MULTISCALED ANALYSIS OF IMPULSE/ENERGY TRANSPORT IN 1D NONHOMOGENEOUS HERTZIAN CHAIN

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The transmission of energy impulse through the low-dimmensional systems of discrete particles which interact with each others by elastic forces shows us the possibility of specific transformation which happens between dispersive and nonlinear modes (like solitons). We show how already in the linear approximation in nonhomogeneous systems the formation of nonlinear modes can be induced with the help of disordering which interplay with other parameters like nonlinearity and dimmension. We found a set of rigorous solutions of governed equations for wave transmition through 1D Hertzian chains either under the finite length or infinite length including the case of specific decoration.
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The study of wave transport in complex inhomogeneous media belong either to traditionally actual or complex problems where solid state physics, theory of dynamic systems and numerical modeling are combine their efforts. In the series of papers the mentioned problem have been studied in many details [1-12]. Within mentioned problems the nonlinear character of wave transport is superimposed with nonlinearity of medium where this phenomena has been occurred. Particular focus of research has been paid to low-dimensional media (like, for instance, 1D chains) where the above mentioned problem can be relatively easy solved numerically or even analytically. It is known after [1-13] that linear limit of above mentioned problem characterized by normal mode solutions as well as in the continuous limit one has a soliton. In what follows we focus on study of perturbation at intermittency between rigorous nonlinear discrete formulation of problem and linear inhomogeneous, and continuous form of governing equations. As it is shown here we have a specific wave-modes which approach the familiar solutions in the relevant limits of studied interval.

We perform numerical solution of general equation

$$\frac{d^2 z_n}{dt^2} = \gamma\left[d - \left(z_n - z_{n-1}\right)\right]^{\delta} - C_n\left[d - \left(z_{n+1} - z_n\right)\right]^{\delta} + g\,, \qquad (1)$$

which govern the evolution of displacement of the n-th grain in vertical column subject into gravity. Here: $\gamma = \frac{E\sqrt{d}}{3m\left(1-\nu^2\right)}$ - is a force constant, m - is a mass of an individual grain, d - is the diameter of an unloaded particle, E - is a Young elastic modulus, ν - is the Poisson ratio [14]. The exponent δ, in Eq.(1), could approach different values. For instance Hertzian contacts between beads gives rise to $\delta = 3/2$. Note, that in what follows we will ignore the role of dissipation in the present study. Equations like (1) normally called Hertzian.

Consider current displacement of the n-th grain z_n by use of molecular dynamics implementing 4-order predictor-corrector method. On Fig.1 we plot results of numerical simulations in case when system became weakly perturb by mechanical impulse which came from highest outmost particle. We observe a typical bell-like form of developed wave with a negligible decay and almost conserved dispersion. This characters normally belong to soliton mode.

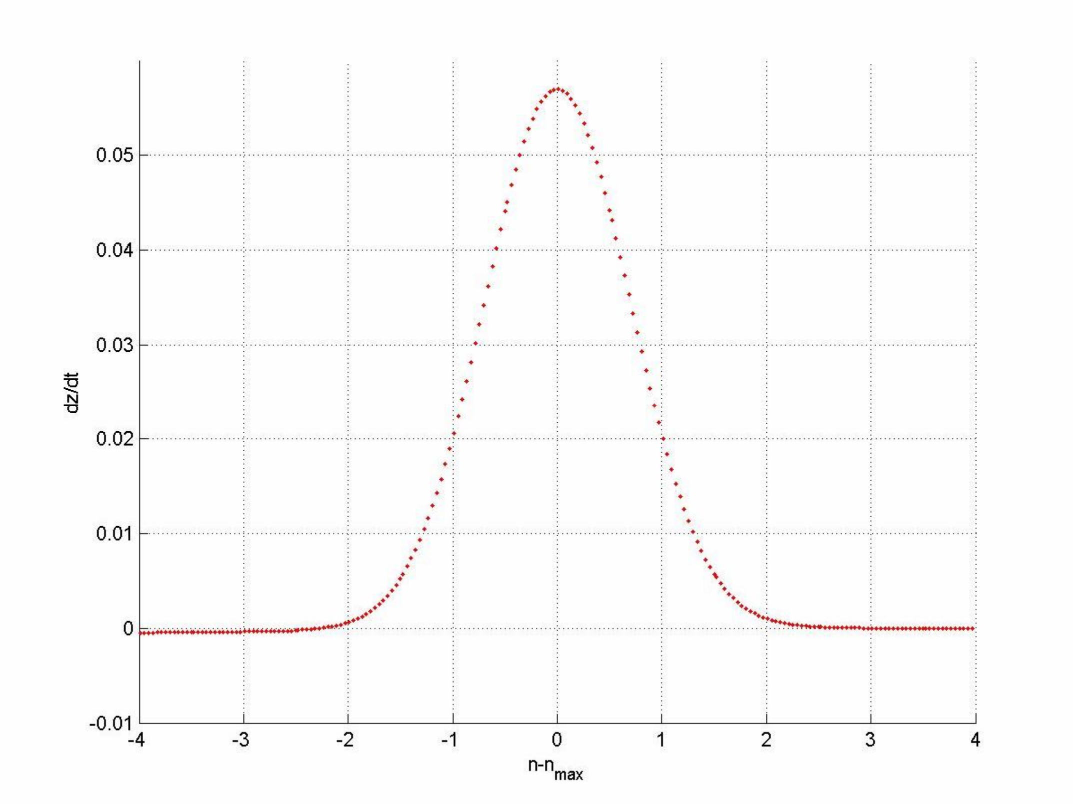

Fig.1 – Result of numerical calculation of Eq.(1).

Linearization of Eq.(1) as it is known [10-13] lead to wave-diffusion scenario of impulse transport. On Fig.2 we plot the results of numerical simulations of the linear form of Eq.(1).

Obtained results shows a complex multiscaled character of the wave transport in 1D inhomogeneous systems of the force centers which happens at intermittency between discrete and continuous limits for governed equations.

The applicability of WKB approach for linking the solutions related to the different scales of wave transport in inhomogeneous media has been discussed.

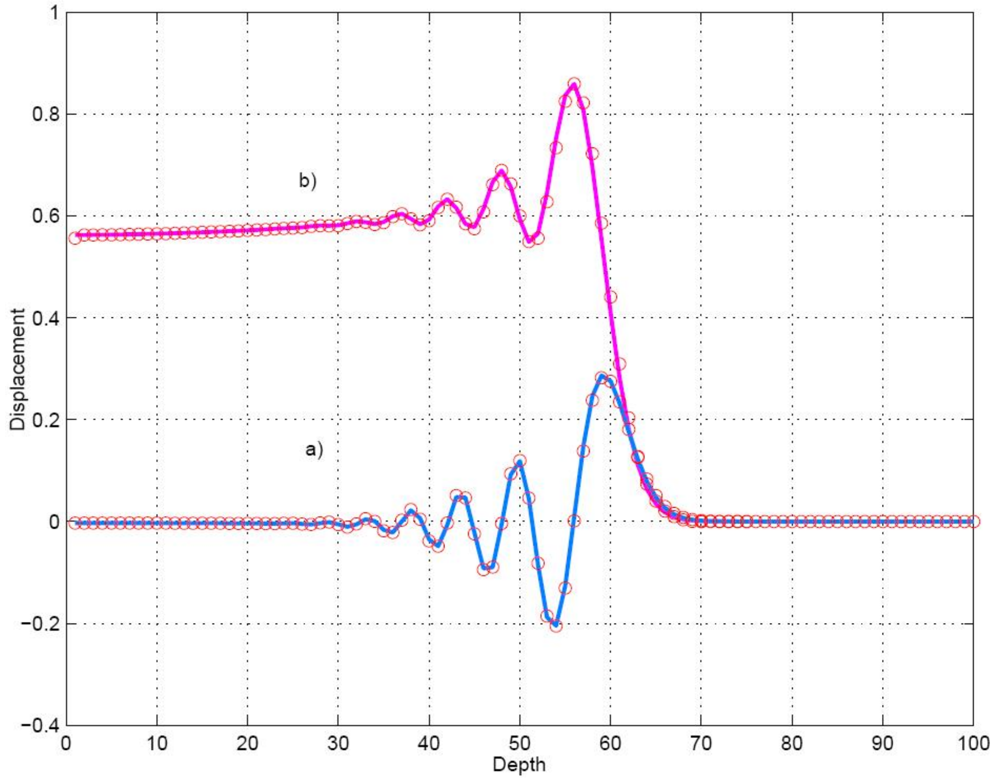

Fig.2 – Solutions for linearized form of Eq.(1): (a) – “static” excitation and (b) – “dynamic” excitation.

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