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## **CALCULATING THE RADIATIVE VACUUM POLARIZATION CONTRIBUTION TO THE ENERGY SHIFT OF 2P-2S TRANSITION IN PIONIC DEUTERIUM**

Calculating the radiative contribution due to the vacuum polarization effect to energy value for the 3p-1s transition in pionic deuterium has been carried out using the modified generalized Uehling-Serber potential which includes effectively the higher order vacuum polarization corrections. Comparison with analogous theoretical data by Indelicato is performed.

Due to the significant progress in the modern experimental technologies now a great interest attracts studying spectra of heavy and super heavy elements atoms, exotic atomic systems, including hadronic and leptonic atoms [1-15]. Especial problem is connected with précised calculating the radiative corrections to the transition energies of the low-Z exotic (pionic, kaonic, muonic) atoms, namely, hydrogen and deuterium. Naturally, it is provided by necessity of further developing the modern as atomic and as nuclear theories. From the other side, detailed information about spectra of the exotic atomic systems (kaonic, pionic, muonic atoms) can be very useful under construction of the new X-ray standards. One could remind a great importance of the muonic chemistry, muonic spectroscopy. Very attractive perspective of the thermonuclear fission through the mechanism of the muonic catalysis is still interesting and widely studied.

The standard Dirac approach is traditionally used as starting basis in calculations of the heavy ions [2]. The problem of accounting the radiative corrections, in particular, self-energy part of the Lamb shift and vacuum polarization contribution is mostly treated with using the expansions on the natural physical parameters  $1/Z$ ,  $aZ$  ( $a$  is fine structure constant) [5,10]. It permits evaluations of the relative contributions of different expansion energy terms: non-relativistic, relativistic ones, as functions of  $Z$ . For high  $Z$  ( $Z$  is a nuclear charge) it should be necessary to account for the high-order QED corrections and the nuclear finite

size correction etc [1-3,10-12,16]. Further improvement of this method in a case of the heavy ions is linked with using gauge invariant procedures of generating relativistic orbital bases and more correct treating nuclear and QED effects [1-3]. In a case of the low-Z exotic atomic systems such as an exotic hydrogen (deuterium) a great interest attracts estimation of the radiative, in particular, vacuum polarization, correction. In refs. [17-19] it has been proposed a precise scheme to calculating spectra of heavy systems with account of nuclear and radiative effects, based on the relativistic many-body perturbation theory (see also [3]) and advanced effective procedures for accounting the radiative corrections.

In this paper we present the results of calculating the contribution due to the vacuum polarization effect to energy shift for 3p-1s transition in pionic deuterium. The obtained result is compared with calculation data by Indelicato [19]. The calculation of the radiative vacuum polarization shift in the pionic deuterium should be performed using the Dirac approximation as a zeroth one. Further, the expectation value of the radiative vacuum polarization operator gives the corresponding correction. The total electromagnetic interaction potential:

$$V(r) = V_n(r) + U(r). \quad (1)$$

includes the electrical  $V_n$  and polarization  $U(r)$  potentials of a nucleus with accounting the finite

size correction. As usually, the Coulomb potential for spherically symmetric density  $\rho(r|R)$  can be written as follows:

$$V_n(r|R) = -\left(\frac{1}{r}\right) \int_0^r dr' r'^2 \rho(r'|R) + \int_r^\infty dr' r' \rho(r'|R). \quad (2)$$

The details of the determination of this potential can be, for example, found in ref. [21,22]. The vacuum polarization part is usually accounted in the first PT order by using the Uehling potential [1,8,16,17]:

$$U(r) = -\frac{2\alpha}{3\pi r} \int_1^\infty dt \exp(-2rt/\alpha Z) \left(1 + \frac{1}{2t^2}\right) \bullet$$

$$\bullet \frac{\sqrt{t^2-1}}{t^2} \equiv -\frac{2\alpha}{3\pi r} C(g)$$

$$g = \frac{r}{\alpha Z}. \quad (3)$$

The corresponding expectation value of this operator gives the corresponding vacuum polarization correction. In the scheme [12] this potential is approximated by quite precise analytical function (see details in refs. [3,16,17]).

The most advanced version of the such potential ( $C \rightarrow \tilde{C}$ ) is presented by Khetselius [12] and written as follows:

$$\tilde{C}(g) = \tilde{C}_1(g) \tilde{C}_2(g) / \left( \tilde{C}_1(g) + \tilde{C}_2(g) \right),$$

$$\tilde{C}_2(g) = \tilde{C}_2(g) f(g),$$

$$\tilde{C}_2(g) = -1.8801 \exp(-g) / g^{3/2}$$

$$\tilde{C}_1(g) = h(g/2) + 1.410545 - 1.037837g,$$

$$f(g) = \left( (1.1024/g - 1.3361) / g + 0.8027 \right) \quad (4)$$

The using this formula permits one to decrease the calculation errors for this term down to ~0.1%. Error of usual calculation scheme is ~10%. We carried out the calculation of the vacuum polarization contribution to the energy shift for 3p-1s transition in pionic deuterium. It should be noted that the energy levels of exotic (pionic) muonic atoms are very sensitive to effects of QED, nuclear structure and recoil since the pion is heav-

ier than the electron. As usually the fundamental constants from the CODATA 1998 are used in the numerical calculations. The most QED effect for pionic atoms is the virtual production and annihilation of a single  $e^+e^-$  pair (the Uehling-Serber contribution).

We have evaluated the modified Uehling-Serber potential expectation values and obtained the value for the vacuum-polarization correction: 3.73940 eV. It is interesting to compare this result with the result by Indelicato, namely, the total vacuum polarization contribution: 3.74844 eV. Further, using data by Indelicato [24] for the main Coulomb contribution (3074.1597 eV), the self-energy and finite size correction (-0.0022 eV), the relativistic recoil correction is -0.00029 eV (recoil 1 correction: 0.00293 eV) we can evaluate the pure electromagnetic 3p-1s transition energy in the pionic deuterium  $E_{QED} = 3077.8995$  eV. The corresponding value, presented by Indelicato is 3077.90858 eV. The physically reasonable agreement between the theories can be easily explained by the fact that pionic deuterium is the low-Z atomic system and in this case the expansion on the parameter  $aZ$  works sufficiently well. From the other side, the approach used by us is not perturbative one and can be applied to the non low-Z atomic systems.

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### **Abstract**

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**Key words:** pionic deuterium, radiative corrections

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## **РАСЧЕТ РАДИАЦИОННОГО ВКЛАДА ЗА СЧЕТ ЭФФЕКТА ПОЛЯРИЗАЦИИ ВАКУУМА В СДВИГ ЭНЕРГИИ 3P-1S ПЕРЕХОДА В ПИОННОМ ДЕЙТЕРИИ**

### **Резюме**

Проведен расчет радиационного вклада за счет эффекта поляризации вакуума в величину энергии 3p-1s перехода в пионном дейтерии с использованием модифицированного потенциала Юлинга-Сербера, эффективно учитывающего вакуум - поляризационные поправки высших порядков. Проведено сравнение с аналогичными данными теории Indelicato.

**Ключевые слова:** пионный дейтерий, радиационные поправки

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## **РОЗРАХУНОК РАДІАЦІЙНОГО ВНЕСКУ ЗА РАХУНОК ЕФЕКТУ ПОЛЯРИЗАЦІЇ ВАКУУМУ У ЗСУВ ЕНЕРГІЇ 3P-1S ПЕРЕХОДУ У ПІОННОМУ ДЕЙТЕРІЇ**

### **Резюме**

Виконано розрахунок радіаційного внеску за рахунок ефекту поляризації вакууму у величину енергії 3p-1s переходу у піонному дейтерії з використанням модифікованого потенціалу Юлінга-Сербера який ефективно враховує вакуум - поляризаційні поправки вищих порядків. Проведено порівняння з аналогічними даними теорії Indelicato.

**Ключові слова:** піонний дейтерій, радіаційні поправки