МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ОДЕСЬКИЙ ДЕРЖАВНИЙ ЕКОЛОГІЧНИЙ УНІВЕРСИТЕТ

Methodical instructions for practical work, test performance, distance learning of PhD students in the discipline "Computational Methods of optics and dynamics of quantum and laser systems. Part 3". (Training of PhD students of the specialty: 104 –"Physics and astronomy")

«Затверджено»
на засіданні групи забезпечення спеціальності
Протокол №1 від 28/08/2021 Голова групи ________ Свинаренко А.А.

«Затверджено»
на засіданні кафедри вищої та прикладної математики
Протокол №1 від 28/08/2021 Зав. кафедри ________ Глушков О.В.

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THE MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
ODESSA STATE ENVIRONMENTAL UNIVERSITY

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PREFACE

The discipline "Computational methods of optics and dynamics of quantum and laser systems" is a mandatory discipline in the cycle of professional training of graduate students (3rd level of education) in the specialty 104- Physics and astronomy.

It is aimed at mastering (providing) a number of planned competencies, including the study of the modern apparatus of optics and dynamics of quantum and laser systems, as well as the development of new computational methods and algorithms for numerical study of energy and spectroscopic characteristics of atoms, molecules, solids , the main properties of quantum and laser systems, systems in general in the field of optics and laser physics and new developed computational methods in order to achieve scientific results that create potentially new knowledge in computational applied mathematics.

The place of the discipline in the structural and logical scheme of its teaching: the knowledge gained in the study of this discipline is used in writing dissertations, the subject of which is related to the development of new computational methods and algorithms for studying energy and spectroscopic characteristics of atoms, molecules, solids, basic properties quantum and laser systems. The basic concepts of the discipline are the basic tools of a specialist in the field of optics and laser physics.

The purpose of the discipline is to master (provide) a number of competencies, in particular, to achieve relevant knowledge, understanding and ability to use methods of data analysis and statistics at the latest level, the ability to use standard and build new software based on new mathematical approaches.
-study, adapt, improve computational methods and algorithms for numerical study of the characteristics of linear and nonlinear processes in complex quantum and classical systems.

The total amount of study time for the study of the discipline is 150 hours. for full-time and part-time education, respectively.

After mastering this discipline, the graduate student must be able to use modern or personally developed new computational methods, in particular, to analyses, model, predict, program the dynamics of classical and quantum systems with the formulation of appropriate computer experiments.

The main topics: Numerical methods in the problems of modeling the processes of heat transfer, convection, radiation, energy and mass transfer.

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Топіки: Чисельні методи в задачах моделювання процесів теплообміну, конвекції, випромінювання, переносу енергії і маси. (ЗБ- Л2.3)

1. Introduction.

One of the most actual and important problems of the modern physics of aerodispersed systems, atmospheric and climate systems is study of an energy-, heat-, mass-transfer in natural continuous environments such as atmosphere or other geospheres. The most of different simplified approaches that allow to estimate the temporal and spatial structure of air ventilation in an atmosphere, significantly use as the simple molecular diffusion models as system of regression equations [1-14].

Disadvantages of these approaches are well known and became very critical if the atmosphere contains elements of convective instability. In our previous papers [7, 15-18] we have developed an advanced approach to the simulation of heat and air ventilation in atmosphere of an industrial region (so called local scale atmospheric circulation complex-field (LACCF) approach). The approach includes an improved theory of atmospheric circulation in combination with the hydrodynamic forecast model (with quantitatively correct account of turbulence in the atmosphere at local scales) and the Arakawa-Schubert model of cloud convection. Here we present a new theoretical approach to dynamics of heat-mass-transfer, thermal turbulence (as in a heat island zone as in a city's periphery) and air ventilation in atmosphere of an industrial city.

Spectrum of thermal turbulence of an industrial city's zone. The modified approximation of "shallow water" is used, but, in contrast to the standard difference methods of solution, in further we will use the spectral expansion algorithm [7, 15].

The necessary solution, for example, for the vx-ivy component for the city's heat island has the form of expansion into series on the Bessel functions. As usually, we attribute the movement to the polar coordinates (r,θ) in the area located within the zone of action of the thermal "cap" (or "heat island") of the city [7]. Flowchart of the ventilation over the urban region territory by air flows in a presence of the cloud's convection is presented in Figure 1 and explains the key physical processes [16].

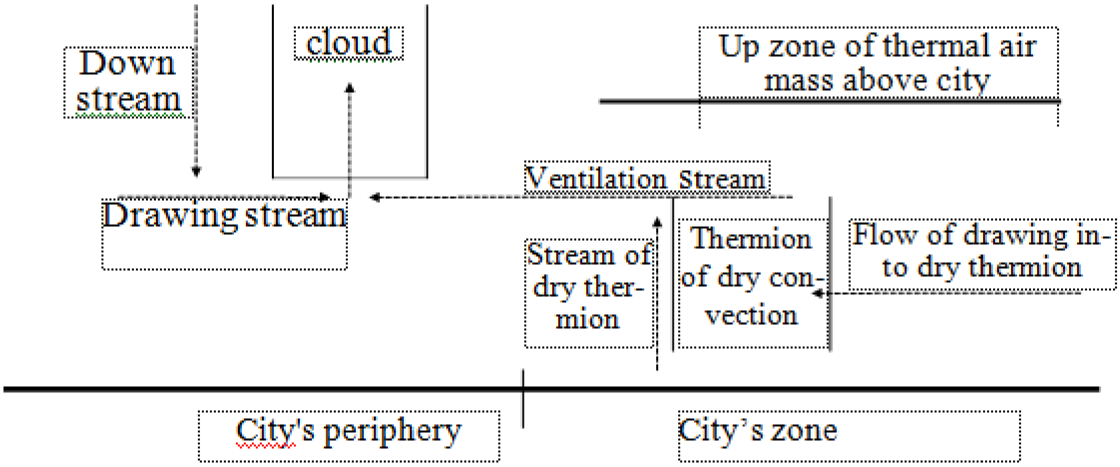

Figure 1. Flowchart of air mass transfer between the city and its periphery

The system of equations of motion is as follows [7]:

$$\frac{\partial u}{\partial t} - 2\omega v + g\frac{\partial \varsigma}{\partial r} = 0,$$

$$\frac{\partial v}{\partial t} - 2\omega u + \frac{g}{r}\frac{\partial \varsigma}{\partial \theta} = 0,$$

(1)

$$\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\frac{\partial w}{\partial z} = 0,$$

$$w = \frac{\partial \varsigma}{\partial t}$$

(where u,v,w - components of wind speed, ω - angular velocity of rotation of the circulation ring around the city heat island; g is acceleration of gravity, ζ is free surface
level of the shallow water equations (1)) with boundary conditions:

$$\frac{\partial \zeta}{\partial t}|_{z=0} = u\frac{\partial H}{\partial r} + \frac{v}{r}\frac{\partial H}{\partial \theta},$$

$$\frac{\partial \zeta}{\partial t}|_{z=H} = \frac{1}{gp}\frac{\partial P}{\partial t} - \int_0^H \left[\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right] dz + u\frac{\partial \zeta}{\partial r} + \frac{v}{r}\frac{\partial \zeta}{\partial \theta}, \qquad (2)$$

where P - is the atmospheric pressure, H - height of the thermal head or free level.
Lateral boundary condition: $\zeta|_{r=1} = 0$ corresponds to the absence of disturbances at the boundary of the circulation ring. In a single-layer fluid approximation:

$$\frac{\partial \omega}{\partial z} \approx \frac{\frac{\partial \zeta}{\partial t}|_{z=\bar{H}} - \frac{\partial \zeta}{\partial t}|_{z=0}}{H}$$

Equation (2) can be rewritten as follows:

$$\frac{\partial \zeta}{\partial t} = \frac{1}{gp}\frac{\partial P}{\partial t} - \int_0^{\bar{H}} \left[\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right] dz + u\frac{\partial \zeta}{\partial r} + \frac{v}{r}\frac{\partial \zeta}{\partial \theta} - \left[u\frac{\partial H}{\partial r} + \frac{v}{r}\frac{\partial H}{\partial \theta}\right] \quad (4)$$

Equation (1) are transformed into independent ones with respect to u and v:

$$\left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right)u = -g\frac{\partial^2 \zeta}{\partial r \partial t} - 2\omega g\frac{\partial \zeta}{r\partial \theta};$$

(5)

$$\left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right)v = 2\omega g\frac{\partial \zeta}{\partial r} - g\frac{\partial^2 \zeta}{r\partial \theta \partial t}$$

Applying the operator $\left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right)$ to equation (5), and excluding u and v, taking into account advection, we obtain a nonlinear differential equation with respect to ζ:

$$\left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right)\frac{\partial \zeta}{\partial t} = g\bar{H}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta}\right]\frac{\partial \zeta}{\partial t} + \left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right) \times$$

$$\times\left\{-\frac{1}{gp}\frac{\partial P}{\partial t} - \int_0^{\bar{H}} \left[\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right] dz + u\frac{\partial \zeta}{\partial r} + \frac{v}{r}\frac{\partial \zeta}{\partial \theta} - \left[u\frac{\partial H}{\partial r} + \frac{v}{r}\frac{\partial H}{\partial \theta}\right]\right\} \quad (6)$$

Here:

$$\frac{\partial P}{\partial t} = -u\frac{\partial P}{\partial r} - \frac{v}{r}\frac{\partial P}{\partial \theta} + \frac{C_p}{C_v}RT\frac{\partial p}{\partial z}$$

where T and $\frac{\partial P}{\partial t}$ set the temperature and baroclinic modes. The solution of equation (6) is divided into the solution of a homogeneous differential equation:

$$\left(\frac{\partial^2}{\partial t^2} + 4\omega^2\right)\frac{\partial \zeta}{\partial t} - g\bar{H}\left[\frac{\partial^2}{\partial} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]\frac{\partial \zeta}{\partial t} = 0 \quad (7)$$

and further determination of a total solution in the form as a superposition of particular solutions of the equation (7). The natural definition of particular solution is as follows: $\zeta = \zeta'(r)\cos(m\theta - \sigma t)$. Its substitution to Eq.(7) allows to obtain:

$$\frac{\partial^2 \zeta}{\partial r^2} + \frac{1}{r}\frac{\partial \zeta}{\partial r} + \left(x^2 - \frac{m^2}{2}\right)\zeta = 0 \tag{8}$$

where

$$x^2 = \frac{\sigma^2 - 4\omega^2}{H} \tag{9}$$

Equation (8) coincides with the Bessel equation and, as it is known, its own solutions will also be the Bessel functions: ζ'(r)=Jm(xr). The total solution can be presented as follows:

$$\varsigma(r,\theta,t) = \sum_{m=0}^{M}\sum_{n=0}^{M}\left(A_{m,n}\cos m\theta\cos\sigma_{m,n}t + B_{m,n}\sin m\theta\sin\sigma_{m,n}t\right)J_m\left(\lambda_{m,n}r\right) \tag{10}$$

where λm,n is "n" root of the function Jm(r). It is known that it is directly related to frequency σm,n if x in a particular solution is identified with λm,n in a complete solution. The constants Am,n, Bm,n are then determined either from the initial level for t=0 for the function ζ, which in this case is a solution of the homogeneous equation (7), or according to the method of solving the inhomogeneous equation (6), but without satisfying, in the general case, the initial condition [19].

The series (10) is simultaneously the Fourier-Bessel series for the function ζ(r,θ). Further, one can easily get:

$$u = \sum_{m=0}^{R}\sum_{n=0}^{R}\left[\frac{g}{\sigma_{m,n}^2 - 4\omega^2}\left(\sigma_{m,n}\frac{dJ_n\left(\lambda_{m,n}r\right)}{dr} - \frac{2m\omega J_n\left(\lambda_{m,n}r\right)}{r}\right)\times\right.$$
$$\left.\times\left(A_{m,n}\sin m\theta\cos\sigma_{m,n}t + B_{m,n}\cos m\theta\sin\sigma_{m,n}t\right)\right];$$
$$v = \sum_{m=0}^{R}\sum_{n=0}^{R}\left[\frac{g}{\sigma_{m,n}^2 - 4\omega^2}\left(-2\omega\frac{dJ_n\left(\lambda_{m,n}r\right)}{dr} + \frac{m\sigma_{m,n}J_n\left(\lambda_{m,n}r\right)}{r}\right)\times\right.$$
$$\left.\times\left(A_{m,n}\cos m\theta\cos\sigma_{m,n}t + B_{m,n}\sin m\theta\sin\sigma_{m,n}t\right)\right] \tag{11}$$

The initial function of the thermal relief of an industrial city and the fields associated with it are determined by the following Fourier-Bessel series:

$$F(r,\theta) = \sum_{m=0}^{M}\sum_{n=0}^{M}\left(C_{m,n}\cos m\theta\cos\sigma_{m,n}t_j + D_{m,n}\sin m\theta\sin\sigma_{m,n}t_j\right)J_{m,n}\left(\lambda_{m,n}r\right) \tag{12}$$

$$C_{m,n} = \frac{2\cos\sigma_{m,n}t_j}{\pi J_m'^2(\lambda_{m,n})}\int_0^1\int_0^{2\pi} F(\theta,r)\cos m\theta J_m(\lambda_{m.n}r)rd\theta dr, \tag{13}$$

$$D_{m,n} = \frac{2\cos\sigma_{m,n}t_j}{\pi J_m^{'2}(\lambda_{m,n})}\int_0^1\int_0^{2\pi} F(\theta, r)\sin m\theta J_m(\lambda_{m.n}r) r d\theta dr. \quad (14)$$

It is interesting to note that further, for example, the diffusion of impurities in atmosphere of industrial city inside the city's heat island from some point sources is read by the method of simple advection according to the values of the velocity projection calculated by formulas (11).

One could also directly use the equation of molecular (wave) diffusion. The vertical rise of the impurity is calculated by the formula (2b).

Application of the theory of a plane complex field for calculating air circulation in an industrial city's periphery.

Within the new geophysical approach [16-18], an air flux speed over a city's periphery in a case of convective instability can be found by method of plane complex field theory (in analogy with the Karman vortices chain model) [6,7]:

$$v_x - iv_y = \frac{df}{d\varsigma} = \frac{\Gamma}{2\pi i}\left\{\frac{1}{\varsigma - \varsigma_0} + \sum_{k=1}^{\infty}\left(\frac{1}{\varsigma - \varsigma_0 - kl} + \frac{1}{\varsigma - \varsigma_0 + kl}\right)\right\} + \frac{d}{d\varsigma}\left[\sum_{k=1}^{n}\Gamma_k \ln(\varsigma - b_k)\right]; \quad (15)$$

Here Γ_k –circulation on the vortex elements, created by clouds, b_k – co-ordinates of these elements, Γ – circulation on the standard Karman chain vortices of, l – distance between standard vortices of the Karman chain, ς - co-ordinate of the convective perturbations line (or front divider) centre, $\varsigma_0 - kl$ – co-ordinate of beginning of the convective perturbation line, $\varsigma_0 + kl$ – co-ordinate of end of this line. The indicated parameters are the input model ones and explained in details in Ref. [7].

Naturally, we further assume that possible convective disturbances on the periphery of an industrial city approach it in the form of convective ridges. The required ridges of cloudiness can be set in the problem in the field of the velocity of vertical currents and associated currents of involvement by formula (15).

The model stability of a front segment or a convective line in the general dynamics of the atmosphere can be formulated by combining solution (11) with the formula of the theory of a plane complex field (15):

$$\sum_{m=0}^{R}\sum_{n=0}^{R}\left[\frac{g}{\sigma_{m,n}^2-4\omega^2}\left(\sigma_{m,n}\frac{dJ_n(\lambda_{m,n}r^/)}{dr}-\frac{2m\omega J_n(\lambda_{m,n}r^/)}{r}\right)(A_{m,n}\sin m\theta^/\cos\sigma_{m,n}t+B_{m,n}\cos m\theta^/\sin\sigma_{m,n}t)\right]-$$

$$i\sum_{m=0}^{R}\sum_{n=0}^{R}\left[\frac{g}{\sigma_{m,n}^2-4\omega^2}(-2\omega\frac{dJ_n(\lambda_{m,n}r^/)}{dr}+\frac{m\sigma_{m,n}J_n(\lambda_{m,n}r^/)}{r})(A_{m,n}\cos m\theta^/\cos\sigma_{m,n}t+B_{m,n}\sin m\theta^/\sin\sigma_{m,n}t)\right]=$$

$$=\frac{\Gamma}{2\pi i}\left\{\frac{1}{\varsigma-\varsigma_0}+\sum_{k=1}^{\infty}\left(\frac{1}{\varsigma-\varsigma_0-kl}+\frac{1}{\varsigma-\varsigma_0+kl}\right)\right\}+\frac{d}{d\varsigma}\left[\sum_{k=1}^{n}\Gamma_k\ln(\varsigma-b_k)\right]+$$

$$+\frac{C_m}{(z-a_m)^m}+\frac{C_{m-1}}{(z-a_{m-1})^{m-1}}+...+\frac{C_1}{z-a_1}. \tag{16}$$

Here the coordinates $r^/,\theta^/$ are located in the zone of action of the functional ensemble of the front or the line of convective instability; the coordinates $a_m,a_{m-1},...,a_1$, "contouring" the said section of the frontal section include the coordinates $r^/,\theta^/$ in the immediate vicinity. The consistency of the coefficients $A_{m,n},B_{m,n}$ with the similarity coefficients $\sigma_m,\sigma_{m-1},...,\sigma_1$ of the Laurent series in a certain neighborhood with nearby coordinate points $a_m,a_{m-1},...,a_1$ determines the stability at the time point of the physical process specified by the complex velocity potential in the process of thermal circulation at the boundary of the city's thermal ring with the general solution of the thermal circulation model given by series (2).

The area of solution of the problem (1) – (2) in this case belongs to the line of maximum speeds in the zone of the thermal circulation ring of the city. The multipole coefficients $\sigma_m,\sigma_{m-1},...,\sigma_1$ specify the total contribution of focal convection beyond the lines of convective perturbations. Next, one could find the spectral agreement between the wave numbers that define the functional element in the Fourier-Bessel series with the element source of the theory of a plane field:

$$\frac{g}{\sigma_{M,N}^2-4\omega^2}\left(\sigma_{M,N}\frac{dJ_N(\lambda_{M,N}r^/)}{dr}-\frac{2M\omega J_N(\lambda_{M,N}r^/)}{r}\right)\exp[i(M\theta^/-\sigma_{M,N}t)](A_{N,M}+iB_{N,M})-$$

$$-i\frac{g}{\sigma_{M,N}^2-4\omega^2}(-2\omega\frac{dJ_N(\lambda_{M,N}r^/)}{dr}+\frac{M\sigma_{M,N}J_N(\lambda_{M,N}r^/)}{r})\exp[i(M\theta^/+\sigma_{N,M}t)]\times$$

$$\times(A_{N,M}+iB_{N,M})=\frac{\Gamma}{2\pi i}\left\{\frac{1}{\varsigma-\varsigma_0}+\left(\frac{1}{\varsigma-\varsigma_0-kl}+\frac{1}{\varsigma-\varsigma_0+kl}\right)\right\}+\frac{d}{d\varsigma}[\Gamma_k\ln(\varsigma-b_k)]+\frac{C_1}{z-a_1}; \tag{17}$$

In this case, the linearity of the circulation model is actually used. Further one can write:

$$C_1 = \frac{1}{2\pi i}\oint_0 \frac{1}{(z-a_1)^2}\left\{\frac{g}{\sigma_{M,N}^2 - 4\omega^2}(\sigma_{M,N}\frac{dJ_N(\lambda_{M,N}r^{/})}{dr} - \frac{2M\omega J_N(\lambda_{M,N}r^{/})}{r})\times\right.$$
$$\times\exp[i(M\theta^{/} - \sigma_{M,N}t)](A_{N,M} + iB_{N,M}) -$$
$$-i\frac{g}{\sigma_{M,N}^2 - 4\omega^2}(-2\omega\frac{dJ_N(\lambda_{M,N}r^{/})}{dr} + \frac{M\sigma_{M,N}J_N(\lambda_{M,N}r^{/})}{r})\exp[i(M\theta^{/} + \sigma_{M,N}t)]\times$$
$$\times(A_{N,M} + iB_{N,M}) - \frac{\Gamma}{2\pi i}\{\frac{1}{\varsigma - \varsigma_0} + (\frac{1}{\varsigma - \varsigma_0 - kl} + \frac{1}{\varsigma - \varsigma_0 + kl})\} + \frac{d}{d\varsigma}[\Gamma_k \ln(\varsigma - b_k)]\}d\varsigma;$$
(18)

Here M, N are the wave numbers of the two-dimensional harmonic: $e^{i(m\varphi + \sigma_{n,m}t)}J_n(\lambda_{m,n}r^{/})$, which best approximates the functional field of the dipole in the circle of convergence of the Taylor series. As a result, the spectrum in the Fourier-Bessel series is consistent with the source with weight C_1. Of course, here we are talking about coordinate wise matching of the spectral mode with a source in a small subdomain of the total solution. In other subareas, the desired solution may be inconsistent. Nevertheless, it is possible to achieve fairly good agreement across the formula (17) with the coefficients $\sigma_m, \sigma_{m-1}, ..., \sigma_1$ and spectral modes: $A_{m,n}, B_{m,n}$. Thus, in fact, the problem is not solved at a specific point, but on average along the convergence ring of the Laurent series. Coordinates $r^{/}, \theta^{/}$ are recalculated when calculating into a coordinate ς on a complex plane. An additional way to clarify the stability of the front section or the line of convective disturbance in the field of action of the thermal circulation of the city is based on the formula (theory of complex variable functions):

$$f(z) = -\frac{1}{2\pi i}\oint_{G'}\frac{f(\zeta)d\zeta}{\zeta - z} = \frac{C_{-1}}{z-a} + \left(\frac{C_{-2}}{(z-a)^2} + ... + \frac{C_{-n}}{(z-a)^n} + ...\right) ;$$
(19)

Here, obviously, the Laurent series with convergence in the ring in the neighborhood of the point (a) is already applied. Further, we can represent the vortex chain formula in the form of successive vortex sources in the field of the complex velocity potential ω in the complex plane with the coordinate z:

$$\frac{d\omega}{dz} = \frac{\Gamma}{2\pi i}\{\ln[\frac{(z-z_0)}{l}] + \sum_{k=1}^{\infty}[\ln(\frac{z-z_k}{-lk}) + \ln(\frac{z-z_{-k}}{lk})]\} + const;$$
(20)

Here l is the distance between the vortices in the Karman chain; $z_0, z_2, ..., z_k$ are the complex coordinates of the centers of the vortices, the coordinates z_{-k} were introduced by Karman, but can be eliminated in relation to atmospheric disturbances if the coordinate z_0 coincides with a certain center of intense convection of the intramass manifestation: $\Gamma = \Omega\sigma$ – circulation along the contour of an individual element of the vortex chain, where Ω is a projection of the vortex element onto the normal to the surface of a certain section separating the zone of intramass convection from the rest of the solution area; σ is the area of the normal section of the elementary vortex in the chain.

Specific model applications of the presented approach will be considered in subsequent works. It is interesting to remind that the processes in the thermal "cap" or heat island zone can be defined by analogy with the known soliton of fogging as a "locale", which has its own wave and turbulent (or chaotic) structure. These structures are rigidly connected to each other. Namely, the energy spectra of harmonics of the Fourier or Fourier-Bessel transforms can be understood both as a wave spectrum and as a spectrum of turbulent vortices (c.g. [19-21]).

Tests performance

Task Option 1.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of "shallow water" , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field

2) Find the solution of the modified system of the "shallow water" equations using the spectral expansion algorithm and standard difference methods.

3).To apply the spectral algorithm to numerical realization of the problem 2**.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Supersystem" for computational solving this equation. **Use the data set N1.**

Task Option 2.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of "shallow water" , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field

2) Find the solution of the modified system of the "shallow water" equations using the spectral expansion algorithm and standard difference methods.

3).To apply the spectral algorithm to numerical realization of the problem 2**.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Supersystem" for computational solving this equation. **Use the data set N2**

Task Option 3.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of “shallow water” , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field

2) Find the solution of the modified system of the “shallow water” equations using the spectral expansion algorithm and standard difference methods.

3).To apply the spectral algorithm to numerical realization of the problem 2. To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: “Supersystem” for computational solving this equation. **Use the data set N3.**

Task Option 4.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of “shallow water” , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field

2) Find the solution of the modified system of the “shallow water” equations using the spectral expansion algorithm and standard difference methods.

3).To apply the spectral algorithm to numerical realization of the problem 2. To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: “Supersystem” for computational solving this equation. **Use the data set N4**

Task Option 5.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of “shallow water” , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field
2) Find the solution of the modified system of the “shallow water” equations using the spectral expansion algorithm and standard difference methods.
3).To apply the spectral algorithm to numerical realization of the problem 2**.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: “Supersystem” for computational solving this equation. **Use the data set N5.**

Task Option 6.

1). Give the key definitions of numerical methods in problems of modeling of processes of heat exchange, convection, radiation, heat transfer, energy and mass. Explain: the definitions and key features : i). The numerical details of the standard difference methods of solution of the master system ii) The numerical details of the modified approximation of “shallow water” , iii) The numerical details of the spectral expansion algorithm iv) elements of theory of a plane complex field; iv). The numerical details of the geophysical approach, method of plane complex field
2) Find the solution of the modified system of the “shallow water” equations using the spectral expansion algorithm and standard difference methods.
3).To apply the spectral algorithm to numerical realization of the problem 2**.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: “Supersystem” for computational solving this equation. **Use the data set N6**

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