LASER MULTIPHOTON SPECTROSCOPY OF ATOM EMBEDDED IN DEBYE PLASMAS:
MULTIPHOTON RESONANCES AND TRANSITIONS

The consistent relativistic energy approach to atom in a realistic laser field, based on the Gell-Mann and Low S-matrix formalism, is applied to studying the resonant multiphoton transitions in atoms embedded in the Debye plasmas. There is considered a new scheme to calculating the multiphoton transitions characteristics, shifts and widths of multiphoton resonances. An approach is used for treating the three-photon transitions in krypton embedded in the Debye plasmas.

1. The physics of multiphoton phenomena is one of the very quickly developed branches of the modern optics and spectroscopy, photophysics. Studying of multiphoton phenomena in atoms, molecules etc has a great progress that is stimulated by development of new laser technologies (see Refs. [1-10]). The appearance of the powerful laser sources allowing to obtain the radiation field amplitude of the order of atomic field in the wide range of wavelengths results to systematic investigations of the nonlinear interaction of radiation with atomic and molecular systems [1-14].

At the same time a direct laser-nucleus interactions traditionally have been dismissed because of the well known effect of small interaction matrix elements [9-11]. Some exceptions such as an interaction of x-ray laser fields with nuclei in relation to alpha, beta-decay and x-ray-driven gamma emission of nuclei have been earlier considered. With the advent of new coherent x-ray laser sources in the near future, however, these conclusions have to be reconsidered.

At present time a great interest has been connected with studying atomic processes in plasma environments because of the plasma environment screening effect on the plasma-embedded atomic systems. One should remind that the screening effects have play a important and significant part in the investigation of plasma environments over the past several decades.

Different theoretical methods have been employed along with the Debye screening to study plasma environments.

The interaction of atoms with the external alternating fields, in particular, laser fields, has been the subject of intensive experimental and theoretical studied (see, for example, Refs. [1-8, 12-24]). A definition of the k-photon emission and absorption probabilities and atomic levels shifts, study of dynamical stabilization and field ionization etc are the most actual problems to be solved.

Above methods which are usually used one should mention such approaches as the standard perturbation theory (surely for low laser filed intensities), Green function method, the density-matrix formalism, time-dependent density functional formalism, direct numerical solution of the Schrödinger (Dirac) equation, multi-body multiphoton approach, the time-independent Floquet formalism etc (see [1-8,12-24] and Refs. therein).

Earlier the relativistic energy approach to studying the interaction of atom with a realistic strong laser field, based on the Gell-Mann and Low S-matrix formalism, has been developed. Originally, Ivanov has proposed an idea to describe quantitatively a behaviour of an atom in a realistic laser field by means studying the radiation emission and absorption lines and further the theory of interaction of an atom with the Lorentz laser pulse and calculating the corresponding
lines moments has been in details developed in Ref. [19-25]. It has been checked in numerical simulation of the multiphoton resonances shifts and widths in the hydrogen and caesium. Theory of interaction of an atom with the Gauss and soliton-like laser pulses and calculating the corresponding lines moments has been in details presented in Refs. [23,26,27].

Here the consistent relativistic energy approach to atom in a realistic laser field, based on the Gell-Mann and Low S-matrix formalism, is applied to studying the resonant multiphoton transitions in atoms embedded in the Debye plasmas. There is considered a new scheme to calculating the multiphoton transitions characteristics, shifts and widths of multiphoton resonances. An approach is used for treating the three-photon transitions in krypton embedded in the Debye plasmas.

The relativistic energy approach in the different realizations and the radiation lines moments technique is in details presented in Refs. [19-30]. Here the consistent relativistic energy approach to atom in a realistic laser field, based on the Gell-Mann and Low S-matrix formalism, is applied to studying the resonant multiphoton transitions in atoms embedded in the Debye plasmas. There is considered a new scheme to calculating the multiphoton transitions characteristics, shifts and widths of multiphoton resonances. An approach is used for treating the three-photon transitions in krypton embedded in the Debye plasmas.

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2. The relativistic energy approach in the different realizations and the radiation lines moments technique is in details presented in Refs. [19-30]. So, here we are limited only by presenting the master elements. In the theory of the non-relativistic atom a convenient field procedure is known for calculating the energy shifts \( dE \) of degenerate states. This procedure is connected with the secular matrix \( M \) diagonalization. In constructing \( M \), the Gell-Mann and Low adiabatic formula for \( dE \) is used [20-23,31]. In relativistic theory, the Gell-Mann and Low formula \( dE \) is connected with electrodynamical scattering matrix, which includes interaction with as a laser field as a photon vacuum field. A case of interaction with photon vacuum is corresponding to standard theory of radiative decay of excited atomic states. Surely, in relativistic theory the secular matrix elements are already complex in the second perturbation theory (PT) order. Their imaginary parts are connected with radiation decay possibility. The total energy shift is usually presented in the form [23]:

\[
\delta E = \text{Re} \delta E + i \text{Im} \delta E ,
\]

where \( P \) is the level width (decay possibility). Let us describe the interaction “atom-laser field” by the Ivanov potential [21,23]:

\[
V(r,t) = V(r) \delta \omega f(\omega - \omega_0) \sum_{k} \cos(\omega_0 t + \omega_0 k t) \tag{2}
\]

Here \( \omega_0 \) is the central laser radiation frequency, \( n \) is the whole number. The function \( f(\omega) \) is a Fourier component of the laser pulse. The condition \( \delta \omega f(\omega) = 1 \) normalizes potential \( V(rt) \) on the definite energy in the pulse. Usually one could consider the pulses with Lorentz shape (coherent 1-mode pulse): \( f(\omega) = b/[(\omega^2 + D^2)] \), Gaussian one (multi-mode chaotic pulse): \( f(\omega) = b \exp[\ln 2(\omega^2/D^2)] \) and the soliton-like pulse: \( f(t) = b \cosh^{-1}[t/D] \) (b -normalizing multiplier).

The master program results in the calculating an imaginary part of energy shift \( \text{Im} E_a(\omega_n) \) for any atomic level as the function of the central laser frequency \( \omega_n \). An according function has the shape of the resonance, which is connected with the transition a-p (a, p-discrete levels) with absorption (or emission) of the “k” number of photons. For this transition the following values are determined [20-23]:

\[
\delta \omega \left( \omega_0 ; k \right) = \int \delta \omega f(\omega) \text{Im} E_a(\omega) \left( \omega - \omega_0 /k \right) / N ,
\]

\[
\mu_m = \int \delta \omega f(\omega) \text{Im} E_a(\omega) \left( \omega - \omega_0 /k \right)^m / N ,
\]

where

\[
\int \delta \omega f(\omega) \text{Im} E_a(\omega) \left( \omega - \omega_0 /k \right)^m / N ,
\]

is the normalizing multiplier; \( \omega_n \) is position of the non-shifted line for transition a-p, \( \delta \omega f(\omega) \) is the line shift under k-photon absorption; \( \text{Im} E_a(\omega_0) \) is the first moments \( m \), \( m \), and \( m \) determine the atomic line centre shift, its dispersion and the asymmetry.

To find \( \mu_m \), we need to get an expansion of \( E_a \) to PT series:

\[
E_a = \sum E_a^{(2k)}(\omega_0) .
\]

One may use here the Gell-Mann and Low adiabatic formula for \( dE_a \) [20-23]. The consideration can be simplified by account of the k-photon absorption contribution in the first two PT orders. Besides, summation on laser pulse is exchanged by integration. The corresponding \( (l+2k+1) \)-times integral on \( (l+2k) \) temporal variables and \( r \) \( (l=0,2) \) integral \( I_g \) are calculated [19-23]. Finally, after some cumbersome transformations one can get the expressions for the line moments. The corresponding expressions for the Gaussian laser pulse are as follows:

\[
129
\]
\[ \delta \omega(p\alpha | k) = \{\pi \Delta(k+1)k\} [E(p,\omega_{pd}/k) - E(\alpha,\omega_{pd}/k)], \quad (9) \]

\[ \mu_2 = \Delta^2/k \]

\[ \mu_3 = \{4\pi \Delta^3/[k(k+1)]\} [E(p,\omega_{pd}/k) - E(\alpha, \omega_{pd}/k)], \quad (10) \]

The summation in (10) is over all atomic states. Let us note that these formulas for the Gaussian pulse differ of the Lorenz shape pulse expressions [21-23]. For the soliton-like pulse it is necessary to carry out the numerical calculation or use some approximations to simplify the expressions [27].

In order to calculate (10), one should use the technique [28,29] of calculating sums of the QED PT second order, which has been earlier applied by us in calculations of some atomic and mesoatomic parameters [26,27,30-32].

Finally the computational procedure results in a solution of the ordinary differential equations system for above described functions and integrals. In concrete numerical calculations the PC “Superatom-ISAN” package is used. The construction of the operator wave functions basises within the QED PT, the technique of calculating the matrix elements in Eqs. (9,10) and other details is are presented in Refs. [19-30].

3. In order to take into account the plasmas screening effect one could use the known Debye shielding model. As it is well known (c.f.[33-35] and refs there) in the classical theory of plasmas developed by Debye and Hückel, the interaction potential between two charged particles in a plasma is modelled by a Yukawa-type potential as follows:

\[ V(r_a, r_b) = (Z_aZ_b/|r_a-r_b|) \exp(-\mu|r_a-r_b|), \quad (11) \]

where \( r_a, r_b \) represent respectively the spatial coordinates of particles A and B and \( Z_a, Z_b \) denote their charges. A difference between the Yukawa type potential and standard Coulomb potential is in account for the effect of plasma, which is modeled by the shielding parameter \( \mu \) [33]. The parameter \( \mu \) is linked with the plasma parameters such as the temperature \( T \) and the charge density \( n \) as follows:

\[ \mu = e^2n/k_B T, \]

where \( e \) is the electron charge and \( k_B \) is the Boltzman constant. The density \( n \) is given as a sum of the electron density \( N_e \) and the ion density \( N_i \) of the \( k \)-th ion species having the nuclear charge \( q_k \).

Let us remind [35] that under typical laser plasma conditions of \( T \sim 1 \text{keV} \) and \( n \sim 10^{23} \text{ cm}^{-3} \) the parameter \( m \) is of the order of 0.1 in atomic units. By introducing the Yukawa-type electron-nuclear attraction and electron-electron repulsion potentials, the electronic Hamiltonian for \( N \)-electron multicharged ion in a plasma is given in atomic units as follows:

\[ H = \sum_{i} [\alpha p^2 - Z \exp(-\mu r_i) / r_i] + \sum_{i>j} \left( \frac{1}{r_{ij}} \right) \exp(-\mu r_{ij}) \quad (12) \]

A difference between the Hamiltonian (12) and analogous model Hamiltonian with the Yukawa potential of ref. [33] is in using the relativistic approximation, which is obviously necessary for adequate description of relativistic systems [35].

4. In ref. [36] there were presented the results of the numerical simulation for the three-photon resonant, four-photon ionization profile of atomic krypton (the 4p \( \otimes \) 5d[1/2], and 4p \( \otimes \) 4d[3/2], three photon Kr resonances are considered; intense uv (285-310 nm) laser radiation with intensity range \( 3 \times 10^{12}-10^{14} \text{ W/cm}^2 \) studied) in a free state (in sense of the absence a plasmas environment). There have been determined the corresponding parameters of the 4p \( \otimes \) 5d[1/2] (ii) and 4p \( \otimes \) 4d[3/2] (ii) three photon Kr resonances. The resonance shift is proportional to intensity with a width dominated by lifetime broadening of the excited state.

The numerical simulation [36] results for the 4p \( \otimes \) 5d[1/2], (i) and 4p \( \otimes \) 4d[3/2], (ii) three photon Kr resonances are as follows: (i) the shift \( \Delta \omega(p)a_l = a_l \exp = 3.95 \text{ meV/(Tw/cm}^2) \)

and width \( b_l = 1.5 \text{ meV/(T}^2 \text{w/cm}^2) \); (ii) shift \( \Delta \omega(p)a_l = a_l \exp = 8.1 \text{ meV/(T}^2 \text{w/cm}^2) \)

and width \( b_l = 4.2 \text{ meV/(T}^2 \text{w/cm}^2) \). We have chosen the Debye length parameter values \( \lambda_D = 50 \) (25) and the corresponding computed coefficients are as
follows: (i) \( a = 3.76 \text{ meV}/(\text{Tw} \times \text{cm}^{-2}) \) \( (a = 3.2 \text{ meV}/(\text{Tw} \times \text{cm}^{-2}) \); (ii) \( a = 7.8 \text{ meV}/(\text{Tw} \times \text{cm}^{-2}) \) \( (a = 6.5 \text{ meV}/(\text{Tw} \times \text{cm}^{-2}) \).

The presented results show that Debye plasma environments have an effect on the multiphoton transitions. Nevertheless, one should keep in mind some important facts the (see, for example, [33,34]). It is clear the static screening result considered above is subject to the condition that the plasma is a thermodynamically equilibrium plasma and neglects the contributions from ions in plasma since electrons provide more effective shielding than ions.

Obviously with the changing the plasma conditions (parameters) in principle there can be taken a place a significant variations. Besides, one should remember about the conditions of applicability of the Debye approximation.

References

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Abstract.
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Key words: electromagnetic interactions, laser field, multiphoton resonances, plasmas
резонансных многофотонных переходов атомов в дебаевской плазме. Предложена новая схема вычисления характеристик многофотонных переходов, энергий и ширин многофотонных переходов. Подход использован для описания трехфотонных переходов в криптоне в плазме Дебая.

Ключевые слова: электромагнитное, лазерное поле, многофотонные резонансы, плазма

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ЛАЗЕРНА МУЛЬФОТОНА СПЕКТРОСКОПІЯ АТОМІВ У ДЕБАЄВСЬКОЙ ПЛАЗМІ: БАГАТОФОТОННІ ПЕРЕХОДИ І РЕЗОНАНСИ

Резюме.
Релятивістський енергетичний підхід до опису спектроскопії атома в лазерному полі, що грунтується на S-матричному формалізмі Гелл-Манна і Лоу, застосовується для вивчення резонансних багатофotonних переходів атомів в дебаєвській плазмі. Запропоновано нову схему обчислення характеристик багатофотонних переходів, енергій і ширин багатофотонних переходів. Підхід використаний для опису трьох фотонних переходів у криптоні в плазмі Дебая.

Ключові слова: електромагнітна взаємодія, лазерне поле, багатофотонні резонанси, плазма