DOI: http://dx.doi.org/10.15673/2072-9812.2/2015.51584
Proc. Intern. Geom. Center 2015 **8**(2) 58-64

Geometry of Chaos: Advanced computational approach to treating chaotic dynamics of some hydroecological systems II

A.V. Glushkov, V.M. Kuzakon, Yu.Ya.Bunyakova, V.V.Buyadzhi

Abstract This paper goes on our work on application of the chaos theory and non-linear analysis technique to studying chaotic features of different nature systems. Here we present new results of using an advanced chaos-geometric approach to treating chaotic pollution dynamics in the hydroecological systems, in particular, forested watersheds. Generally, an approach combines together application of the advanced mutual information scheme, Grasberger-Procachi algorythm, Lyapunov exponent's analysis etc.

Keywords geometry of chaos, non-linear analysis, nature system

Mathematics Subject Classification: (2000) 55R01-55B13

1. Introduction

In this paper we go on our work on application of the chaos theory and non-linear analysis technique to studying chaotic features of different nature systems (see, for example [1,2]). The theoretical basis's of the chaos-geometric combined approach to treating of chaotic behaviour of complex dynamical systems are in details in series of ref. [1-10]. Generally, an approach combines together application of the advanced mutual information scheme, Grassberger-Procachi algorythm, Lyapunov exponent's analysis etc [1-23]. It is important to note that our advanced approch has been successfully applied to studying dynamics not only mathmeatical and physical systems. Very impressive application is the investigated dynamics of the atmospheric pollutants concentrations and forecasting their temporal evolution. Besides, in Ref [2,4] we have numerically studied the chaotic features of the pollutants concentration time series for some hydroecological systems, in particular, nitrates (sulphates) pollution concentration for a number of the forested watersheds of the Small Carpathian (for example,Blatina (Pezinok), Parna (Majdan), Ladomirka (Svidnik), Babie (Olsavka) etc.). It has been noted that the successful application of new chaos-geometrical approach to studying dynamics of the different nature systems demonstrates its universal character. Here we present the analogous numerical results of using an advanced chaos-geometric approach to treating the nitrates pollution dynamics for other forested watersheds with revealing the chaos elements in the temporary sets of the nitrates and sulphates concentrations.

2. An advanced chaos-geometrical approach to hydroecological system dynamics: Short review

As all details of a new chaos-geometric approach have been descibed in our previous papers (see, for example,[1-8]) below we shortly present only the key positions, which are important for the further listing numerical results. As usually, following to [1-10], further we formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is a start time, Δt is time step, and n is number of the measurements. In a general case, $s(n)$ is any time series (f.e. nitrates pollution concentration). As processes resulting in a chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using as well as possible information contained in $s(n)$. Such reconstruction results in set of d-dimensional vectors $\mathbf{y}(n)$ replacing scalar measurements. The main idea is that direct use of lagged variables $s(n+\tau)$, where τ is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a collection of time lags to create a vector in d dimensions, $\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau), .., s(n+(d-1)\tau)]$, the required coordinates are provided. In a nonlinear system, $s(n+j\tau)$ are some unknown nonlinear combination of the actual physical variables. The dimension d is the embedding dimension, d_E.

Let us remind that following to [2,10], the choice of proper time lag is important for the subsequent reconstruction of phase space. If τ is chosen too small, then the coordinates $s(n+j\tau)$, $s(n+(j+1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n+j\tau)$, $s(n+(j+1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under-or overestimated. One needs to choose some intermediate position

between above cases. First approach is to compute the linear autocorrelation function $C_L(\delta)$ and to look for that time lag where $C_L(\delta)$ first passes through 0. This gives a good hint of choice for τ at that $s(n+j\tau)$ and $s(n+(j\ +1)\tau)$ are linearly independent. It's better to use approach with a nonlinear concept of independence, e.g. an average mutual information. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i,\ b_k)$. In ref. [4] it is suggested, as a prescription, that it is necessary to choose that τ where the first minimum of $I(\tau)$ occurs.
In [1-4,10] it has been stated that an aim of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E\ >\ d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. If the time series is characterized by an attractor, then correlation integral $C(r)$ is related to a radius r as $d = \lim_{r\to 0, N\to\infty} \frac{\log C(r)}{\log r}$, where d is correlation exponent.
The fundamental problem of theory of any dynamical system is in predicting the evolutionary dynamics of a chaotic system. Let us remind following to [1-,2,10] that the cited predictability can be estimated by the Kolmogorov entropy, which is proportional to a sum of positive LE. As usually, the spectrum of LE is one of dynamical invariants for non-linear system with chaotic behaviour. The limited predictability of the chaos is quantified by the local and global LE, which can be determined from measurements. The LE are related to the eigenvalues of the linearized dynamics across the attractor. Negative values show stable behaviour while positive values show local unstable behaviour. For chaotic systems, being both stable and unstable, LE indicate the complexity of the dynamics. The largest positive value determines some average prediction limit. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and hence the choice of trajectory, and they do comprise an invariant measure of the attractor. An estimate of this measure is a sum of the positive LE. The estimate of the attractor dimension is provided by the conjecture d_L and the LE are taken in descending order. The dimension d_L gives values close to the dimension estimates discussed earlier and is preferable when

estimating high dimensions. To compute LE, we use a method with linear fitted map, although the maps with higher order polynomials can be used too. Non-linear model of chaotic processes is based on the concept of compact geometric attractor on which observations evolve. Since an orbit is continually folded back on itself by dissipative forces and the non-linear part of dynamics, some orbit points [1,10] $\mathbf{y}^r(k)$, $r = 1, 2, .., N_B$ can be found in the neighbourhood of any orbit point $\mathbf{y}(k)$, at that the points $\mathbf{y}^r(k)$ arrive in the neighbourhood of $\mathbf{y}(k)$ at quite different times than k. One can then choose some interpolation functions, which account for whole neighbourhoods of phase space and how they evolve from near $\mathbf{y}(k)$ to whole set of points near $\mathbf{y}(k+1)$. The implementation of this concept is to build parameterized non-linear functions $\mathbf{F}(\mathbf{x}, \mathbf{a})$ which take $\mathbf{y}(k)$ into $\mathbf{y}(k+1) = \mathbf{F}(\mathbf{y}(k), \mathbf{a})$ and use various criteria to determine parameters $\mathbf{a}$. Since one has the notion of local neighbourhoods, one can build up one's model of the process neighbourhood by neighbourhood and, by piecing together these local models, produce a global non-linear model that capture much of the structure in an attractor itself.

3. The numerical results and conclusions

We continued the investigation of the pollution dynamics of the hydrological systems, in particular, variations of the nitrates concentrations in the forested watersheds of the the Small Carpathian (Slovakia) by using the non-linear prediction approaches and chaos theory method (in versions) [1-10]. As in Refs. [2,4] the initial data have been taken from empirical observations on a number of the watersheds in the region of the Small Carpathians, carried out by coworkers of the Institute of Hydrology of the Slovak Academy of Sciences [11]. The temporal changes in the concentrations of nitrates in the catchment areas are listed in [11]. In Ref. 2 we have listed data on values of the autocorrelation function C_L , the first minimum of mutual information I_{min1} , the correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for time series of the concentration of nitrates in some watersheds of the Small Carpathians, for example,Blatina (Pezinok), Parna (Majdan), Ladomirka (Svidnik), Babie (Olsavka) etc. Here we have maken a numerical analysis of time series for other watersheds, namely, Gidra (Pila) Vydrica (Spariska) Ondava (Stropkov) Manelo (Gribov).

As usually, the first step is in computing the values of the autocorrelation function C_L , the first minimum of mutual information I_{min1} for the concentration of nitrates in four another watersheds (Blatina,Parna, Ladomirka, Babie). The values, where the autocorrelation function first crosses 0.1, can be chosen as τ, but in [6,9] it's showed that an attractor cannot be adequately reconstructed for very large values of τ. So, before making up final decision we calculate the dimension of attractor for all values. The large values of τ result in impossibility to determine both the correlation exponents and attractor dimensions using Grassberger-Procaccia method [1,16]. Here the outcome is explained not only inappropriate values of τ but also shortcomings of correlation dimension method. If algorithm [14] is used, then a percentages of false nearest neighbours are comparatively large in a case of large τ. If time lags determined by average mutual information are used, then algorithm of false nearest neighbours provides $d_E = 6$ for all water pollutants.

In Table 1 we firstly present the advanced data on the time lags, the correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for time series of the concentration of nitrates in the above cited watersheds.

Table 1. The Time lag (τ), correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for time series of the concentration of nitrates in the watershed of the Small Carpathians.

	Gidra (Pila)	Vydrica (Spariska)	Ondava (Stropkov)	Manelo (Gribov)
τ	20	18	9	7
(d_2)	5.82	5.66	5.31	3.71
(d_E)	6	6	6	4
d_L	5.17	5.85	4.11	3.66
$\Pr_{max}$	12	13	8	9

As usually, the sum of the positive LE determines the Kolmogorov entropy, which is inversely proportional to the limit of predictability (Pr_{max}. Let us remember [1,4] since the conversion rate of the sphere into an ellipsoid along different axes is determined by the LE, it is clear that the smaller the amount of positive dimensions, the more stable is a dynamic system. Consequently, it increases the predictability of it. As the numerical calculation shows the presence of the two (from six) positive λ_i suggests the system broadens in the line of two axes and converges along four axes that in the six-dimensional space. It is worth also to present data on the known Gottwald-Melbourne [23] chaos test parameter K. For the studied four time series K is in range [0.65;0.75] that confirms an availability of the chaos elements.

Therefore, here we have presented new further results of an effective application of an advanced chaos-geometric approach to treating of non-linear dynamics of the complex nature, namely, hydroecological systems with discovery of an availability of the middle-D chaos elements.

References

1. Glushkov A.V., Bunyakova Yu.Ya., Analysis and estimation of anthropogenic loading influence on industrial city air basin.-Odessa: Ecology, 2011.-290P.
2. Glushkov A.V., Buyadzhi V.V., Ponomarenko E.L., Geometry of Chaos: Advanced approach to treating chaotic dynamics in some nature systems// Proc. Int. Geom. Centre.-2014.-Vol.7,N1.-P.24–29
3. Glushkov A.V., Kuzakon' V.M., Khetselius O.Yu., Prepelitsa G.P. and Svinarenko A.A., Geometry of Chaos: Theoretical basis's of a consistent combined approach to treating chaotic dynamical systems and their parameters determination// Proc. Int. Geom. Centre.-2013.-Vol.6,N1.-P.6–12.
4. Glushkov A.V., Kuzakon V.M., Buyadzhi V.V., Solyanikova E.P., Geometry of Chaos: Advanced computational approach to treating chaotic dynamics of some hydroecological systems// Proc. Int. Geom. Centre.-2015.-Vol.8,N1.-P.93–99.
5. Bunyakova Yu.Ya., Glushkov A.V.,Fedchuk A.P., Serbov N.G., Svinarenko A.A., Tsenenko I.A., Sensing non-linear chaotic features in dynamics of system of couled autogenerators: standard multifractal analysis// Sensor Electr. and Microsyst. Techn.-2007.-N1.-P.14–17.
6. Glushkov A.V., Khokhlov V.N., Loboda N.S., Bunyakova Yu.Ya., Short-range forecast of atmospheric pollutants using non-linear prediction method// Atmospheric Environment (Elsevier).-2008.-Vol.42.-P. 7284–7292.
7. Bunyakova Yu.Ya., Khetselius O.Yu., Non-linear prediction statistical method in forecast of atmospheric pollutants//Proc. of the 8th International Carbon Dioxide Conference.-Jena (Germany).-2009.- P.T2-098.
8. Glushkov A.V., Khokhlov V.N., Loboda N.S., Khetselius O.Yu., Bunyakova Yu.Ya., Non-linear prediction method in forecast of air pollutants CO_2, CO// Transport and Air Pollution. – Zurich: ETH University Press (Switzerland). –2010. – P.131–136.
9. Glushkov A.V., Khetselius O.Yu., Bunyakova Yu.Ya., Prepelitsa G.P., Solyanikova E.P., Serga E.N., Non-linear prediction method in short-range forecast of atmospheric pollutants: low-dimensional chaos// Dynamical Systems – Theory and Applications. – Lodz: Lodz Univ. Press (Poland). –2011.- LIF111 (6p.).
10. Glushkov A.V., Bunyakova Yu.Ya., Zaichko P.A., Geometry of Chaos: Consistent combined approach to treating chaotic dynamics atmospheric pollutants and its forecasting// Proc. of Int. Geometry Center.-2013.-Vol.6,N3.-P.6–14.
11. Pekarova P., Miklanek P., Konicek A., Pekar J.: Water quality in experimental basins. National Report 1999 of the UNESKO.-Project 1.1.-Intern.Water Systems. 1999, 1–98.
12. Kozak K., Saylan L., Sen O., Nonlinear time series prediction of O_3 concentration in CityplaceIstanbul.. *AtmosphericEnvironment* (Elsevier) 34, 2000, 1267–1271.
13. Kuznetsov S.P., Dunamical chaos.-Moscow: Fizmatlit.-2006.-356P.
14. Kennel M., Brown R., Abarbanel H., Determining embedding dimension for phase-space reconstruction using a geometrical construction//Phys Rev A.-1992.-Vol.45.-P.3403–3411.
15. Packard N., Crutchfield J., Farmer J., Shaw R., Geometry from a time series//Phys Rev Lett.-1988.-Vol.45.-P.712–716.
16. Grassberger P., SnplaceProcaccia SnI., Measuring the strangeness of strange attractors//Physica D.-1983.-Vol.9.-P.189–208.
17. Fraser A., Swinney H., Independent coordinates for strange attractors from mutual information// Phys Rev A.-1986.-Vol.33.-P.1134–1140.
18. Takens F (1981) Detecting strange attractors in turbulence. In: Rand DA, Young LS (eds) Dynamical systems and turbulence, Warwick 1980. (Lecture notes in mathematics No 898). Springer, Berlin Heidelberg New York, pp 366–381
19. Mane R (1981) On the dimensions of the compact invariant sets of certain non-linear maps. In: Rand DA, Young LS (eds) Dynamical systems and turbulence, Warwick 1980. (Lecture notes in mathematics No 898). Springer, Berlin Heidelberg N.-Y., p. 230–242

20. Sano M, Sawada Y (1985) Measurement of the Lyapunov spectrum from a chaotic time series//Phys Rev.Lett.-1995.-Vol.55.-P.1082–1085
21. Theiler J., Eubank S., Longtin A., Galdrikian B., Farmer J., Testing for nonlinearity in time series: The method of surrogate data// Physica D.-1992.-Vol.58.-P.77–94.
22. Kaplan J.L., Yorke J.A., Chaotic behavior of multidimensional difference equations, in: Peitgen H.-O., Walter H.-O. (Eds.), Functional Differential Equations and Approximations of Fixed Points. Lecture Notes in Mathematics No. 730. Springer, Berlin.-1979.-pp.204-227.
23. Gottwald G.A., Melbourne I., A new test for chaos in deterministic systems// Proc. Roy. Soc. London. Ser. A.вЂ“ 2004. вЂ“ Vol. 460. вЂ“ P. 603–611.

A.V. Glushkov, V.M. Kuzakon, Yu.Ya.Bunyakova, V.V.Buyadzhi

Mathematics Department

Odessa State Environmental University, Ukraine

E-mail: dirac13@mail.ru