

Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>

# On possible genesis of fractal dimensions in the turbulent pulsations of cosmic plasma – galactic cosmic rays – turbulent pulsation in planetary atmosphere system

V. Rusov<sup>b,d</sup>, A. Glushkov<sup>a,b,\*</sup>, N. Loboda<sup>c</sup>, O. Khetselius<sup>c</sup>, V. Khokhlov<sup>c</sup>,  
A. Svinarenko<sup>c</sup>, G. Prepelitsa<sup>c</sup>

<sup>a</sup> International Innovative Geoscience Centre and Institute for Applied Mathematics, P.O. Box 24a, Odessa-9, 65009, South-East, Ukraine

<sup>b</sup> Odessa National Polytechnical University, P.O. Box 24a, Odessa-9, 65009, Ukraine

<sup>c</sup> Odessa State Environmental University, P.O. Box 24a, Odessa-9, 65009, Ukraine

<sup>d</sup> Ukrainian Antarctic Centre, Kiev, Ukraine

Received 20 December 2006; received in revised form 30 April 2008; accepted 4 May 2008

## Abstract

The spectrum of turbulent pulsations induced in the atmosphere by the galactic cosmic rays is defined. A possible manifestation of genesis of fractal dimensions in the system of “spectrum of turbulent pulsations of cosmic plasma – galactic cosmic rays’ spectrum – spectrum of atmospheric turbulent pulsations” is analyzed.

© 2008 COSPAR. Published by Elsevier Ltd. All rights reserved.

**Keywords:** Fractal spectrum; Cosmic plasma; Galactic cosmic rays; Atmospheric turbulence

## 1. Introduction

The previous experiments of Pudovkin and Raspopov (1992) have detected that processes in the atmosphere at heights 10–20 km are extremely influenced by the galactic cosmic rays (GCR) with the protons’ energies of  $10^{11} \div 10^{15}$  eV. Strong variations of these rays (a few tens of percents) coincide with the solar activity cycles and atmospheric perturbation variations induced by the separate flares on the Sun. Pudovkin and Raspopov (1992) have also shown that both the value of incoming energy from the GCR spectrum in the magnetosphere and the magnitude of consequent processes in the magnetosphere–ionosphere coincide with values of actual energy for

atmospheric processes ( $\sim 10^{19} \div 10^{20}$  J day<sup>-1</sup>). Due to this fact, traditional “energy-balance” argument of opponents that external factors influence inefficiently the Earth’s climate variations can be rejected.

It is known (Trubnikov, 1990) that the GCR spectrum is striking stable and, in the limits of  $10^{11} \div 10^{15}$  eV, is defined by the following power law:

$$\frac{dN}{dE} = CE^{-\nu}, \quad \nu \approx 2.4 \div 2.74. \quad (1)$$

It is also known that interaction of cosmic particles with the gas in the Earth’s homosphere, where the density is essential, results in the intensive turbulent-mode energy/substance exchange at the interface of homosphere and upper atmosphere (Ebeling et al., 1990).

The present work is aimed to define of the spectrum of the turbulent GCR-induced pulsations in the atmosphere and to determine possible manifestation of genesis of fractal dimensions in the system of “spectrum of turbulent

\* Corresponding author. Address: International Innovative Geoscience Centre and Institute for Applied Mathematics, P.O. Box 24a, Odessa-9, 65009, South-East, Ukraine.

E-mail addresses: [glushkov@paco.net](mailto:glushkov@paco.net) (A. Glushkov), [loboda@paco.net](mailto:loboda@paco.net) (N. Loboda).

pulsations of cosmic plasma – GCR spectrum – spectrum of atmospheric turbulent pulsations”.

## 2. Analysis

It is obvious that according to Eq. (1) the integral GCR spectrum in the limits of  $10^{11} \div 10^{15}$  eV is (Trubnikov, 1990)

$$N \sim E^\mu, \quad \mu = 1.7. \quad (2)$$

Assume that the GCR energy is absolutely absorbed by the atmosphere; then the average energy,  $E_g$ , transferred to the atmospheric gas is evaluated as

$$E_g \sim NE \sim E^{1-\mu}. \quad (3)$$

Let's consider that each cosmic particle induces the initiation of an eddy with a size of  $\lambda$  in the moving gaseous medium. This size is inversely proportional to the energy of particle,  $E$ , i.e.

$$E \sim \lambda^{-1}. \quad (4)$$

Like Landau and Livshitz (1986), we introduce the appropriate spatial “wave numbers” of pulsations (eddies) as  $k \sim 1/\lambda$  instead of scales  $\lambda$ . Then, using Eqs. (3) and (4), the integral spectrum of eddies,  $E_g$ , looks like

$$E_g \sim k^{1-\mu}, \quad (5)$$

and the appropriate spectral density of turbulence is

$$E_g(k) \sim k^{-\mu}, \quad (6)$$

where  $E_g(k)$  is the kinetic energy of gaseous eddy with the spatial wave number  $k$ .

Since  $\mu \approx 5/3$ , it is obvious that the spectrum (6) is the well-known Kolmogorov–Obukhov spectrum (Monin and Yaglom, 1981) describing the dynamics of high-frequency perturbations or, in other words, the structure of small-scale turbulized medium as a skeleton of eddy cluster with the fractal dimension  $D = 5/3$  (Glushkov et al., 1995; Svensmark, 2000).

It is noteworthy that the corresponding scaling laws, scale ratios, and spectral dynamics, in particular within the inertial interval theory that results in the energy spectrum of the Kolmogorov–Obukhov eddies, are conventionally applied at the atmospheric turbulence modelling but for planetary boundary layer only, i.e. for the air layer, in which the interaction of atmosphere with the underlying surface is directly appeared (Monin and Yaglom, 1981; Wyngaard, 1982). This implies that the detected GCR-induced Kolmogorov–Obukhov spectrum (6) differs not only by the cause, but by the principal location of appearance: homosphere – upper atmosphere. The experimental data on the small-scale turbulence structure in this atmospheric area are not known for authors. On the other hand, if the large-scale fractal structure (as skeleton of eddy cluster) occurs in this area, then it is possible to divert some part of energy into “infinity”, e.g. from the area of turbulent motions into the upper atmosphere. There is natural

question: what consequences should be experimentally observed in this case?

Such a time-stable and large-value increment (e.g. as the Joulean heat into upper atmosphere) should essentially revise the “centre of gravity” in the Earth's energy-balance by means of taking into account the turbulent heat flux  $G$  generated by the variations of the galactic and solar cosmic rays:

$$\frac{\partial U}{\partial t} = S[1 - \alpha(T)] - I_a(T) - Q(T) + G(T), \quad (7)$$

where  $\partial U/\partial t$  is the rate of heat generation in the Earth's climate system,  $T$  is the temperature,  $S$  is the solar irradiance onto top of the atmosphere,  $\alpha$  is the albedo of the atmosphere–Earth system,  $I_a$  is the intensity of outgoing long-wave atmospheric radiation,  $Q$  is the heat quantity leaving the considered volume of the climate system due to the horizontal transport of sensible,  $Q_1$ , and latent,  $Q_2$ , heat.

Represent  $I_a$ ,  $Q$ ,  $G$ , and  $\alpha$  as the functions of temperature. First energy term  $I_a$  is responsible for the long-wave radiation of the Earth with the mean temperature  $T$ ; with approximation sufficient for our model it is equal to

$$I_a = \gamma_a \sigma T^4, \quad (8)$$

where  $\sigma$  is the Stephan–Boltzmann constant,  $\gamma_a$  is the coefficient allowing for the area of atmospheric external boundary parallel with the Earth's surface.

The heat quantity  $Q$  is calculated in the following way. It is known that the poleward heat flux defines the equator–pole temperature gradient. In the other words, the larger gradient causes the intensive poleward heat flux. It is assume in our model that in the case of horizontal transport the meridional gradient is proportional to the mean temperature of the Earth's climate system. Consequently, the heat flux  $Q$  is formulated as follows:

$$Q = Q_1 + Q_2 = \gamma_{adv} \mu_{adv} T + \gamma_{adv} m_{wv} c (T_{wv} - T), \quad (9)$$

where  $\mu_{adv}$  is the advection coefficient,  $\gamma_{adv}$  is the coefficient allowing for the total area of the lateral sides of the Earth's climate system,  $m_{wv}$  is the weight velocity of the condensation of water vapour molecules,  $c$  is the specific heat.

The dependence of the effective value of albedo for Earth–atmosphere system from temperature determines essentially the quantity of assimilated direct solar radiation for Earth's climate system. This dependence is taken as the continuous Fegr's parameterization (Budyko, 1969):

$$\alpha(T) = 0.486 - \eta_\alpha (T - 273), \quad (10)$$

where  $\eta_\alpha = 0.0092 \text{ K}^{-1}$ .

Finally, consider the heat transport  $G$  in the turbulent mode. The universal behaviours obtained within the inertial interval theory or, in other words, the Kolmogorov–Obukhov scaling laws (Monin and Yaglom, 1981) were developed to describe the statistical structure of temperature turbulent pulsations when they not essentially affect the structure of flow (Kolmogorov, 1941). It has been also shown that the structure of temperature field for the

urbulent mode is defined not only by the dissipation rate of turbulent kinetic energy per mass unit  $\varepsilon$ , but also by the dissipation rate of intensity of temperature fluctuations,  $N_T$ , which is equal in order of magnitude to

$$N_T \cong (\Delta T)^2 \Delta u L^{-1}, \tag{11}$$

where  $\Delta u$  and  $L$  are the typical size for the velocity and length of main energy-bearing eddies,  $\Delta T$  is the typical temperature variation in the flow at its external scale  $L$ . It is also easily to show that the integral spectrum of eddies  $E_g$  looks like:

$$E_T = C_{1r} (\Delta T)^2. \tag{12}$$

The analysis of Eq. (12) allows to write lossless the common form of dependence for the heat flux in the turbulent mode from the typical temperature variation  $\Delta T$  in the flow at its external scale  $L$ . Assuming  $\Delta T \sim \beta T$ , where  $\beta < 1$ , this dependence can be expressed as

$$G \sim g(\Delta T)^2 \sim (g\beta)T^2, \tag{13}$$

where  $g$  is the dimensional coefficient,  $W K^{-2}$ .

Finally, summing all contributions of the fluxes (8)–(10) and (13) into the resulting energy-balance Eq. (7), we obtain:

$$S - \frac{\partial U}{\partial t} = \gamma_a \sigma T^4 - (g\beta)T^2 + (\gamma_{adv} \mu_{adv} + \eta_a S - \gamma_{adv} m_{wv} c) T + \text{free term}, \tag{14}$$

or, rewriting Eq. (14) in suitable form for further presentation,

$$\frac{1}{4\gamma_a \sigma} \left( S - \frac{\partial U}{\partial t} \right) = F(T, a, b) = \frac{1}{4} T^4 + \frac{1}{2} a T^2 + b T + \text{free term}, \tag{15}$$

where

$$a = -\frac{g\beta}{2\gamma_a \sigma}, \tag{16}$$

$$b = \frac{\gamma_{adv} \mu_{adv} + \eta_a S - \gamma_{adv} m_{wv} c}{4\gamma_a \sigma}. \tag{17}$$

Here, we assume that the power  $F(T, a, b)$  is not time-dependent, which seems as physically lawful. It is obviously that Eq. (15) describes the family of functions  $F(T, a, b)$  depending on the two control parameters,  $a$  and  $b$ . The so-called potential of assembly-type catastrophe is easily recognized in this equation (Gilmore, 1993).

In our opinion, it is interesting to model the long-period behaviour of mean value,  $\langle T \rangle$ , and dispersion,  $\langle \Delta T^2 \rangle$ , of temperature for the manifold of assembly-type catastrophe. To reveal the nontrivial capabilities of the proposed method, let us consider two cases of cyclic path. The first case is modelled under the condition that  $a = -1$  and  $b(t) = b \cos \omega t$ ;  $T_1^{-1} \gg \omega$ . The time evolutions of  $\langle T \rangle$  and  $\langle \Delta T^2 \rangle$  are shown in Fig. 1a (the symmetric cyclic path C). The second case, which corresponds to the asymmetric

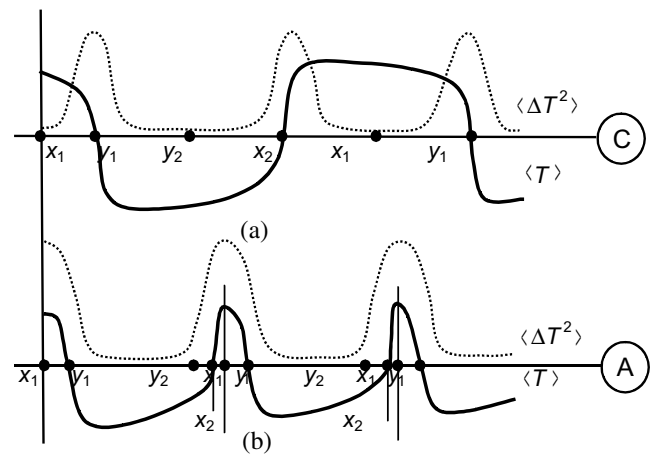


Fig. 1. Modelling long-period oscillations for the mean value  $\langle T \rangle$  and dispersion  $\langle \Delta T^2 \rangle$  of temperature corresponding to symmetric (a) and asymmetric (b) paths in the space of controlling parameters (a, b).

(with regard to coordinate axis  $a$ ) cyclic path A in plane (a–b), is modelled under following conditions:  $a = -0.5$ ,  $b(t) = [-b \cos \omega t + \Delta]$ , where  $\Delta = (b - 2|b_c|)/4$ , and  $b_c$  is determined by the equation of semi-cubical parabola describing the bifurcation set of assembly-type catastrophe (Fig. 1b).

It is noteworthy that the analysis of well-known experimental data from the Antarctic station Vostok, which are related to the temperature variations during the last 420 ky (Petit et al., 1999), confirms the existence of period  $\sim 120$  ky (Rusov et al., 2007); the latter is in agreement with the data of earlier works (Broecker and van Donk, 1970; Hays et al., 1976). Also, this analysis demonstrates the fact that the reason that organize such an periodical behaviour of control parameter  $b$  is the periodical variations of Earth’s orbit geometry (eccentricity) initiating the variations of solar radiation or, in other words, the physical mechanism for “controlling” of global climate, which was long time ago concerned in Milankovitch’s (1930) theory of ice age rhythms.

### 3. Conclusions

The considered possibility for the existence of spectrum of Kolmogorov–Obukhov turbulent kinetic energy dissipation induced by the GCR in the atmosphere establishes the attractive problem associated with the genesis of scaling invariance and scaling representation of turbulent spectrums. It becomes evidently apparent under the simple analysis of Eqs. (1)–(6); to derive the connection of fractal dimensions in the GCR spectrum with Kolmogorov–Obukhov spectrum, the procedure could be carried out in inverse manner. The latter means that this wonderful behaviour of the GCR spectrum allows not only suppose certain mechanism of initiation for these particles (Trubnikov, 1990; Rusov et al., 2003), but predetermine the conclusion that this mechanism should also describe the initiation of cosmic plasma’s large-scale fractal structures

stipulated by the turbulence with Kolmogorov–Obukhov spectrum.

### Acknowledgements

The authors thank anonymous referees for useful comments.

### References

- Broecker, W.S., van Donk, J. Insolation changes ice volumes and the  $O^{18}$  record in deep-sea cores. *Rev. Geophys. Space Phys.* 8, 169–198, 1970.
- Budyko, M.I. The effect of solar radiation variations on the climate of the Earth. *Tellus* 21, 611–619, 1969.
- Ebeling, W., Engel, A., Feistel, R. *Physic der Evolutionsprozesse*. Akademie-Verlag, Berlin, 1990.
- Gilmore, R. *Catastrophe Theory for Scientists and Engineers*. Dover Publications Inc., New York, 1993.
- Glushkov, A.V., Makarov, E.S., Nikiforova, E.S., Pravdina, M.I. Muon component of EAS with energies above  $10^{17}$  eV. *Astropart. Phys.* 4, 531–538, 1995.
- Hays, J.D., Imbrie, J., Shackleton, N.J. Variations in the Earth's orbit: pacemaker of the Ice Ages. *Science* 194, 1121–1132, 1976.
- Kolmogorov, A.N. The local structure of turbulence in incompressible viscous fluid for very large Reynolds number. *C.R. Acad. Sci. URSS* 30, 301–305, 1941.
- Landau, L.D., Livshitz, E.M., *Theoretical Physics vol. 6, Hydrodynamics*. Nauka, Moscow, 1986. (in Russian).
- Milankovitch, M., *Mathematische Klimalehre und Astronomische. Theorie der Klimaschwankungen*, in: Koppen, W., Geiger, R. (Eds.), *Handbuch der Klimatologie*, Band 1, Teil A. Gebruder Borntrager, Berlin, pp. 1–176, 1930.
- Monin, A.S., Yaglom, A.M. *Statistical Fluid Mechanics: Mechanics of Turbulence*. MIT Press, Cambridge, 1981.
- Petit, J.R., Jouzel, J., Raynaud, D., Barkov, N.I., Barnola, J.M., Basile, I., Bender, M., Chappellaz, J., Davis, J., Delaygue, G., Delmotte, M., Kotlyakov, V.M., Legrand, M., Lipenkov, V., Lorius, C., Pepin, L., Ritz, C., Saltzman, E., Stievenard, M. Climate and atmospheric history of the past 420,000 years from the Vostok Ice Core Antarctica. *Nature* 399, 429–436, 1999.
- Pudovkin, M.I., Raspopov, O.M. The mechanism of action of solar activity on the state of the lower atmosphere and meteorological parameters (A review). *Geomagn. Aeronomy* 32, 593–608, 1992.
- Rusov, V.D., Glushkov, A.V., Vaschenko, V.N., *Astrophysical model of global climate of the Earth*. Nauka Kiev, 2003.
- Rusov, V., Glushkov, A.V., Vaschenko, V., Mavrodiiev, S., Vachev, B., Kosenko, S., Mihalus, O., Eremenko, V., Kolos, A. Galactic cosmic rays – cloud effect and bifurcation model of Earth global climate. *Bound Vol. Observatorie Montagne de Moussalla* 12, 80–90, 2007.
- Svensmark, H. Cosmic rays and Earth's climate. *Space Sci. Rev.* 93, 155–166, 2000.
- Trubnikov, B.A. On the possible generation of cosmic rays in plasma pinches. *Sov. Phys. Uspekhi* 33, 1061–1071, 1990.
- Wyngaard, J.C. Boundary-layer modelling, in: Nieuwstadt, F.T.M., van Dop, H. (Eds.), *Atmospheric Turbulence and Air Pollution Modelling*. D. Reidel Publishing, Dordrecht, pp. 69–106, 1982.