)}{2!}(x-1)^2 + \frac{y'''(1)}{3!}(x-1)^3 + \ldots$$

Assuming $y(1) = 0$, and putting in differential equation $x = 1$, we will find that $y'(1) = 1$.

Let us differentiate resulting equation and will get $y'' = y^2 + 2xyy'$, therefore, with $x = 1$, $y''(1) = 0$. Then let us differentiate second order equation and will get $y''' = 2yy' + 2yy' + 2x(y')^2 + 2xyy''$, which with $x = 1$ gives $y'''(1) = 2$. In the same way, $y^{IV} = 6(y')^2 + 6yy'' + 6xy'y'' + 2xyy'''$; with $x = 1$ gives $y''(1) = 6$ etc. Assuming found values of solution of y and its derivatives at point $x = 1$, we will have

$$y(x) = (x-1) + \frac{2(x-1)^3}{3!} + \frac{6(x-1)^4}{4!} + \ldots$$

or

$$y(x) = (x-1) + \frac{1}{3}(x-1)^3 + \frac{1}{4}(x-1)^4 + \ldots$$

Example 10. Expand in Fourier series a function with period of 2π, which on interval $-\pi < x < \pi$ can be shown as $f(x) = |x|$.

Solution. Function $f(x)$ is double, therefore $b_n = 0$,

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \pi,$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx = \frac{2}{\pi}\left(\frac{x\sin nx}{n}\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi}\sin nx\,dx\right) = \frac{2}{\pi n^2}\cos nx\Big|_0^{\pi} = \frac{2}{\pi n^2}\left((-1)^n - 1\right).$$

Fourier series for the given function is written in following form

$$f(x) = |x| = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots + \frac{\cos(2n-1)x}{(2n-1)^2} + \ldots\right)$$

Or in short form

$$f(x)=|x|=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}.$$

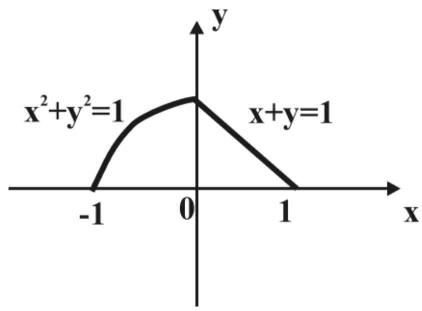
MULTIPLE INTEGRALS. CURVILINEAR AND SURFACE INTEGRALS

Example 1. Change the order of integration in the integrated integral

$$\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx$$

Solution. Let us make integration area according to integration borders: $\psi_1=-\sqrt{1-y^2}$, ψ_2=1-y, y=0, y=1. The area is limited from above by the following curve

$$\varphi_2=\begin{cases}\sqrt{1-x^2}\, npu & -1\le x\le 0\ ,\\ 1-x \quad npu & 0<x\le 1\end{cases}$$



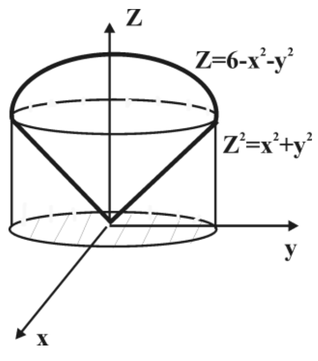
And from below with the line *y=0,* therefore

$$\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx=$$

$$=\int_{-1}^{0} dx \int_0^{\sqrt{1-x^2}} f(x,y)dy+\int_0^1 dx \int_0^{1-x} f(x,y)dy$$

Example 2. Calculate using triple integration volume of a body limited with following surfaces: $Z=6-X^2-Y^2$; $Z^2=X^2+Y^2$; $Z=0$.

Solution. Given body is limited from below with a cone $Z^2=X^2+Y^2$, and from above with paraboloid $Z=6-X^2-Y^2$. Let us determine D area, which is projection of the body to the surface XOУ. Solving together equations of surfaces which limit the body,



$$\begin{cases}Z^2=X^2+Y^2,\\ Z=6-X^2-Y^2\end{cases} \quad \rightarrow \qquad Z_1=2,\quad Z_2=3,$$

We will have surface equation $Z=2$, where surfaces are crossing, i.e. equation of the line which limit area D, is $X^2+Y^2=4$. Let us use formula

$$V=\iiint\limits_V dxdydz=\iint\limits_D dxdy\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}dz \quad ,$$

where $Z=\Psi_1(x,y)$ and $Z=\Psi_2(x,y)$ - are equations of lower and upper limiting surfaces.

$$V=\iint\limits_D dxdy\int\limits_{\sqrt{x^2+y^2}}^{6-x^2-y^2}dz=\iint\limits_D(6-x^2-y^2-\sqrt{x^2+y^2})dxdy$$

As surface D- is a circle, let us switch to polar coordinates *x=rcosφ; y=rsinφ; dxdy=rdrdφ,* which are located inside integration area, therefore coordinate φ is changes from 0 to 2π, and coordinate *r* – from 0 to 2.

$$V=\int\limits_0^{2\pi}d\varphi\int\limits_0^2(6-r^2-r)rdr=\int\limits_0^{2\pi}d\varphi(6-\frac{r^2}{2}-\frac{r^4}{4}-\frac{r^3}{3})\Big|_0^2=$$
$$=\frac{16}{3}\int\limits_0^{2\pi}d\varphi=\frac{16}{3}\varphi\Big|_0^{2\pi}=\frac{32\pi}{3}$$

Example 3. Calculate the line integral

$$I=\int\limits_{(1,1)}^{(4,4)}(x^3+4x^2y)dx+(y^3+\frac{4}{3}x^3)dy$$

Solution. Let us check is the condition fulfilled $\dfrac{\partial P(x,y)}{\partial y}=\dfrac{\partial Q(x,y)}{\partial x}$:

$$\frac{\partial(x^3+4x^2y)}{\partial y}=4x^2\ , \qquad \frac{\partial(y^3+\frac{4}{3}x^3)}{\partial x}=4x^2$$

It is fulfilled, so integral is independent from the form of the integration path. Let us choose as path of integration from point *A(1,1)* to point *B (4,4)* at first along the line *AC*, then along *CB*.

Equation of *AC: y= I; dy=0; Equation* of *CB*: *x= 4; dx=0.* Resulting integral looks following:

$$I=\int\limits_1^4(x^3+4x^21)dx+\int\limits_1^4(y^3+\frac{4}{3}4^3)dy=(\frac{x^4}{4}+4\frac{x^3}{3})\Big|_0^4+(\frac{y^4}{4}+\frac{4^4}{3}y)\Big|_0^4=467{,}5$$

4 Tasks for control work №2

In tasks 111-120 find and depict determination areas of following functions:

111. $z=\sqrt{9-x^2-y^2}$
112. $z=\ln\left(y^2-4x+8\right)$
113. $z=\arcsin\left(x^2+y^2-5\right)$
114. $z=\ln\left(x+y\right)$
115. $z=\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{x-y}}$
116. $z=x+\arccos y$
117. $z=\ln\left(x^2+y\right)$
118. $z=\sqrt{1-x^2}+\sqrt{1-y^2}$
119. $z=\arcsin\frac{y}{x}$
120. $z=\sqrt{x^2-4}+\sqrt{4-y^2}$

In tasks 121-130 find the second order partial derivatives of following functions:

121. $z=e^{3x-y}$
122. $z=e^{xy}$
123. $z=\ln\left(x^2+y^2\right)$
124. $z=\arctan\frac{y}{x}$
125. $z=\sin\left(xy\right)$
126. $z=\arcsin\sqrt{\frac{x-y}{x}}$
127. $z=\sqrt{2xy+y^2}$
128. $z=\arctan\frac{x+y}{1-xy}$
129. $z=x^{n}\cdot y^m$
130. $z=e^{3x}\cos 2y$

In tasks 131-140 find minimum and maximum of the function $z=f\left(x,y\right)$ in given area. Draw the area.

131. $z=x^2+y^2-9xy+27,\quad 0\le x\le 3;\quad 0\le y\le 3$
132. $z=x^2+2y^2+1,\quad x\ge 0;\quad y\ge 0;\quad x+y\le 3$
133. $z=3-2x^2-xy-y^2,\quad x\le 1;\quad y\ge 0;\quad y\le x$
134. $z=x^2+3y^2+x-y,\quad x\ge 1;\quad y\ge -1;\quad x+y\le 1$
135. $z=x^2+2xy+2y^2,\quad -1\le x\le 1;\quad 0\le y\le 2.$
136. $z=5x^2-3xy+y^2+4,\quad x\ge -1;\quad y\ge -1;\quad x+y\le 1.$
137. $z=10+2xy-x^2,\quad 0\le y\le 4-x^2.$
138. $z=x^2+2xy-y^2+4x,\quad x\le 0;\quad y\le 0;\quad x+y+2\ge 0.$
139. $z=x^2+xy-2,\quad 4x^2-4\le y\le 0.$
140. $z=x^2+xy,\quad -1\le x\le 1;\quad 0\le y\le 3.$

In tasks 141-150 solve problems:

141. Material point with mass $m = 2$ g with no initial velocity slowly drawing in a liquid. Resistance of the liquid in proportional to drowning velocity with proportion coefficient $k = 2$ m/s. Find velocity of the point after one second after drowning start.
142. Motorboat moves in still water with velocity $v_0 = 12$ km/h. At full speed it's engine was turned off and after 10 seconds velocity of the boat decreased to $v_1 = 6$ km/h. Water resistance is proportional to boat movement velocity. Find boat velocity after 1 minute after engine stop.
143. A bullet, moving with velocity $v_0 = 400$ m/s, pierce relatively thick wall. Wall resistance gives the bullet negative acceleration proportional toy its squared velocity with coefficient $k = 7\,\text{m.}^{-1}$ Find the bullet velocity after 0,001 seconds after hitting the wall.
144. Material point with mass $m = 1$g is moving straightly. Force acting to it is pointing to the movement direction and is proportional to the time which have passed from the time when velocity of the point was zero with coefficient $k_1 = 2\,\text{m/s}^3$; Also point is affected by environment resistance proportional to the velocity with coefficient $k_2 = 3$m/s. Find the point velocity after 3 seconds after movement start.
145. Container contains 100 liters of salt-water solution. Clear water runs into container with speed of $q = 5$l/m, the solution is running out with same speed, and concentration is kept with uniform mixing. At the beginning solution contained $m_0 = 10$ kg of salt. How much salt will container have inside after 20m of the process?
146. Curve goes through point (2;–1) and have a property according to which angular coefficient of the tangent at any point is proportional to the square of the ordinate point of touching with aspect ratio $k = 3$. Find the curve equation.
147. Curve goes through point (1; 2) and have a property according to which the product of the angular coefficient of the tangent at any point on sum of coordinates of the point is twice touching ordinate this point. Find the curve equation
148. Curve goes through point (1;2) and have a property according to which related ordinates of any point of abscissa proportional to angular coefficient of the tangent to the curve drawn at the same point, with a coefficient of proportionality $k = 3$. Find the curve equation.
149. Curve goes through point (1;5) and have a property according to which segment that is cut on the vertical axis by any tangent, is triple abscissa of the touch point. Find the curve equation.
150. Curve goes through point (2;4) and have a property according to which segment that is cut on the vertical axis by any tangent, is cube of abscissa of the touch point. Find the curve equation.

In tasks 151-160 find common solution of differential equations

151. $yy'' + (y')^2 = 1$

152. $xy'' = y'$

153. $y'' = y' + x$

154. $y^n = \frac{1}{4\sqrt{y}}$

155. $(y'')^2 = y'$

156. $(y')^2 + 2yy'' = 0$

157. $y'' - 2ctg\, x \cdot y^1 = \sin^3 x$

158. $y^3 y^n = -1$

159 y"=y'/x+x

160 2xy'y"=$(y')^2$+1

In tasks 161-170 find partial solution of differential equation $y'' + py' + qy = f(x)$, which meets initial requirements $y(0) = y_{0_1}$, $y'(0) = y_0$.

161. $y'' + 4y' - 12y = 8\sin 2x;\ \ y(0) = 0,\ y'(0) = 0.$

162. $y'' - 6y' + 9y = x^2 - x + 3;\ y(0) = 4/3,\ y'(0) = 1/27$

163. $y'' + 4y = e^{-2x};\ \ y(0) = 0,\ \ y'(0) = 0$.

164. $y'' - 2y' + 5y = xe^{2x};\ y(0) = 1,\ \ y'(0) = 0$

165. $y' + 5y + 6y = 12\cos 2x;\ \ y(0) = 1,\ y'(0) = 3$

166. $y'' - 5y'' + 6y = (12 - 7)e^{-x};\ \ y(0) = 0,\ y'(0) = 0$

167. $y'' - 4y' + 13y = 26x + 5;\ \ y(0) = 1,\ y'(0) = 0$

168. $y' - 4y' = 6x^2 + 1;\ \ y(0) = 2,\ \ y'(0) = 3$

169. $y' - 2y' + y = 16e^x;\ \ y(0) = 1,\ \ y'(0) = 2$

170. $y'' + 6y' + 9y = 10e^{-3x};\ \ y(0) = 3;\ \ y'(0) = 2$

In tasks 171-180 find common solution of the system using characteristic equation. Write down the solution in matrix form.

171. $\begin{cases} \dfrac{dx}{dt} = y - 7, \\ \dfrac{dy}{dt} = -2x - 5y. \end{cases}$

172. $\begin{cases} \dfrac{dx}{dt} = x + 5y \\ \dfrac{dy}{dt} = -x - 3y. \end{cases}$

176. $\begin{cases} \dfrac{dx}{dt} = -3x - y, \\ \dfrac{dy}{dt} = x - y. \end{cases}$

177. $\begin{cases} \dfrac{dx}{dt} = x - 3y, \\ \dfrac{dy}{dt} = 3x + y. \end{cases}$

173. $\begin{cases} \dfrac{dx}{dt} = 4x + 6y, \\ \dfrac{dy}{dt} = 4x + 2y. \end{cases}$

174. $\begin{cases} \dfrac{dx}{dt} = 2x + y, \\ \dfrac{dy}{dt} = 3x + 4y. \end{cases}$

175. $\begin{cases} \dfrac{dx}{dt} = 3x + y, \\ \dfrac{dy}{dt} = 8x + y. \end{cases}$

178. $\begin{cases} \dfrac{dx}{dt} = 3x - 2y, \\ \dfrac{dy}{dt} = 2x + 8y. \end{cases}$

179. $\begin{cases} \dfrac{dx}{dt} = -x - 5y, \\ \dfrac{dy}{dt} = -7x - 3y. \end{cases}$

180. $\begin{cases} \dfrac{dx}{dt} = 6x + 3y, \\ \dfrac{dy}{dt} = -8x - 5y. \end{cases}$

In tasks 181-190 explore the convergence of numerical series.

181. $\sum\limits_{n=1}^{\infty} \dfrac{3n+1}{\sqrt{n3^n}}$;

182. $\sum\limits_{n=1}^{\infty} \dfrac{e^{-\sqrt{n}}}{\sqrt{n}}$;

183. $\sum\limits_{n=1}^{\infty} \dfrac{5^n}{3^n(2n+1)}$;

184. $\sum\limits_{n=1}^{\infty} \dfrac{3^n}{(2n)!}$;

185. $\sum\limits_{n=1}^{\infty} \dfrac{n^3}{e^n}$;

186. $\sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln^2 n}$;

187. $\sum\limits_{n=1}^{\infty} \dfrac{1}{(2n+1)^2 - 1}$;

188. $\sum\limits_{n=1}^{\infty} \dfrac{n^2}{(3n)!}$;

189. $\sum\limits_{n=1}^{\infty} \left(\dfrac{3n}{2n+1} \right)^n$;

190. $\sum\limits_{n=1}^{\infty} \dfrac{n^{n+1}}{(n+1)!}$.

In tasks 191-200 explore the convergence of alternating numerical series.

191. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{n}{6n-5}$;

192. $\sum\limits_{n=1}^{\infty} \dfrac{\sin n\alpha}{n\sqrt{n}}, \quad \alpha = const$;

193. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \sin \dfrac{\pi}{n}$;

194. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{\ln(n+1)}{\sqrt{n}}$;

196. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{1}{n^3\sqrt{n}}$;

197. $\sum\limits_{n=1}^{\infty} (-1)^{n} \dfrac{1}{\sqrt{2n+1}}$;

198. $\sum\limits_{n=1}^{\infty} (-1)^n \cos \dfrac{\pi}{5n}$;

199. $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{1}{n(n+1)}$;

195. $\sum_{n=1}^{\infty} \frac{\sin n\alpha}{n!}, \quad \alpha = const;$

200. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3 - 1};$

In tasks 201-210 find the interval of convergence of power series.

201. $\sum_{n=1}^{\infty} \frac{3^n}{n!} x^n;$

202. $\sum_{n=1}^{\infty} \frac{2^n}{n(n+1)} x^n;$

203. $\sum_{n=1}^{\infty} \frac{n^2}{n+1} x^n;$

204. $\sum_{n=1}^{\infty} \frac{1}{n \ln^2 n} x^n;$

205. $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt[n]{n}} x^n;$

206. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^n} x^n;$

207. $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{(n+1)^n} x^n;$

208. $\sum_{n=1}^{\infty} \frac{n+2}{n(n+1)} x^n;$

209. $\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{(3n-1)2^n}} x^n;$

210. $\sum_{n=1}^{\infty} \frac{n}{3^n (n+1)} x^n;$

In tasks 211-220 calculate the definite integral $\int_{\alpha}^{\beta} f(x)dx$ with precision up to 0,001, expanding the integrand function in power series and integrating it term by term.

211. $\int_0^1 e^{-x^2} dx;$

212. $\int_0^{0,5} \cos \frac{x^2}{4} dx;$

213. $\int_0^1 \frac{\sin x}{x} dx;$

214. $\int_0^{0,1} \frac{\ln(1+x)}{x} dx;$

215. $\int_0^{0,5} \arctan x^2 dx;$

216. $\int_0^{0,5} \sqrt{1+x^3} dx;$

217. $\int_0^{0,5} \frac{\arctan x}{x} dx;$

218. $\int_0^1 \sin x^2 dx;$

219. $\int_0^{0,5} \frac{dx}{\sqrt{1+x^4}};$

220. $\int_0^{1/3} x \cos \sqrt{x} dx.$

In tasks 221-230 Find the first three nonzero member expansions in power series solution of differential equations $y = y(x)$, which meets initial conditions:

221. $y'' = xy, y(0) = 1, \; y'(0) = 0;$

222. $y' = x^2 + y^2, \; y(0) = 1;$

226. $y' = x + x^2 + y^2, \; y(0) = 5;$

227. $y' = y^2 + xy, \; y(0) = 1;$

223. $y'' = -xy' - y,\ y(0) = 0,\ y'(0) = 1;$

224. $y' = e^x + y^2,\ y(0) = 0;$

225. $y'' = xyy',\ y(0) = 1,\ y'(0) = 1;$

228. $(1 - x)y' = 1 + x - y,\ y(0) = 0;$

229. $y'' = yy' - x^2,\ y(0) = 1,\ y'(0) = 1;$

230. $y' = e^y + xy,\ y(0) = 0.$

In tasks 231-240 expand following function $f(x)$ in Fourier series at the interval (a, b):

231. $f(x) = x^2 + 1;\ (-2,\ 2);$

232. $f(x) = \dfrac{\pi - x}{2};\ (-\pi,\ \pi);$

233. $f(x) = 1 + |x|;\ (-1,\ 1);$

234. $f(x) = |1 - x|;\ (-2,\ 2);$

235. $f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ 2x, & 0 < x < \pi \end{cases};$

236. $f(x) = |x|;\ (-3,\ 3);$

237. $f(x) = x - 1;\ (-1,\ 1);$

238. $f(x) = \begin{cases} 1, & -\pi < x \le 0 \\ 2, & 0 < x < \pi \end{cases}$

239. $f(x) = \begin{cases} x + 1, & -\pi < x \le 0 \\ 0, & 0 < x < \pi \end{cases}$

240. $f(x) = |1 - x|;\ (-2,\ 2);$

In tasks 241-250 calculate using double integral in the polar system, area of the figure bounded by the curve given by the equation in Cartesian coordinates (a>0)

241. $(x^2+y^2)^3 = a^2x^2y^2$

242. $(x^2+y^2)^2 = a^2(4x^2+y^2)$

243. $(x^2+y^2)^2 = a^2x^2(4x^2+3y^2)$

244. $(x^2+y^2)^2 = a^2(3x^2+2y^2)$

245. $x^4 = a^2(3x^2-y^2)$

246. $x^6 = a^2(x^4-y^4)$

247. $x^4 = a^2(x^2-3y^2)$

248. $y^6 = a^2(y^4-x^4)$

249. $(x^2+y^2)^2 = a^2(2x^2+3y^2)$

250. $y^6 = a^2(3y^2-x^2)(x^2+y^2)$

In tasks 251-260 calculate body volume using triple integral, body is limited by specified surfaces. Make a drawing of the body and its projection on a plane XOУ.

251. $z=0, z=x, y=0, y=4, x=\sqrt{25-y^2}$.

252. $z=0,\ z=9-y^2,\ x^2+y^2=9.$

253. $z=0,\ z=4-x-y,\ x^2+y^2=4.$

254. $z=0,\ z=y^2,\ x^2+y^2=9.$

255. $z=0,\ z+y=2,\ x^2+y^2=4.$

256. $z=0,\ 4z=y^2,\ 2x-y=0,\ x+y=9.$

257. $z=0,\ x^2+y^2=z,\ x^2+y^2=4.$

258. $z=0,\ z=1-y^2,\ x=y^2,\ x=2y^2+1.$

259. $z=0,\ z=1-x^2,\ y=0,\ y=3-x.$

260. $z=0,\ z=4\sqrt{y},\ x=0,\ x+y=4.$

In tasks 261-270 calculate the line integral and make a drawing area of integration:

261. $\int\limits_L (x^2 - y)dx - (x - y^2)dy$ Along the arch L of the circle *x=5cost, y=5sint,* in the counter clock wise direction from point A (5; 0) to point *B(0,* 5).

262. $\int\limits_L (x \; + y)dx - (x - y)dy$ Along broken line L- *OAB*, де *0(0;0),A(2;0),B(4;5)*

263. $\int\limits_L \frac{ydx - xdy}{x^2 + y^2}$ Along triangle border *ABC,* in counter clock wise direction, assuming *A(1; 0), B(1, 1), C(0; 1).*

264. $\int\limits_L (x^2 - 2xy)dx + (y^2 - 2xy)dy$ Along arch L of parabola $y=x^2$ from point *A(-1; 1)* to point *B(1; 1).*

265. $\int\limits_L (x^2 y - 3x)dx + (y^2 x + 2y)dy$ Along upper half of the ellipse L *x=3cost, y =2 sin t* (0<t<π)

266. $\int\limits_L (x^2 + y)dx - (y^2 x + x)dy$ Along broken line L-*ABC,*where *A(1;2), B(l;5), C(3;5)*

267. $\int\limits_L ydx + \frac{x}{y}dy$ Along arch *L* of the curve y=e^{-x} from point *A(0;1)* to point *B(-1; e).*

268. $\int\limits_L \frac{y^2 + 1}{y}dx - \frac{x}{y^2}dy$ Along segment L=*AB* of the line from point *A(1;2)*to point *B(2;4).*

269. $\int\limits_L (xy - x^2)dx + xdy$ Along arch L of parabola $y=2x^2$ from point *A(0;0)*to point *B(1;2).*

270. $\int\limits_L \frac{y}{x}dx + xdy$ Along arch L of the curve *y=lnx*from point *A(1; 0)* to point *B(e; 1).*

In tasks 271-280 given a vector field *F=Xi+Yj+Zk* and a surface *Ax+By-Cz+D=0* (p), which along with coordinate surfaces make a pyramid *V*. Let put σ-base of the pyramid, belonging to surface (p); λ-contour, limitingσ, *n* – outer normal for σ. Calculate following:

1) vector field flow *F* trough surface σ in direction of normal *n;*

2) vector field circulation *V* for closed contour λ implicitly and use Stocks theorem for contour λ and surface σ limited by it with normal *n*;
3) vector field flow *F* trough full surface of the pyramid *V* in direction of outer normal to its surface and using Ostrogradskiy Theorem. Make drawing.

271	F=(x+z)i	(p)	x+y+z-2=0
272	F=(y-x+z)i	(p)	2x-y+2z-2=0
273	F=(x+7z)k	(p)	2x+y+z-4=0
274	F=(x+2y-z)i	(p)	-x+2y+2z-4=0
275	F=(2x+3y-3z)j	(p)	2x-3y+2z-6=0
276	F=(2x+4y+3z)k	(p)	3x+2y+3z-6=0
277	F=(x-y+z)i	(p)	-x+2y+z-4=0
278	F=(3x+4y+2z)j	(p)	x+y+2z-4=0
279	F=(5x+2y+3z)k	(p)	x+y+3z-3=0
280	F=(x-3y+6z)i	(p)	-x+y+2z-4=0

In tasks 281-290 check will a vector field *F=Xi+Yj+Zk* be potential and solenoidal. In case of potentiality – find the potential.

281	F=(6x+7yz)i+(6y+7xz)j+(6z+7xy)k
282	F=(8x-5yz)i+(8y-5xz)j+(8z-5xy)k
283	F=(10x-3yz)i+(10y-3xz)j+(10z-3xy)k
284	F=(12x+yz)i+(12y+xz)j+(12z+xy)k
285	F=(4x-7yz)i+(4y-7xz)j+(4z-7xy)k
286	F=(x+2yz)i+(y+2xz)j+(z+2xy)k
287	F=(5x+4yz)i+(5y+4xz)j+(5z+4xy)k
288	F=(7x-2yz)i+(7y-2xz)j+(7z-2xy)k
289	F=(3x-yz)i+(3y-xz)j+(3z-xy)k
290	F=(9x+5yz)i+(9y+5xz)j+(9z+5xy)k

Table for submission of examinations
(Distance learning)

№	Subject	Job number	Deadline for submission control task
1	Function of several variables	111-150	10.12-20.12
2	Ordinary differential equations and their systems	151-180	25.01-05.02
3	Series	181-240	10.03 -20.03
4	Multiple integrals, curvilinear and surface integrals and theory of field	241-290	01.05 – 10.05

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Methodical instructions
for self-sufficient work of students and tests performance
in the discipline **«Higher Mathematics» Part 2**

for the 2nd year students of the extramural studies and distance learning (all directions of training)

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