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МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ УКРАИНЫ  
ОДЕССКИЙ НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ  
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# **Физика аэродисперсных систем**

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**МЕЖВЕДОМСТВЕННЫЙ НАУЧНЫЙ СБОРНИК**

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## **Inclusion of condensation heating into the atmospheric energy cycle based on the transformed Eulerian mean**

A formulation is proposed to analyze the atmospheric energetics. To see the effect of eddy forcing due to the condensation heating, we reformulate the original atmospheric energy cycle based on the transformed Eulerian mean and make numerical experiments with original and improved equations for the Northern mid-latitudes in the winter and summer. Inclusion of condensation heating changes the values of terms containing the components of the residual meridional circulation and the effective momentum flux. It is shown that the energy cycle based on the improved equations, in comparison with the original ones, possesses the larger balance ability, which is concerned with more reliable values of the energy fluxes.

### **1. Introduction**

The attention, which was focused during the development of atmospheric physics on the investigation of energetics for the atmospheric processes, can be explained that the energetics allows to consider the dynamical processes taking place in the atmospheric systems with different scales in all variety. Almost single conceptual assumption, both fully realistic and experimentally and/or theoretically proven, is the presence of conversion between different energies. This process leads to the energy «replenishment» of atmospheric processes with some scale by the other ones. Thus the problem lies in the most precise definition, from the physical view, of mentioned conversions.

Lorenz [1] offered the zonal mean and eddy kinetic and available potential energies (hereafter  $K$  denotes a kinetic energy,  $P$  denotes an available potential energy, subscript  $Z$  denotes a zonal mean component, and subscript  $E$  denotes an eddy component) as the main components for the atmospheric energy cycle. Also, he showed the physical interpretation for the conversion of some kind of energy to the other one. During the ensuing years other reformulations were offered [2-6] but the concepts of mentioned energies were not changed.

With a focus on the zonally symmetric structure of the global atmosphere, Lorenz [1] divided each of available potential energy and kinetic energy into its zonal mean and eddy components based on the conventional Eulerian mean meridional circulation. The Eulerian mean view defines zonal means as those on isobaric surfaces and eddies as departures from the zonal means. However it has been recognized that this approach faces some difficulties. For example, the Eulerian mean view cannot express the non-acceleration theorem of steady wave propagation.

To cope with these difficulties, Andrews and McIntyre [7] proposed the transformed Eulerian mean (hereafter TEM) approach, which adds the Stokes correction to the conventional Eulerian mean circulation. Plumb [2] and Kanzawa [3] formulated the energetics based on the TEM. They showed that the zonal mean available potential energy does not exchange with the eddy potential energy in the TEM energetics.

Though the energy cycle based on the TEM had advantages in comparison with the conventional formulation offered by Lorenz, it is not free of shortcoming. In the presence of mean heating, the eddies not only force the mean circulation and modify both the zonal flow and the mean temperature but also alter the mean heating, which forces, in turn, the mean circulation [8]. Furthermore, the eddies modify the mean heating due to the condensation owing to both the eddy flux of moisture and the Eliassen-Palm flux divergence [9].

In this paper, we solve partially the problem of mean heating by introducing the condensation heating into the budget equations of energies.

The structure of this short paper is follows. First we rewrite original formulation of the TEM energy cycle offered by Plumb [2]. Then we include the condensation heating into this energy cycle. Further, we make numerical experiments for the Northern mid-latitudes. Finally we present main conclusions about proposed improvement.

## 2. The TEM energy cycle

For simplicity, we take the pressure ( $p$ ) as a vertical coordinate and a Cartesian coordinate in the horizontal, which has a cyclic lateral boundary with a length of  $Lx$  in the zonal direction and rigid wall conditions of  $v = 0$  at  $y = 0$  and  $Ly$  in the meridional direction. Such lateral boundary conditions make it possible to translate implications on wave-mean-flow interactions into those in spherical coordinate. In this case, zonal means are defined as function of ( $y, p$ )

$$\bar{A} \equiv \frac{1}{Lx} \int A(y, p) \quad (2.1)$$

and eddies are defined as departures from the zonal means

$$A' \equiv A - \bar{A} \quad (2.2)$$

The TEM energy cycle is based on two concepts: a) a residual meridional circulation ( $\bar{v}_*$ ,  $\bar{\omega}_*$ ), where

$$\left. \begin{aligned} \bar{v}_* &= \bar{v} + \frac{\partial}{\partial p} \left( \frac{p}{R} b_2 \overline{T'v'} \right) \\ \bar{\omega}_* &= \bar{\omega} - \frac{p}{R} b_2 \frac{\partial}{\partial y} \overline{T'v'} \end{aligned} \right\}, \quad (2.3)$$

and b) an effective momentum flux  $\mathbf{B}$ , which is the negative of the Eliassen-Palm flux and has a form

$$\mathbf{B} = \begin{bmatrix} B_y \\ B_p \end{bmatrix} = \begin{bmatrix} \overline{u'v'} + \frac{p}{R} b_2 \overline{T'v'} \frac{\partial \bar{u}}{\partial p} \\ \overline{u'\omega'} + \frac{p}{R} b_2 \overline{T'v'} \left( f - \frac{\partial \bar{u}}{\partial y} \right) \end{bmatrix}. \quad (2.4)$$

In Eqs. (2.3) and (2.4)  $u$  and  $v$  are the zonal and meridional winds,  $\omega = dp/dt$  is the «vertical velocity»,  $T$  is the air temperature,  $R$  is the gas constant,  $f$  is the Coriolis parameter, and

$$b_2 = \frac{R}{p} \left( \kappa \frac{\tilde{T}}{p} - \frac{\partial \tilde{T}}{\partial p} \right)^{-1}, \quad (2.5)$$

where the tilde over  $T$  denotes the averaging on latitudinal belts and  $\kappa$  is the ratio of specific heat ( $\approx 0.29$ ).

The budget equations of the TEM energy cycle (Plumb, 1983) can be written as follows

$$\frac{\partial K_Z}{\partial t} = -C(K_Z K_E) + C(P_Z K_Z) + F(K_Z) + S(K_Z), \quad (2.6)$$

$$\frac{\partial P_Z}{\partial t} = -C(P_Z P_E) - C(P_Z K_Z) + F(P_Z) + S(P_Z), \quad (2.7)$$

$$\frac{\partial K_E}{\partial t} = C(K_Z K_E) + C(P_E K_E) + F(K_E) + S(K_E), \quad (2.8)$$

$$\frac{\partial P_E}{\partial t} = C(P_Z P_E) - C(P_E K_E) + F(P_E) + S(P_E), \quad (2.9)$$

where

$$C(K_Z K_E) = \bar{u} \left( \frac{\partial B_y}{\partial y} + \frac{\partial B_p}{\partial p} \right), \quad C(P_Z P_E) = 0, \quad C(P_Z K_Z) = -\frac{R}{p} \bar{T} \bar{\omega}_*, \quad C(P_E K_E) = -\frac{R}{p} \overline{T' \omega'},$$

$$\begin{aligned} F(K_Z) &= -\frac{\partial}{\partial y} \bar{\phi} \bar{v}_* - \frac{\partial}{\partial p} \bar{\phi} \bar{\omega}_*, \quad F(P_E) = 0, \quad F(P_Z) \\ &= \frac{1}{\kappa} \frac{\partial}{\partial p} \left\{ b_2 p \left[ \overline{T'v'} \frac{\partial \bar{T}}{\partial y} + \overline{T' \omega'} \left( \frac{\partial \bar{T}}{\partial p} - \frac{\kappa \bar{T}}{p} \right) \right] \right\}, \end{aligned}$$

$$F(K_E) = -\frac{\partial}{\partial y} \left( \overline{\phi'v'} + \bar{u}B_y \right) - \frac{\partial}{\partial p} \left( \overline{\phi'\omega'} + \bar{u}B_p \right).$$

In the last relations  $\phi$  is the geopotential,  $(L_x, L_y)$  are the components of frictional force per unit mass, and  $Q$  is the diabatic heating rate;  $C$ ,  $F$ , and  $S$  denote the terms of conversion, flux, and sink/source, respectively. We not give relations for the energies and sink/source terms since they are well-known.

### 3. Including the condensation heating into the TEM energy cycle

Stone and Salustri [9] defined a more general form of the quasi-geostrophic Eliassen-Palm flux. This form allows to restate the Eliassen-Palm and non-acceleration theorems in ways that explicitly recognize the effect of eddy forcing on condensation heating. Such generalization preserves the form of the relationship between the stationary wave energy flux and Eliassen-Palm flux.

Their calculations of the modified flux divergence showed that the eddy forcing is much stronger than was indicated by calculations using the original form of the flux. They also found that condensation effects lead to stronger eddy forcing in summer than in winter. The changes in the flux divergence are due to the enhancement of eddy heat flux component of the Eliassen-Palm flux. In an overall sense this component is much more important than the eddy momentum flux component in forcing the zonal wind and temperature fields [9].

Nevertheless one must keep in mind that modified form of Eliassen-Palm flux still omits other implicit effects of eddy forcing; in particular, the effect on the frictional forces in mid-latitudes near surface may also be important.

By using the formulations of Stone and Salustri [9], we write following form for the components of residual meridional circulation and effective momentum flux

$$\begin{aligned} \bar{v}_* &= \bar{v} + \frac{\partial}{\partial p} \left( \frac{p}{R} b_2 \overline{v' (T' + q' L/c_p)} \right) \\ \bar{\omega}_* &= \bar{\omega} - \frac{p}{R} b_2 \frac{\partial}{\partial y} \overline{v' (T' + q' L/c_p)} \end{aligned} \quad (3.1)$$

$$\begin{bmatrix} B_y \\ B_p \end{bmatrix} = \begin{bmatrix} \overline{u'v'} + \frac{p}{R} b_2 \overline{v' (T' + q' L/c_p)} \frac{\partial \bar{u}}{\partial p} \\ \overline{u'\omega'} + \frac{p}{R} b_2 \overline{v' (T' + q' L/c_p)} \left( f - \frac{\partial \bar{u}}{\partial y} \right) \end{bmatrix}. \quad (3.2)$$

In Eqs. (3.1)-(3.2)  $q$  is the specific humidity,  $L$  is the latent heat of vaporization, and  $c_p$  is the specific heat at constant pressure.

Thus the formulations both for the budget equation in the TEM energy cycle and

for the energies is not modified but the physical meaning for some terms in these equations is changed.

#### 4. Main results and conclusions

In this section we give the results of numerical experiments with the atmospheric energy cycle for the mid-latitudes of Northern Hemisphere. The NCEP-NCAR reanalysis [10] is used as the data for the estimations of budget terms. Calculations are carried out for the latitudinal belts 40-65°N. As the seasonal variability for the energy contents is significant [11, 12], we make separately experiments both for the winter and for the summer. It can be noted that the integration of energy contents over the whole atmosphere at latitudinal belt gives a typical value of  $10^5$  for  $K_z$ ,  $K_E$ , and  $P_E$ ,  $10^7$  for  $P_z$ , and a dimensionality  $J m^{-2}$ . Analogous procedure applied for the terms of energy budget gives a dimensionality  $W m^{-2}$ .

Table 1 describes the energy budget and Figure 1 shows the energy cycles calculated by using the original and improved TEM-based equations for January 2002 and July 2002.

In the January mid-latitudes the processes of energy conversion result in the increase of kinetic energy (eddy part especially) and decrease of available potential energy. To the contrary, in the warm month the latter increases and kinetic energy decreases.

Since we calculate the energy budget for the geographical region with open lateral boundaries, the total budget of energy is not equal to the zero. At the same time, the budget calculated with improved TEM-based equations possesses the larger balance ability (see Table 1). As this disbalance is defined by the flux terms, for the energy cycle based on the improved TEM-equations these terms seem to be more reliable.

Thus, from our point of view, the inclusion of condensation heating into the atmospheric energy cycle based on the transformed Eulerian mean eliminates to some extent ambiguities originating during calculations by using the concept of quasi-geostrophic Eliassen-Palm flux.

Table 1

**The energy budget calculated with the original and improved equations for the Northern mid-latitudes in January 2002 and July 2002; values are in units  $W m^{-2}$**

	$\partial P_z / \partial t$	$\partial K_z / \partial t$	$\partial K_E / \partial t$	$\partial P_E / \partial t$	total
January 2002					
original	-1.39	2.95	5.85	-0.93	6.48
improved	-6.12	0.69	9.26	-0.93	2.90
July 2002					
original	11.04	-6.28	-1.54	0.36	3.58
improved	6.56	-3.11	-1.23	0.36	2.58

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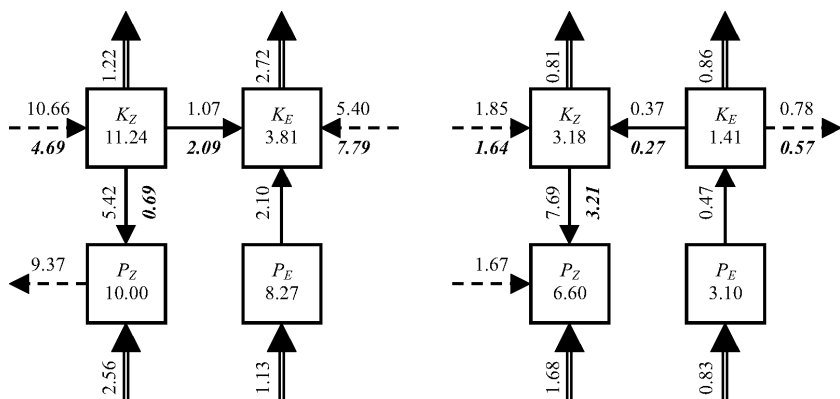


Fig. 1. The energy cycle for the Northern mid-latitudes in January 2002 (left) and July 2002 (right). The bold-italic type indicates the values calculated with improved TEM-based equations. Single arrows are the conversion terms; dashed arrows are the flux terms, and double arrows are the sink/source terms. Values for the energies are in  $10^5 \text{ J m}^{-2}$  and values for the budget terms are in  $\text{W m}^{-2}$ .

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**Залучення тепла конденсації у атмосферний цикл енергії,  
що ґрунтується на трансформованих Ейлеревих середніх**

**АНОТАЦІЯ**

Пропонується нове формулювання для аналізу атмосферної енергетики. Для того, щоб висвітлити ефект змушення вихорів внаслідок реалізації тепла конденсації, модифікується оригінальна схема циклу енергії в атмосфері, що ґрунтується на трансформованих Ейлеревих середніх та здійснюються чисельні експерименти за допомогою оригінальної та удосконаленою схем для помірних широт північної півкулі взимку та влітку. Залучення тепла конденсації змінює величини складових, що містять компоненти залишкової меридіональної циркуляції та потік ефективної кількості руху. Показано, що цикл енергії, який ґрунтується на удосконалених рівняннях, порівняно з оригінальним, виказує більшу балансність, яка є наслідком більш достовірних значень потоків енергії.

**Введение тепла конденсации в атмосферный цикл энергии,  
основанный на трансформированных Эйлеровых средних**

**АННОТАЦИЯ**

Предлагается новая формулировка для анализа атмосферной энергетики. Для того, чтобы показать эффект вынуждения вихрей вследствие реализации тепла конденсации, модифицируется оригинальная схема цикла энергии в атмосфере, основанная на трансформированных Эйлеровых средних и осуществляются численные эксперименты с помощью оригинальной и усовершенствованной схем для умеренных широт северного полушария зимой и летом. Введение тепла конденсации изменяет величины составляющих, которые содержат компоненты остаточной меридиональной циркуляции и поток эффективного количества движения. Показано, что цикл энергии, основанный на усовершенствованных уравнениях, по сравнению с оригинальным, выказывает большую балансность, которая является результатом более достоверных значений потоков энергии