THE MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
ODESA STATE ENVIRONMENTAL UNIVERSITY

Methodical instructions for self-sufficient work of students and tests performance in the discipline «Higher Mathematics» Part 1

for the 1st year students of the extramural studies and distance learning (all directions of training)

Odessa 2015

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Methodical instructions for self-sufficient work of students and tests performance in the discipline «**Higher Mathematics**» (Part 1) for the 1st year students of the extramural studies and distance learning (all directions of training)

Compilers:
Glushkov O.V., D.f.-m.s.(Hab.Dr.), prof.,
Khetselius O.Yu., D.f.-m.s.(Hab.Dr.), prof.,
Sukharev D.E., C.f.-m.s.(PhD), assoc.-prof.,
Buyadzhi V.V., assoc.-lect.

Editor:
Glushkov O.V., d.f.-m.s. (Hab.Dr.), prof., Head of the department of higher and applied mathematics OSENU

PREFACE

Mathematics is one of the main subjects of the fundamental cycle, which is aimed at the study of fundamental principles of differential and integral calculus, including multiple and curvilinear integrals , field theory, numerical and functional series, differential equations, theory of functions of a complex variable, equations of mathematical physics, probability theory and synthesis of the possibilities of practical application of the studied methods in solving practical problems in specific scientific and practical activities.

The curriculum is a normative discipline “Higher mathematics” is designed to perform standard “Regulations on the organization of educational process in higher educational institutions” (order of Ministry of Ukraine №161 of 02.06.93 G.)

The aim of the course is to provide fundamental learning of the theoretical course of higher mathematics , to foster the development of skills in the application of known methods of higher mathematics in different areas, skills creative research and mathematical modeling tasks. The total amount of the educational process in hours, equations knowledge and skills in the discipline determined by educational professional programs.

Tasks of the discipline “Higher mathematics” is to teach students to correctly use learned techniques when solving problems and to correctly analyze the results of mathematical calculations. The study of discipline “Higher mathematics” is based on the principles of integration of theoretical and practical knowledge acquired by students in General educational educational institutions.

The volume of learning of individual sections and topics are defined work programs, developed on the basis of this program taking into account the professional direction flows separate groups. After studying the course “Higher mathematics” students are (1-3 courses) exam.

After studying the course the student should acquire basic knowledge and skills; he should know the basic definitions, terms and theorems of linear and vector algebra, differential and integral calculus of functions of one and several variables, theory of differential equations, multiple and curvilinear integrals, field theory, numerical and functional series, differential equations, theory of functions of a complex variable, equations of mathematical physics; ability to use theoretical knowledge and skills in solving linear algebraic problems, problems of mathematical analysis, calculus derivatives and integrals, solution of ordinary and partial differential equations, to apply a range of practical skills in the implementation of the methods of mathematics to solve applied math problems.

THE COURSE OF HIGHER MATHEMATICS (FIRST COURSE)

1. The main course

1.1. Elements of linear algebra and analytical geometry

Coordinate systems on a line, plane and space. Space R^2 i R^2. Linear operations on vectors. The projection of the vector on an axis. The guides of the cosines and vector length. The concept of vector diagrams in science and engineering (diagrams of forces, moments of forces, electric currents, voltages, etc.). The coordinates of the center of mass.

Scalar product of vectors and its properties. Vector length and the angle between two vectors in coordinate form. The condition of orthogonality of two vectors. The mechanical meaning of the dot product.

Determinants of the second and third order, and their properties. Algebraic complement i minors. Determinants of the nth order. The computation of the determinant of the decomposition of a row (column).

Vector product of two vectors, properties. The condition of collinearity of two vectors. The geometric meaning of the determinant of the second order. The simplest application of vector product in science and technology: moments of forces, the force acting on a current carrying conductor in a magnetic field, the velocity of a point rotating body, the direction of propagation of electromagnetic waves, the concept about the phenomenon of a gyroscope.

The mixed product of two vectors. The geometric meaning of the determinant of the third order.

Equations of lines in the plane. Various forms of equations of a straight line in the plane. The angle between lines. The distance from a point to a straight line.

Curves of the second order circle, ellipse, hyperbola, parabola, their geometrical properties and equations. Technical applications of geometrical properties of curves (focal properties, mathematical models of morphogenesis of biological, technical and other object).

Equations of plane and straight line in space. The angle between the planes. The angle between lines. The angle between the straight line and plane.

The equation of the surface in space. The cylindrical surface of the sphere. Cones. Ellipsoid. Hyperboloids. Paraboloids. Geometric properties of these surfaces, the study of their forms by the method of sections. Technical applications of geometric properties of surfaces (focal properties, models of building structures, physical models of elements, etc.).

Polar coordinates in the plane. The Spiral of Archimedes.

Cylindrical and spherical coordinates in space. Different ways of setting lines and surfaces in space.

Matrices and operations on them. The concept of inverse matrix.

Systems of two and three linear equations. Matrix recording system of linear equations. The Cramer's Rule. A system of n linear equations with n unknowns. Gauss Method. Finding inverse of a matrix by Gauss method.

Space R^n. Linear operations on vectors. Different standards in R^n

The scalar product in R^n.

Linear and quadratic forms in R^n. Condition sign-definiteness of quadratic forms.

The concept of linear (vector) space. Vector as an element of a linear space. Examples. Linear operators. Examples of linear operators. Examples of linear operators for modeling of various processes.

1.2. Introduction to mathematical analysis

Elements of mathematical logic: necessary and sufficient conditions. Direct and inverse theorems. The symbols of mathematical logic and their use. Binom of Newton. Abridged multiplication formula.

The set of real numbers. Basic elementary functions, their properties and graphs.

Numerical limits and their role in computing processes. The limit of the numerical sequence. Stabilization of decimal places for the members of a sequence having a limit. The existence of the limit of monotone bounded sequence.

Complex and inverse functions, their graphs. The class of elementary functions.

The limit of functions at a point. The limit of the functions of infinity. The limit of monotone functions.

The continuity of functions at a point. The continuity of elementary functions.

Infinitely small point functions, their properties. The comparison is infinitely small. Symbols ∞ and 0.

Properties of continuous functions on an interval: the boundedness, the existence of the highest and lowest values, the existence of intermediate values. Method bisects.

1.3. Differential calculus of functions of one variable.

The notion of a function, differentiable at a point, its geometric meaning. Differential functions. A General idea for how linearization.

The derivative of the function, its meaning in different tasks. The rule of finding derivative and differential. Derivative complex, and inverse functions. The invariance of differential forms. Differentiation of functions defined parametrically.

The extremum point of the function. The Theorem The Farm. The theorem of Rolle, Lagrange, Cauchy, for use. Derivative and differentials of higher orders. The Rule of L'Hopital.

The Taylor formula with the residual number in the form of Piano and Lagrangian form. Representation of functions e^x, $\sin x, \cos x, \ln(1+x), (1+x)^\alpha$ by the Taylor formula. The application of the Taylor formula in computational mathematics.

1.4. Application of differential calculus to investigation of functions and plotting their graphs.

The conditions of monotonicity of the function. The extremes of the function, a necessary condition. Sufficient conditions. Finding maximum and minimum values of functions, differentiable on the interval. Study the convexity of the function. The inflection point. Asymptotes of functions. The concept of the asymptotic expansion. The General scheme of the study of functions and the construction timetable. The concept of the curve. Examples. The equation of the tangent to the curve at the given point.

1.5 Indefinite integral.

Initial (indefinite integral), and its elementary properties. Table of basic antiderivatives.

Direct integration of functions. Integration, decomposition, parts and substitution. Examples of integration of rational functions by decomposition into partial fractions. Examples of trigonometric substitutions and the method of "rationalization" of the integrals. Examples of recurrent formulas of integration. The initial examples are not elementary functions. The use of tables, reference books, primary.

1.6 Definite integral

Tasks, leading to the concept of the integral. The integral of continuous and piecewise continuous functions as the limit of the sum theorem statement about its existence. The elementary properties of the integral, the mean value theorem. The average value of the function.

The derivative of the integral over the upper limit. The relationship between integrals and antiderivatives, the formula of Newton-Leibniz.

The evaluation of the integral using integration by parts and substitution variable.

Integral calculation by applying the Taylor formula. The use of reference integrals. Approximate calculation of integrals by the formulas trapezoidal rule and Simpson; the magnitude of the error.

Improper integrals with infinite limits, with unlimited under the integral function. Basic properties, convergence tests, absolute and don't absolute convergence. Approximate evaluation of improper integrals.

Gamma function and the functional equation for her. Beta function and its expression through gamma function

Integrals depending on parameter. Their continuity, Leibniz rule. The concept of improper integrals depending on parameter.

2. General advice student to work on a course of higher mathematics.

The main form of student learning-external students is part-time work on academic material, which consists of the following elements: studying material from textbooks, problem solving, self-test, the control works. To help extramural institutions organize lectures, practical classes and laboratory work. In addition, the student can contact the teacher with questions to obtain verbal or written advice. Instruct the student in the current work are also in the process of reviewing the control works. However, the student must remember that only systematic and persistent and independent work using the Institute will be sufficiently effective. The final stage of completion of individual parts of a mathematics course is passing the tests and examinations according to the curriculum.

Test. Preliminary results of the work of students on study course sums up reference work. The work must be done independently and serve to some extent to be a guarantee that this section is learned by the student. According to the new regulation on the subject of the audit work with the student are interviewed for the purpose of establishing the independence of its performance and identify skills in solving problems. Therefore, to perform work should be taken very seriously. The table below contains the numbers of tasks within a test. The student must perform control tasks on the options, the number of which coincides with the last digit of its training rooms (cipher).

variant	№ of test									
	1				2			3		
1	1	11	21	31	41	51	61	71	81	91	101
2	2	12	22	32	42	52	62	72	82	92	102
3	3	13	23	33	43	53	63	73	83	93	103
4	4	14	24	34	44	54	64	74	84	94	104
5	5	15	25	35	45	55	65	75	85	95	105
6	6	16	26	36	46	56	66	76	86	96	106
7	7	17	27	37	47	57	67	77	87	97	107
8	8	18	28	38	48	58	68	78	88	98	108
9	9	19	29	39	49	59	69	79	89	99	109
10	10	20	30	40	50	60	70	80	90	100	110

TEST №1

"ELEMENTS OF LINEAR VECTOR ALGEBRA AND ANALYTICAL GEOMETRY"

Methodical instructions to performance test papers No. 1.

To solve problems 1-10, you must understand the concept of a linear transformation of the space.

Example 1. Given two linear transformations of coordinates

$$\begin{cases} x_1' = 2x_1 - x_2 + 4x_3 \\ x_2' = x_1 + 3x_2 - x_3 \\ x_3' = x_1 - 2x_2 + 3x_3 \end{cases} \quad \text{and} \quad \begin{cases} x_1'' = x_1' + 2x_2' - 2x_3' \\ x_2'' = 5x_1' - x_2' - x_3' \\ x_3'' = 2x_1' + 3x_2' + x_3' \end{cases}$$

By means of matrix calculus to find the transformation that expresses X_1'', X_2'', X_3'' through X_1, X_2, X_3.

Solution. Denote the transformation matrix X в X' through A, a X' в X'' through B.

$$A = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & -1 \\ 1 & -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -2 \\ 5 & -1 & -1 \\ 2 & 3 & 1 \end{pmatrix},$$

a X, X', X'' through

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad X' = \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}, \quad X'' = \begin{pmatrix} x_1'' \\ x''_2 \\ x''_3 \end{pmatrix}.$$

Then convert X in X'' will look: $X'' = BAX$, that is

$$\begin{pmatrix} X_1'' \\ X_2'' \\ X_3'' \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 5 & -1 & -1 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & -1 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 & 7 & -1 \\ 8 & -6 & 18 \\ 8 & 5 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

or in another form

$$\begin{cases} x_1'' = 2x_1 + 7x_2 - x_3 \\ x_2'' = 8x_1 - 6x_2 + 18x_3 \\ x_3'' = 8x_1 + 5x_2 + 8x_3 \end{cases}$$

When solving problems 11-20 (systems of linear equations), is good to learn the concepts and be able to compute the rank of a matrix, to understand the theorem of Kronecker – Kapelly, to master the successive elimination of unknowns (Gauss method), to be able to construct the inverse matrix and use it to solve systems of equations.

Example 2. Given a system of linear equations

$$\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ 3x_1 + 2x_2 - 2x_3 = 1 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

To prove its consistency, and to solve it in two ways: by means of matrix calculus and the Gauss method.

Solution. Prepare a matrix A and B determine their ranks.

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & -1 & 1 \\ 3 & 2 & -2 & 1 \\ 1 & -1 & 2 & 5 \end{pmatrix}.$$

To determine the grade A use the "border". Choose one of the determinants of the matrix A. Let it

$$\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1 \quad (\neq 0)$$

(if the determinant was zero, you could take any non-zero; if all the determinants of the second order are equal to zero, then the matrix would have rank 1). Add to this the determinant of one row and one column ("Oberheim" it). Get

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 1 \quad (\neq 0) \ .$$

Because the biggest determinant of the matrix is not equal to zero, and its order is 3, then the matrix A has rank equal to three.

Rank of the matrix B no need to count, because it contains the considered determinant, and the determinant of the fourth order item to be build. So, rank of a matrix B also three, and the system (in accordance with the theorem of Kronecker – Kapelly) has a unique solution, because the number of unknowns is also three.

Will find a solution by Gauss method. We choose for the master equation is the third and swap them.

Get:

$$\begin{cases} x_1 - x_2 + 2x_3 = 5 \\ 2x_1 + x_2 - x_3 = 1 \\ 3x_1 + 2x_2 - 2x_3 = 1 \end{cases}$$

Subtract from the second equation is first multiplied by 2, and then from the third to the first multiplied by 3. We will come to an equivalent system

$$\begin{cases} x_1 - x_2 + 2x_3 = 5 \\ \quad 3x_2 - 5x_3 = -9 \\ \quad 5x_2 - 8x_3 = -14 \end{cases}$$

Multiply the second equation on $\frac{3}{5}$ and subtract from the third

$$\begin{cases} x_1 - x_2 + 2x_3 = 5 \\ \quad 3x_2 - 5x_3 = -9 \\ \qquad \frac{1}{3}x_3 = 1 \end{cases}$$

Sequentially from the third equation, we find

$x_3 = 3$, $x_2 = \frac{-9+15}{3} = 2$, $x_1 = 5 - 6 + 2 = 1$.

Thus, the solution of the original system will be:

$x_1 = 1$; $x_2 = 2$; $x_3 = 3$.

Tasks 21-30 of finding the eigenvalues and eigenvectors of the matrix actually has the same problems on solving systems of linear (but homogeneous) equations.

Example 3. To find eigenvalues and eigenvectors of the linear transformation defined by matrix A.

$$A = \begin{pmatrix} 8 & -2 & 2 \\ -2 & 5 & -5 \\ 2 & -5 & 5 \end{pmatrix} .$$

Solution. Will make the characteristic equation.

$$\begin{vmatrix} 8-\lambda & -2 & 2 \\ -2 & 5-\lambda & -5 \\ 2 & -5 & 5-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 18\lambda^2 + 72\lambda = 0$, where $\lambda_1 = 0$; $\lambda_2 = 6$; $\lambda_3 = 12$.

Substituting turn eigenvalues λ_1, λ_2, λ_3 in system

$$\begin{cases} (8-\lambda)x_1 - 2x_2 + 2x_3 = 0 \\ -2x_1 + (5-\lambda)x_2 - 5x_3 = 0 \\ 2x_1 - 5x_2 + (5-\lambda)x_3 = 0 \end{cases} ,$$

we will find $\lambda = \lambda_1 = 0$

$$\begin{cases} 8x_1 - 2x_2 + 2x_3 = 0 \\ -2x_1 + 5x_2 - 5x_3 = 0 \\ 2x_1 - 5x_2 + 5x_3 = 0 \end{cases} .$$

In this system, the third equation coincides with the second, so it can be skipped and solve the system

$$\begin{cases}4x_1-x_2+x_3=0\\2x_1-5x_2+5x_3=0\end{cases}.$$

She joint, and the solution obtained by well-known methods

$$x_1=\begin{vmatrix}-1 & 1\\-5 & 5\end{vmatrix}\cdot k=0\ ;\ x_2=\begin{vmatrix}1 & 4\\5 & 2\end{vmatrix}\cdot k=-18k;\ x_3=\begin{vmatrix}4 & -1\\2 & -5\end{vmatrix}\cdot k=-18k.$$

Thus, the first vector will be $x_1=(0;18;18)\cdot k$. In this way we find (for $\lambda=6$) $x_2=(-6;6;3)\cdot k$ i (for $\lambda=12$), $x_3=(12;-12;12)\cdot k=(1;-1;1)\cdot k$.

Solving problems 31 – 40 requires one to master the methods of vector algebra and analytic geometry. In particular, we need to remember how to find the module of the vector from the known coordinates of its beginning and end, the angle between the vectors, the angle between the straight line and plane, etc. Let set four points of space $A_1(x_1;y_1;z_1)$, $A_2(x_2;y_2;z_2)$, $A_3(x_3;y_3;z_3)$, $A_4(x_4;y_4;z_4)$. Where:

1) A_1A_2 is calculated by the formula

$$A_1A_2=\left|\overrightarrow{A_1A_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\ ;$$

2) the angle between the ribs A_1A_2 i A_1A_4 calculated by the formula

$$\cos\left(\overrightarrow{A_1A_2}\hat{,}\overrightarrow{A_1A_4}\right)=\cos\varphi=\frac{X_1X_2+Y_1Y_2+Z_1Z_2}{\sqrt{X_1^2+Y_1^2+Z_1^2}\cdot\sqrt{X_2^2+Y_2^2+Z_2^2}},$$ where

$X_1=x_2-x_1$; $Y_1=y_2-y_1$; $Z_1=z_2-z_1$; $X_2=x_4-x_1$; $Y_2=y_4-y_1$; $Z_2=z_4-z_1$;

3) equation verge $A_1A_2A_3$ (that is, the plane passing the point A_1, A_2, A_3), has the form

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\x_2-x_1 & y_2-y_1 & z_2-z_1\\x_3-x_1 & y_3-y_1 & z_2-z_1\end{vmatrix}=0;$$

4) if the equation (1.14) lead to the form $Ax+By+Cz+D=0$, the angle between the plane passing the point A_1, A_2, A_3, and direct A_1A_4,

$$\sin\varphi=\frac{|A(x_4-x_1)+B(y_4-y_1)+C(z_4-z_1)|}{\sqrt{A^2+B^2+C^2}\sqrt{(x_4-x_1)^2+(y_4-y_1)^2+(z_4-z_1)^2}};$$

5) the area of the face $A_1A_2A_3$ find out half of the module of vector product

$$S=\frac{1}{2}\left|\overrightarrow{A_1A_2}\times\overrightarrow{A_1A_3}\right|\ ;$$

6) the volume of the pyramid $A_1A_2A_3A_4$ is determined by the formula

$$V=\frac{1}{6}\left(\overrightarrow{A_1A_2}\times\overrightarrow{A_1A_3}\right)\cdot\overrightarrow{A_1A_4}\ ;$$

7) equation ribs A_1A_2 there is an equation of a line that goes through these points A_1 and A_2

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}\ ;$$

8) to find the equation of the altitude dropped from the point A_4 on the plane $A_1A_2A_3$, need point A_4 draw a straight line perpendicular to the plane $Ax+By+Cz+D=0$, that passes through three points A_1, A_2, A_3, that is, make the equation

$$\frac{x-x_4}{A}=\frac{y-y_4}{B}=\frac{z-z_4}{C}\ .$$

TEST №2

INTRODUCTION TO MATHEMATICAL ANALYSIS
DIFFERENTIAL CALCULUS OF FUNCTIONS OF ONE VARIABLE.
APPLICATION OF DIFFERENTIAL CALCULUS TO INVESTIGATION OF FUNCTIONS AND PLOTTING THEIR GRAPHS.

Literature. [4]. ch. II, § 1—5, ex. 1, 4, 6, 8—14, 18, 19;
§ 6, ex. 31—33, 35.37—40; § 7, 8, ex. 41—44, 46, 48, 49; § 9. ex. 2, 3, 21—23, 25—30, 45, 47, 57, 59; § 10, 11
ch. III, § 1, 2, ex 1, 3.4; § 3. ex. 7, 8;
§ 4-8. ex. 10, 12, 15, 16.20—22, 24.27, 29.42, 45, 71; § 9, ex. 33— 40. 43, 46—48, 50. 52, 54. 56. 58. 59.61, 64—68, 72, 74, 75, 78, 80 § 10, ex. 51, 53, 60.62, 63.79, 81; § 11, ex. 142. 143. 147, 149—151;
§ 12, ex. 83, 85, 90, 100, 101, 108, 110. 113; § 13, 14, ex. 116. 118. 120. 134,
You can also use [5], Ch. II, § 1—4; [9], Ch. I, Ch. VII

Methodical instructions to performance test №2

To solve problems it is necessary to understand the concept of limit of a function, to learn the definition of derivative, geometric and mechanical sense. Consider a typical example.

Example 1 Find $\lim\limits_{x\to\infty}\dfrac{4x^3-5x^2+3x-2}{3x^3-4x+5}$.

Solution

$$\lim_{x\to\infty}\frac{4x^3-5x^2+3x-2}{3x^3-4x+5}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{4\frac{x^3}{x^3}-5\frac{x^2}{x^3}+3\frac{x}{x^3}-\frac{2}{x^3}}{3\frac{x^3}{x^3}-4\frac{x}{x^3}+\frac{5}{x^3}}=$$

$$=\frac{4}{3}$$

Example 2 . Find $\lim\limits_{x\to 0}\frac{\sin 5x}{2x}$

Solution

$$\lim_{x\to 0}\frac{\sin 5x}{2x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\sin 5x}{5x}\bullet\frac{5x}{2x}=\frac{5}{2}\lim_{x\to 0}\frac{\sin 5x}{5x}=\frac{5}{2}$$

Here for the disclosure of uncertainty $\left(\frac{0}{0}\right)$ used the first remarkable trait.

However, more rational to use the infinitely small is equivalent to: $\lim\limits_{x\to 0}\frac{\sin 5x}{2x}$

$$=\left(\frac{0}{0}\right)=\left|\sin 5x\sim 5x\right|=\lim_{x\to 0}\frac{5x}{2x}=\frac{5}{2}$$

Example 3 . Find $\lim\limits_{x\to\infty}\left(\frac{x+2}{x-3}\right)^{2x+1}$

Solution

$$\lim_{x\to\infty}\left(\frac{x+2}{x-3}\right)^{2x+1}=\left(1^{\infty}\right)=\lim_{x\to\infty}\left(1+\frac{x+2}{x-3}-1\right)^{2x+1}$$

$$=\lim_{x\to\infty}\left(1+\frac{5}{x-3}\right)^{2x+1}=\lim_{x\to\infty}\left(1+\frac{5}{x-3}\right)^{\frac{x-3}{5}\cdot\frac{5}{x-3}\cdot(2x+1)}$$

$$=\lim_{x\to\infty}e^{\frac{10x+5}{x-3}}==e^{10}$$

Example 4 Find derivative of a function y=(sinx+4)^3
Solution. To find the derivative use the first rule of finding derivative of a complex exponential function, then the sum of two functions sinx+4:

$$y'=3(\sin x+4)^2(\sin x+4)'=3(\sin x+4)^2\cos x$$

Example 5 Find derivative of a function y'x

$$\begin{cases} x(t)=\ln ctgt \\ y(t)=\dfrac{1}{\cos^2 t} \end{cases}$$

Solution.

Find $x'_t=\dfrac{1}{ctgt}\cdot\left(-\dfrac{1}{\sin^2 t}\right)$=-$\dfrac{1}{\sin t\cos t}$

$$y'_t=-2\cos^{-3}t(-\sin t)=\frac{2\sin t}{\cos^3 t}$$

$$y'_x=\frac{y'_t}{x'_t}=\frac{2\sin t\sin t\cos t}{\cos^3 t}=-2tg^2t$$

Self-assessment

1. Specify the definition of limit of a sequence, limit of a function as the argument to some finite boundaries and limits functions when the argument approaches infinity.
2. As related to the concept of limit of a function with the concepts of limits left and right?
3. Specify the definition of a bounded function. Prove the theorem on boundedness of a function that has a limit.
4. What function is called infinitely small and what are its main properties?
5. What function is called infinitely great and what's her connection with the infinitesimal?
6. Prove basic theorems about limits of functions.
7. Write down and prove the first of a remarkable trait
8. Specify the definition of the number e (the second remarkable limit").
9. Specify the definition of continuity of a function at a point and on an interval. Specify the basic properties of functions continuous on the interval, and give a geometric interpretation to these properties.
10. Specify the definition of the order of one infinitesimal infinitesimal relative to another.
11. Specify the definition of the derivative. What is its mechanical and geometric meaning?
12. Print the formula amounts of derivative works, share two features. Bring Examples
13. Formula of differentiation of complex functions. Bring Examples.
14. Print formulas derivative of a constant and permanent works to the function.
15. Print the formula of differentiation of trigonometric and logarithmic functions.
16. Specify the rule, logarithmic differentiation. Bring Examples.
17. Print the formula of differentiation of the power function with any valid indicator, exponential functions, complex exponential functions.
18. Prove the theorem about the derivative of inverse functions. Print the formula of differentiation of inverse trigonometric functions
19. Specify the definition of the differential of the function.

Test №3

INDEFINITE AND DEFINITE INTEGRAL

Antiderivatives, the overall look primitive for this function. Indefinite integral and its properties. Table of basic integrals.The integration function. Methods of integration: substitution, retail. The decomposition of rational fractions in simplest. Integration of rational fractions, trigonometric expressions, irrationalities. Example integrals are not elementary functions.

DEFINITE INTEGRAL

Tasks, leading to the concept of the definite integral. Definite integral of a continuous function as the limit of the sum, the wording of the theorem of existence and uniqueness of the definite integral, mean value theorem.

The relationship between definite and indefinite integrals. The Rule Of Newton-Leibniz.

Calculating a definite integral using the change of variable, integration by parts.

Calculating areas in polar and rectangular coordinates. The calculation of the arc lines.

Use the definite integral to calculating volume and surface area of revolution.

The application of the definite integral Solution to problems in mechanics and physics.

To solve task: [4], №№ 692-701; 727-726; 882-885; 869-880; 989-994; 2060-2072; 2259-2263; 2455-2461; 2490-24992; 2519-2522; 2555-2557; 2594-2596; 2649-2651; 2670; 2674; 2686; 2693; 2695].

Methodical instructions to performance test №3

Test " Indefinite and definite integral" is to identify the techniques of the original (calculation of an indefinite integral and a skillful application of the definite integral to calculate some physical, mechanical and geometrical quantities.

The work consists of nine tasks, the first six of which relate to the theme of "Indefinite integral", the other three — on applications of the definite integral. Consider a typical examples.

Example 1. Calculate the integrals:

1. $I=\int \frac{\sqrt{4+\ln x}}{x}dx$

Here is the integral for the table substitute: $U=4+\ln x;\ dU=\frac{1}{x}dx.$

Then $I=\begin{vmatrix} U=4+\ln x \\ dU=\frac{1}{4}dx \end{vmatrix}=\int\sqrt{U}\ dU=\frac{U^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{2}{3}\sqrt{(4+\ln x)^3}+C$.

Hereinafter, when calculating an indefinite integral in the answer, we turn to the original variable.

2. $I=\int e^{x^2+6x}(x+3)dx$.

If we denote $U=x^2+6x$, то $dU=(2x+6)dx=2(x+3)dx$, the original integral will immediately table view

$$I=\begin{vmatrix} U=x^2+6x \\ dU=2(x+3)dx \end{vmatrix}=\frac{1}{2}\int e^u dU=\frac{1}{2}e^U+C=\frac{1}{2}e^{x^{2+6x}}+C\ .$$

3. $I=\int x^2\sin x\ dx$.

The integral is taken using the formula of integration by parts:

$\int udv=uv-\int vdu.$

Apply the formula of integration by parts:

$$I=\begin{vmatrix} u=x^2;\ du=2xdu \\ dv=\sin xdx;\ v=-\cos x \end{vmatrix}=-x^2\cos x+\int 2x\cos xdx$$

To compute these integral formulas of integration by parts should be used again. We denote:

$$\int x\cos xdx\begin{vmatrix} u=x;\ du=dx \\ dv=\sin xdx;\ v=\sin x \end{vmatrix}=x\sin x-\int\sin dx=x\sin x+\cos x+C$$

Substituting, have $I=-x^2\cos x+x\sin x+\cos x+C$

Continuous integration with contains both the constant that appears in a double application of integration by parts.

4. $I=\int \frac{x+4}{x^2+5x+7}dx$

Find the derivative of the denominator $\left(x^2+5x+7\right)'=2x+5$ and select it in the numerator: $x+4=\frac{1}{2}(2x+8)=\frac{1}{2}[(2x+5)+3]$, then

$$I=\frac{1}{2}\int\frac{(2x+5)+3}{x^2+5x+7}dx=\frac{1}{2}\int\frac{2x+5}{x^2+5x+7}dx+\frac{3}{2}\int\frac{dx}{x^2+5x+7}=\frac{1}{2}I_1+\frac{3}{2}I_2\ .$$

When calculating I_1 we denote $u=x^2+5x+7$; $du=(2x+5)dx$

$$I_1=\int\frac{2x+5}{x^2+5x+7}dx=\int\frac{du}{u}=\ln|u|+C=\ln\left|x^2+5x+7\right|+C\ .$$

To calculate I_2 scroll to complete the square in the denominator of the integrand of the expression:

$x^2+5x+7=x^2+2\frac{5}{2}x+\frac{25}{4}-\frac{25}{4}+7=\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$, then

$$I_2=\int\frac{dx}{x^2+5x+7}=\int\frac{dx}{\left(x+\frac{5}{2}\right)^2+\frac{3}{4}}=\begin{vmatrix}u=x+\frac{5}{2}\\ du=dx\end{vmatrix}=\int\frac{du}{u^2+\frac{3}{4}}=\frac{2}{\sqrt{3}}arctg\frac{2u}{\sqrt{3}}+C=$$

$$=\frac{2}{\sqrt{3}}arctg\frac{2(x+\frac{5}{2})}{\sqrt{3}}+C.$$

Combine results:

$$I=\frac{1}{2}\ln\left|x^2+5x+7\right|+\sqrt{3}\arctan\frac{2x+5}{\sqrt{3}}+C.$$

5. $I=\int\sin^7 x\cdot\cos^2 x\ dx=\int\left(1-\cos^2 x\right)^3\cdot\cos^2 x\cdot\sin x\ dx.$

Obvious replacement $u=\cos x$; $du=-\sin x\ dx$.

$$I=-\int\left(1-u^2\right)^3\cdot u^2du=-\int\left(1-3u^2+3u^4-u^6\right)\cdot u^2du=-\frac{u^3}{3}+3\frac{u^5}{5}-3\frac{u^7}{7}+\frac{u^9}{9}+C=$$

$$=-\frac{\cos^3 x}{3}+\frac{3}{5}\cos^5 x-\frac{3}{7}\cos^7 x+\frac{1}{9}\cos^9 x+C.$$

6. $I=\int \frac{dx}{\sqrt{x+3}\cdot\left(1+\sqrt[3]{x+3}\right)}$. The integral contains the irrationality. We denote $x+3=t^6$; $dx=6t^5dt$;

$$I=\int\frac{6t^5dt}{t^3\left(1+t^2\right)}=6\int\frac{t^2}{1+t^2}dt=6\int dt-6\int\frac{dt}{1+t^2}=6t-6\arctan t+C=6\sqrt[6]{x+3}-$$
$$-6\arctan\sqrt{x+3}+C.$$

Example 2 Consider using the definite integral in geometry, mechanics and physics.

Solution. Fig. 2.1 shows that it is easier to calculate the area 1/4 throughout the figures, the parameter t changes from 0 to $\pi/2$. The curve given in parametric form $x=y(t)$; $y=\psi(t)$; $t_1\le t\le t_2$. Calculate the area of a figure bounded by astrodog $x=3\cos^3 t$; $y=3\sin^3 t$.

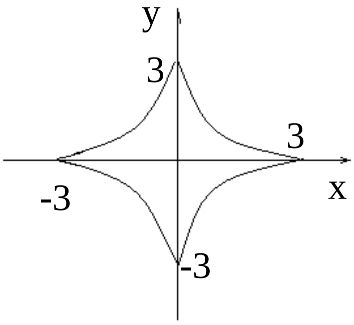

Fig. 2.1.

The formula for calculating the area of plane figures bounded by a curve given in parametric form

$$S=\int_{t_1}^{t_2}\psi(t)\cdot\varphi'(t)dt.$$

In our case $\varphi'(t)=3\cdot3\cos^2 t\cdot(-\sin t)=-9\cos^2 t\sin t$

$$\frac{1}{4}S=-\int_0^{\pi/2}3\sin^3 t\cdot(-9)\cdot\cos^2 t\cdot\sin t\ dt=27\int_0^{\pi/2}\sin^4 t\cos^2 t\ dt=27\int_0^{\pi/2}\left(\frac{1-\cos2t}{2}\right)^2\times$$
$$\times\left(\frac{1+\cos2t}{2}\right)dt=\frac{27}{8}\int_0^{\pi/2}\left(1-2\cos2t+\cos^2 2t\right)\cdot\left(1+\cos2t\right)dt=$$
$$=\frac{27}{8}\int_0^{\pi/2}\left(1-\cos2t-\cos^2 2t+\cos^3 2t\right)dt=$$
$$=\frac{27}{8}t\Big|_0^{\pi/2}-\frac{27}{8}\frac{t}{2}\sin2t\Big|_0^{\pi/2}-\frac{27}{8}\int_0^{\pi/2}\frac{1+\cos4t}{2}dt+\frac{27}{8}\int_0^{\pi/2}\left(1-\sin^2 2t\right)\cdot\cos2t\ dt=$$
$$=\frac{27}{8}\cdot\frac{\pi}{2}-\frac{27}{8}\cdot\frac{1}{2}t\Big|_0^{\pi/2}-\frac{27}{8}\cdot\frac{1}{8}\cdot\frac{1}{4}\sin4t\Big|_0^{\pi/2}+\frac{27}{8}\cdot\frac{1}{2}\sin2t\Big|_0^{\pi/2}-\frac{27}{8}\cdot\frac{\sin^3 2t}{3}\Big|_0^{\pi/2}=\frac{27}{32}\pi.$$

Example 3. Find the coordinates of the center of gravity of the homogeneous shapes bounded by lines: $\sqrt{x}+\sqrt{y}=2$; x=0, y=0.

Solution. In figure 2.2, which shows the figure, it is seen that one is symmetrical about the line y=x (points lying on the parabola symmetrical to the straight line). Therefore, the center of gravity of this figure lies on this line, that is $x_c = y_c$. Use the formula:

$$X_c = \frac{M_y}{M}; \quad Y_c = \frac{M_x}{M};$$

$$M = \int_a^b \gamma(y_b - y_n)dx; \qquad M_y = \int_a^b \gamma(y_b - y_n)dx;$$

In our case $y_b = (2-\sqrt{2})^2;\ Y_m = 0$ the limits of integration a=0, b=4, $\gamma = 1$ (uniform figure).

$$M = \int_0^4 (2-\sqrt{2})^2 dx = \int_0^4 (4-2\sqrt{2}+x)dx = (4x - 2\frac{x^{3/2}}{3/2} + \frac{x^2}{2})|_0^4 = 8/3$$

Calculate the statistical moment about the axis OY.

$$M_y = \int_0^4 x(2-\sqrt{2})^2 = \int_0^4 (4x - 2x^{2/3} + x^2)dx = (4\frac{x^2}{2} - 2\frac{x^{5/2}}{5/2} + \frac{x^3}{3})|_0^4 = 32/15$$

$x_c = \frac{32}{15} : 8/3 = 4/5;\ y_c = x_c = 4/5$ – the coordinates of the center of gravity of the considered shape.

Example 4. A vertical dam has the formula of a triangle with the base α, which coincides with the liquid surface, and height h. To determine the effect of water pressure on the dam.

Solution. Coordinate system position, aligning the OY axis with the base of the dam, and the OX axis is height. (Fig. 2.3). Determine the pressure force to the strip of width dx located at a depth x. We believe that this strip has the shape of a rectangle. Error in this situation was because dx is the thickness of the plate $dp = \gamma x dS$, where $\gamma = C$ - the density of the fluid $dS = MN \cdot dx.$ From the similarity of triangles TWB

$$MN = \frac{a}{h}(h-x),$$

$$dP = \gamma x \frac{a}{h}(H-x)dx.$$

The pressure force on the entire dam will receive, integrating over d 0 to h. Note that we consider the error made when replacing the real elementary form of a rectangular plate:

$$P = \int_0^h \gamma x \frac{a}{h}(h-x)dx = \frac{\gamma a}{h}\int_0^h (hx^2 - x^2)dx = \frac{\gamma a}{h}(h\frac{x^2}{2} - \frac{x^3}{3})\Big|_0^h = 1/6\gamma a h^2.$$

Self-assessment

1. What is the definition antiderivatives (indefinite integral).
2. Prove that any two initial for the same functions differ by an arbitrary constant.
3. Bring a table of indefinite integrals.
4. We will state and prove the properties of the indefinite integral.
5. Methods of replacing a variable in the indefinite integral. Give Examples.
6. Prove the formula for integration by parts an indefinite integral.
7. Specify methods of integrating expressions containing trigonometric functions.
8. Outline of the simplest methods of integration of irrational expressions.
9. Prove the theorem on the decomposition of a proper rational fraction in the simplest case, a simple valid roots of the denominator.
10. Prove the theorem on the decomposition of a proper fraction in simplest in the case of multiple valid roots of the denominator.
11. Describe the rule of integration of rational fractions.
12. Give Examples of tasks that lead to the concept of "definite integral".
13. Specify the theorem of existence and uniqueness of the definite integral.
14. Prove properties of the definite integral and specify their geometric meaning.
15. Formula of Newton-Leibniz.
16. Prove the mean value theorem for definite integral.
17. Formula of change of variable in a definite integral.
18. Formula of integration by parts definite integral.
19. Describe the main geometrical applications of definite integral.
20. Describe the main mechanical applications of the definite integral.
21. Describe the main physical applications of the definite integral.
22. Give the definition of improper integrals of the first and second kind, specify their convergence.

Test task №1

1-10. Given two linear transformations

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ x_2' = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ x_3' = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases} \qquad \begin{cases} x_1'' = b_{11}x_1' + b_{12}x_2' + b_{13}x_3' \\ x_2'' = b_{21}x_1' + b_{22}x_2' + b_{23}x_3' \\ x_3'' = b_{31}x_1' + b_{32}x_2' + b_{33}x_3' \end{cases}$$

By means of matrix calculus find the transformation that expresses X_1'', X_2'', X_3'' X_1, X_2, X_3.

The original data are shown in table 1.1.

Table 1.1.

№ tasks	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
1	4	3	5	6	7	1	9	1	8	-1	3	-2	-4	1	2	3	-4	5
2	1	-1	-1	-1	4	7	8	1	-1	9	3	5	2	0	3	0	1	-1
3	7	0	4	0	4	-9	3	1	0	0	1	-6	3	0	7	1	1	-1
4	0	2	0	-2	3	2	4	-1	5	-3	0	1	0	2	1	0	-1	3
5	3	-1	5	1	2	4	3	2	-1	4	3	1	3	1	2	1	-2	1
6	4	3	2	-2	1	-1	3	1	1	1	-2	-1	3	1	2	1	2	2
7	4	3	8	6	9	1	2	1	8	-1	8	-2	-4	3	2	3	-8	5
8	1	-3	4	2	1	-5	-3	5	1	4	5	-3	1	-1	-1	7	0	4
9	3	0	5	1	1	1	0	3	-6	2	-1	-5	7	1	4	6	4	-7
10	1	2	2	0	-3	1	2	0	3	3	1	0	1	-2	-1	0	3	2

11-20. Given a system of linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}.$$

To prove its consistency, and to solve it in two ways: by Gauss Method and means of matrix calculus

The source data to the tasks listed in table 1.2.

Table 1.2

№ tasks	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	b_1	b_2	b_3
11	2	-4	1	1	-5	3	1	-1	1	3	-1	1
12	2	-3	4	3	-2	6	2	-3	-5	-4	-1	-13
13	4	3	-9	2	3	-5	1	8	-7	9	7	12
14	5	-8	-2	3	-12	-5	2	-6	-1	-6	-4	0
15	3	5	-1	1	-4	2	1	3	-1	0	2	4
16	2	5	-8	4	3	-9	2	3	-5	8	9	7
17	1	-2	-3	2	3	-1	3	-2	-5	6	20	6
18	1	1	-1	8	3	-6	4	1	-3	1	2	3
19	7	-5	0	4	0	11	2	3	4	31	-43	-20
20	1	2	4	5	1	2	3	-1	1	31	20	10

21-30. Find eigenvalues and eigenvectors of the linear transformation defined by matrix A.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

These tasks are given in table 1.3.

Table 1.3

№ tasks	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}
21	2	-1	-1	0	-1	0	0	2	1
22	4	-5	7	1	-4	9	-4	0	5
23	2	-1	2	5	-3	3	-1	0	-2
24	0	1	0	-3	4	0	-2	1	2
25	1	-3	3	-2	-6	13	-1	-4	8
26	5	6	3	-1	0	1	1	2	-1
27	4	-5	2	5	-7	3	6	-9	4
28	0	7	4	0	1	0	1	13	0
29	7	0	0	10	-19	10	12	-24	13
30	3	1	0	-4	-1	0	4	-8	-2

31-40. Given the coordinates of the vertices of the pyramid $A_1A_2A_3A_4$ (Table 1.4). Find: 1) length of rib A_2A_3; 2) the angle between the ribs A_1A_2 and A_1A_4; 3) the plane equation $A_1A_2A_3$; 4) the area of the face $A_1A_2A_3$; 5) the volume of the pyramid; 6) equation ribs A_1A_2; 7) the equation of the altitude dropped from the vertex A_4 on the brink $A_1A_2A_3$.

The source data to the tasks listed in table 1.4.

Table 1.4

№ tasks	A_1	A_2	A_3	A_4
31	-1; 4; 2	2; 0; 3	3; 1; 0	4; 0; 5
32	2; 1; 2	0; 0; 3	1; 2; 4	-1; 1; 3
33	3; 2; 1	0; 2; 0	2; 5; 3	0; 4; 0
34	0; 3; 6	2; -3; 1	1; 2; 1	3; 2; 2
35	-1; 0; 2	0; 4; 0	2; 1; 3	2; 3; 1
36	-2; 1; 1	1; 4; 0	2; 4; 1	0; 2; 0
37	0; 1; -2;	2; 3; 4	5; 1; 1	3; 1; 4
38	3; -1; 0	0; 0; 2	2; -1; 4	3; 1; 5
39	5; 1; 2	1; 0; 5	-2; 1; -3	-4; 0; 5
40	-1; 1; 4	0; -5; -1	3; 1; 0;	2; 2; 5;

Test tasks №2

Find the limit of a function

41. a) $\lim\limits_{x\to\infty}\dfrac{3x^5+4x^2+1}{5x^5-x^3+x^3}$; b) $\lim\limits_{x\to 2}\dfrac{\sqrt{6-x}-\sqrt{2+x}}{x^2-x-2}$;

c) $\lim\limits_{x\to 0}\dfrac{1-\cos^3 2x}{x^2}$; d) $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x-4}\right)^{4+x}$;

42. a) $\lim\limits_{x\to\infty}\dfrac{8x^7-3x^2+5}{1-x^2+x^7}$; b) $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{\sqrt{3x+16}-5}$;

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{1-\cos 3x}$; d) $\lim\limits_{x\to 1}(5-4x)^{\frac{2x}{x-1}}$;

43. a) $\lim\limits_{x\to\infty}\dfrac{3x^{11}-11x^5-5}{5-x-6x^{11}}$; b) $\lim\limits_{x\to 0}\dfrac{x^2+5x}{\sqrt{x+1}-\sqrt{1-x}}$;

c) $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 3x}{1-\cos x}$; d) $\lim\limits_{x\to 1}(3-2x)^{\frac{x}{1-x}}$;

44. a) $\lim\limits_{x\to\infty}\dfrac{5+x+3x^7}{4x^7-x^3+1}$; b) $\lim\limits_{x\to 7}\dfrac{\sqrt{2+x}-3}{x^2-3x-28}$;

c) $\lim\limits_{x\to 0}\dfrac{\sin 2x-\sin x}{\arcsin x}$; d) $\lim\limits_{x\to 2}(3x-5)^{\frac{x}{2x-4}}$;

45. a) $\lim\limits_{x\to\infty}\dfrac{3x^5+7x+1}{2-x+5x^5}$; b) $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{\sqrt{1+3x}-\sqrt{2x+2}}$;

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{x\sin x}$; d) $\lim\limits_{x\to\infty}\left(\dfrac{x+7}{x+5}\right)^{2x+2}$;

46. a) $\lim\limits_{x\to\infty}\dfrac{2x^2+3x+5}{2x^2+x+4}$; b) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+8}-\sqrt{10-x}}{2x^2+5x-7}$;

c) $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{5x\sin 3x}$; d) $\lim\limits_{x\to\infty}\left(x[\ln(x+4)-\ln x]\right)$;

47. a) $\lim\limits_{x\to\infty}\dfrac{9x^4-3x^3+7}{2x^4+5x^2-1}$; b) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-5x+4}$;

c) $\lim\limits_{x\to 0}\dfrac{\tan^2 5x}{\sin^2 x}$; d) $\lim\limits_{x\to 2}(5-2x)^{\frac{x}{2-x}}$;

48. a) $\lim\limits_{x\to\infty}\dfrac{7+5x-4x^3}{5x^3+x^2-1}$; b) $\lim\limits_{x\to 1}\dfrac{\sqrt{1-x^2}-1}{x^3+2x}$;

c) $\lim\limits_{x\to 2}\dfrac{\sin(x-2)}{x^2-x-2}$; d) $\lim\limits_{x\to 2}\left(\dfrac{5x+1}{5x-1}\right)^{2x}$;

49. a) $\lim\limits_{x\to\infty}\dfrac{1-7x^2}{5x^2-x+1}$; b) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt{1-2x}}{7x+x^2}$;

c) $\lim\limits_{x\to 0}\dfrac{x\tan 5x}{\sin^2 x}$; d) $\lim\limits_{x\to\infty}(x-2)[\ln(x-5)-\ln x]$;

50. a) $\lim\limits_{x\to\infty}\dfrac{x^9+9}{3+x^7+5x}$; b) $\lim\limits_{x\to 5}\dfrac{\sqrt{x+4}-3}{x^2-4x-5}$;

c) $\lim\limits_{x\to 0}\dfrac{x^2\tan^{-1}2x}{\sin 3x}$; d) $\lim\limits_{x\to\infty}x[\ln(x+1)-\ln x]$;

51–60. Find derivative $\dfrac{dy}{dx}$ for the given functions a), b), c) and derivative $\dfrac{d^2y}{dx^2}$ – for the functions d).

51. a) $y=\arccos\dfrac{\sqrt{2}}{x}+\dfrac{x}{\sqrt{2}}\sqrt{x^2-2}$; b) $y=(\ln x)^{\sqrt{x}}$;

c) $y=ctg(xy)$; d) $\begin{cases} x=\dfrac{t}{t-1}; \\ y=\dfrac{t^2}{t-1},\ t\neq 1 \end{cases}$

52. a) $y=\ln\cos x+0.5tg^2x$; b) $y=(x^2+1)^{arctgx}$;

c) $(x-y)e^x=e^y$; d) $\begin{cases} x=a\cos^3 t; \\ y=a\sin^3 t,\ t\in(0,\pi). \end{cases}$

53. a) $y=arctg\dfrac{x-1}{x+2}+0.5\ln(x^2+1)$; b) $y=(x^2+1)^{\cos x}$;

c) $\sqrt{y}-\sqrt[3]{x}=2;$

d) $\begin{cases} x=e^{y} \\ y=e^{t}\cos t, \ t\in(0,\pi). \end{cases}$

54. a) $y=e^{2x^2}(3+2x^2+2x^4);$

b) $y=(tg\sqrt{x})^{2x};$

c) $3y=\sin(x-y);$

d) $\begin{cases} x=t+arctgt; \\ y=\ln(1+t^2), \ t\in(0,\infty). \end{cases}$

55. a) $y=\ln\dfrac{\sqrt{x+1}+1}{\sqrt{x+1}-1};$

b) $y=(\arcsin x)^{0.5x};$

c) $\cos y=x-y;$

d) $\begin{cases} x=a\sin 2t; \\ y=b\cos 2t, \ t\in(0,\infty). \end{cases}$

56. a) $y=\dfrac{2}{ctgx-1}+\ln tgx;$

b) $y=(x-5)^{\sin 3x};$

c) $x-y=e^{x+y};$

d) $\begin{cases} x=t^3-1 \\ y=\dfrac{1}{3}(t^2-1), \ t\in(0,\infty). \end{cases}$

57. a) $y=(\arccos\dfrac{1}{x})^3+\sqrt{x^2-1};$

b) $y=\left(\sqrt{x}\right)^{\ln 2x};$

c) $y^2=x\sin y;$

d) $\begin{cases} x=\sin t \\ y=\ln 2t, \ t\in(0,\infty). \end{cases}$

58. a) $y=arcctgx-\ln\sqrt{\dfrac{1+x}{1-x}};$

b) $y=(x^3+1)^{ctg2x}$

c) $e^{y}\sin y=\cos x;$

d) $\begin{cases} x=a\arccos t; \\ y=e^{\sqrt{1-t^2}}, \ t\in[-1, 1]. \end{cases}$

59. a) $y=4\sqrt{x^2-4}-(\arcsin\dfrac{2}{x})^2;$

b) $y=(\ln 3x)^{2/3};$

c) $\cos(xy)-3x=0;$

d) $\begin{cases} x=2ctgt; \\ y=\ln\sin t, \ t\in(0,\pi). \end{cases}$

60. a) $y=(x^2+4)arctg\dfrac{x}{2};$

b) $y=(\sin 2x)^{\sqrt{x}};$

c) $\ln y + y/x = 2;$ d) $\begin{cases} x = 3e^{-t}; \\ y = e^{2t},\ t \in (0, \infty) \end{cases}$

61–70. Investigate the methods of differential calculus function $y = f(x)$; using the results of the study, graph can be plotted.

61. a) $y = \frac{x}{x^2+1};$ b) $y = \frac{\ln x - 1}{x}.$

62. a) $y = x + \frac{x}{2};$ b) $y = x + e^{-x}.$

63. a) $y = \frac{x^2}{x\ \ +1};$ b) $y = \ln(x^2 - 4).$

64. a) $y = \frac{x^2-1}{x^2-4};$ b) $y = xe^{-x/2`}.$

65. a) $y = x + \frac{4}{x^2};$ b) $y = \ln\frac{x-1}{x+1}.$

66. a) $y = \frac{(x-1)^2}{x};$ b) $y = xe^x.$

67. a) $y = \frac{x}{2-x^2};$ b) $y = \ln\frac{x}{x-1}.$

68. a) $y = x^4 - 2x^2 + 3;$ b) $y = \ln(x^2 + 1).$

69. a) $y = x - \frac{1}{x};$ b) $y = xe^{-x}.$

70, a) $y = \frac{x}{(x+1)^2};$ b) $y = \frac{1}{e^x - 1}.$

Test tasks №3

Calculate the indefinite integral:

71. a) $\int \frac{\sin 3x}{\sqrt{\cos^3 3x}}dx$; $\int \frac{5x^2}{5-2x^3}dx$; $\int x\cos 5x dx$; $\int \frac{(2-x)}{\sqrt{8-x^2-2x}}dx$; $\int \frac{dx}{2+tgx}$; $\int \frac{dx}{\sqrt[3]{3x+1}-1}$;

72. $\int \frac{\cos x}{2\sin x-5}dx$; $\int \frac{x^2}{\sqrt{x^3-4}}dx$; $\int \frac{\ln(2x+1)}{x^3}dx$; $\int \frac{x}{x^2-7x+13}dx$; $\int \frac{dx}{1-\sin^2 x}$; $\int \frac{\sqrt{x}}{\sqrt{x}+1}dx$;

73. $\int \frac{x^3}{\sqrt{1-x^3}}dx$; $\int x3^x dx$ $\int \frac{(3x-7)}{x^3+4x^2+4x+16}dx$; $\int \frac{1}{\sqrt{x+3}+\sqrt[3]{(x+3)^2}}dx$;

74. $\int \frac{1}{\cos^2 x(3tgx+1)}dx$; $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx$; $\int \frac{1}{x^3+x^2+2x+2}dx$; $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$;

75. $\int \frac{\cos 3x}{4+\sin 3x}dx$; $\int x^2e^{3x}dx$; $\int \frac{x^2}{x^3+5x^2+8x+4}dx$; $\int \frac{\cos x}{1+\cos x}dx$;

76. $\int \frac{\sin x}{\sqrt[3]{\cos^2 x}}dx$; $\int x\arcsin\frac{1}{x}dx$; $\int \frac{(x+3)}{x^3+x^2-2x}dx$; $\int \frac{\sqrt[4]{x+1}}{(\sqrt{x}+4)\sqrt[4]{x^3}}dx$;

77. $\int \frac{x+arctgx}{1+x^2}dx$; $\int x\ln(x^2+1)dx$; $\int \frac{x^2-3}{x^4+5x^2+6}dx$; $\int \frac{\sqrt{x+5}}{1+\sqrt[3]{x+5}}dx$;

48. $\int \frac{arctg\sqrt{x}}{\sqrt{x}(1+x)}dx$; $\int x\sin x\cos x dx$; $\int \frac{x^2}{x^4-81}dx$; $\int \frac{1}{3\cos x+4\sin x}dx$;

79. $\int \frac{\sin x}{\sqrt[3]{3+2\cos x}}dx$; $\int x2\sin 4x dx$; $\int \arcsin x dx$; $\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}+1}dx$;

80. $\int \frac{ax}{(1+x^2)arctgx}dx$; $\int e^{3-5x}dx$; $\int \frac{x}{\sqrt{5x^2-2x+1}}dx$; $\int \frac{\sin x}{1-\sin x}dx$;

Using the concept of the definite integral to solve problems:

81. Find the area of the figure, bounded by one arch of the cycloid $x=t-\sin t;\ y=1-\cos t$
82. Calculate the area of the figure, bounded by a loop of the curve $x=3t^2;\ y=3t-t^3$
83. Calculate the area of the figure, bounded by a line $r=3+\sin\varphi$
84. Calculate the area of the figure, bounded cardioid $r=3\,(1+\cos\varphi)$ i circle $r=3\cos\varphi$
85. Calculate the area of the figure, bounded curve $r=5(1+\sin\varphi)$
86. Find the area of the figure, bounded snail of Pascal

87. Calculate the area of the figure, bounded curve $x = 5\cos t,\ y = 4\sin t$
88. Calculate the area of the figure, bounded lines $x = 4\cos 3\varphi$ i $r = 2$
89. Calculate the area of the figure, bounded line $r = 5\sin 3\varphi$
90. Calculate the area of the figure, bounded line $r = 4\cos 2\varphi$

Find the coordinates of the center of gravity of a homogeneous plane figures, bounded lines:

91. $\frac{x^2}{4} + \frac{y^2}{1} = 1;\ x = 0;\ y = 0;\ x \geq 0;\ y \geq 0$
92. $y^2 = 4x;\ x2 = 4y$
93. $4y = x^2;\ y = 2$
94. $x^2 + y^2 = 9;\ \ y = 0;\ (y \geq 0)$
95. $x^2 + y^2 + 2x = 0;\ \ y = 0;\ (y \geq 0)$
96. $x^2 = 4y - 16 = 0;\ y = 0$
97. $y^2 = -x + 9;\ x = 0$
98. $y = x^2;\ y = 2 - x^2$
99. $y = \ln x;\ y = 0;\ x = e$
100. $y = \sqrt{4 - x^2;}\ \ y = 0;\ y \leq 0$

In solving tasks 101-110 use a definite integral.

101. Calculate the work that must be expended to pump water from a conical vessel, with its apex directed downward, the base radius equal to R and a height H.
102. Compression of a spring is proportional to the applied force. To calculate the force of compression springs 0.08 m if to compress it on 0.01 m power required 5 H.
103. To determine the water pressure at a vertical parabolic segment whose base is 4 m and is located on the surface of the water, and the top is at a depth of 4 m.
104. A vertical dam has a trapezoid shape. To calculate the force of water pressure on the dam, if it is known that its upper base b=50 m, height 20 m and the upper base coincides with the water level.
105. Find the work required to pump water from a cylindrical tank having a base radius 2 m and height 3 m.
106. What kind of work must be expended to body weight m to raise from the Ground surface to a height of h, assume that the force of attraction of the body of the Earth $F = K\frac{mM}{r^2}2$

107. To estimate the force of fluid pressure on a vertical wall in the shape of a half ellipse with axes 2a and 2b, immersed in the liquid (specific gravity $\gamma = I$) so that the upper border of the plate coincides with the liquid surface.
108. Find work for pumping water from a trough, having the form of a semicylinder, $R = 2$ m, long $l = 6$ m.
109. A cylinder with a movable piston diameter $D = 0{,}2$ m and long filled $l = 0.8$ m with steam at a pressure $P = 10$ kg/cm^2. What kind of job you have to spend that at constant temperature the volume of steam to reduce in 2 times?
110. Find the amount of heat, allocated sinusoidal current $I = I_0 \sin(\omega t + \varphi)$ in a conductor with resistance R during the period $T = 2\pi / \omega$, if it is known that when a constant current allocated per time t the amount of heat is determined by the formula $Q = 0.24 I^2 Rt$

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Compilers:
Glushkov O.V., D.f.-m.s.(Hab.Dr.), prof.,
Khetselius O.Yu., D.f.-m.s.(Hab.Dr.), prof.,
Sukharev D.E., C.f.-m.s.(PhD), assoc.-prof.,
Buyadzhi V.V., assoc.-lect.

Editor:
Glushkov O.V., d.f.-m.s. (Hab.Dr.), prof., Head of the department of higher and applied mathematics OSENU

Odessa State Environmental University
65016, Odessa, L'vovskaya str., 15, Room 408 (1st bld.)

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________________ Хохлов В.М.

«Узгоджено»
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________________ Волошина О.В.

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________________Глушков О.В.

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