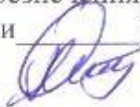



МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ  
ОДЕСЬКИЙ ДЕРЖАВНИЙ ЕКОЛОГІЧНИЙ УНІВЕРСИТЕТ

**Methodical instructions  
for practical work, test performance,  
distance learning of PhD students  
in the discipline “MATHEMATICAL AND PHYSICAL MODELS  
OF QUANTUM AND NEURAL NETWORKS”, Part 3.  
(Training of PhD students of the specialty: 113 –  
“Applied mathematics” and others)**

«Затверджено»  
на засіданні групи забезпечення спеціальності  
Протокол №1 від 28/08/2021 Голова групи  Хецеліус О.Ю.

«Затверджено»  
на засіданні кафедри вищої та прикладної математики  
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**THE MINISTRY OF EDUCATION AND SCIENCE OF  
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**Odessa 2021**

Methodical instructions for practical work, test performance, distance learning of PhD students in the discipline “MATHEMATICAL AND PHYSICAL MODELS OF QUANTUM AND NEURAL NETWORKS, Part 3. (Training specialty: 113 - “Applied mathematics” and others)

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## **PREFACE**

Discipline " MATHEMATICAL AND PHYSICAL MODELS OF QUANTUM AND NEURAL NETWORKS " is an elective discipline in the cycle of professional training of graduate students (third level of education) in the specialty 113- Applied Mathematics.

It is aimed at mastering (providing) a number of planned competencies, including the study of the modern apparatus of quantum and neural networks and on their basis to build new computational algorithms and software systems for mathematical modeling of linear and nonlinear processes in complex systems with regular and chaotic dynamics

The place of the discipline in the structural and logical scheme of its teaching: the knowledge gained in the study of this discipline is used in writing dissertations, the subject of which is related to the study of properties and regular and chaotic dynamics of various [classes of mathematical, physical chemical, cybernetic, socio-economic and environmental systems. The basic concepts of the discipline are the desired toolkit of an experienced specialist in the field of applied mathematics. The purpose of studying the discipline is to master (provide) a number of competencies, in particular, the ability to develop new and improve existing mathematical and physical models of quantum and neural networks and build new computational algorithms and software for mathematical modeling of linear and nonlinear processes in complex systems. regular and chaotic dynamics. After mastering this discipline, the graduate student must be able to improve existing modern mathematical and physical models of quantum and neural networks, as well as build new efficient models, and on their basis to develop new computational algorithms and software for analysis, mathematical modeling and prediction of linear and nonlinear processes in complex systems with regular and chaotic dynamics. The study of the discipline "Mathematical and Physical Models of Quantum and Neural Networks" is conducted in the second year of study (4th semester; full-time and part-time forms of study) and includes lectures and practical classes. Types of control of current knowledge - tests and term papers, surveys, tests.

Topics of these issue: Mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field.

**Topic:** Mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers.

Excitonic system in semiconductor and atom in electric field.

**Topic:** Математичні та фізичні моделі квантових нейромереж. Масив квантових точок у напівпровідниковій гетероструктурі як приклад квантової нейромережі та шлях до побудови адіабатичних квантових комп'ютерів. Excitonic system in semiconductor and atom in electric field.

## ЗБ- Л4

### 1 Introduction

#### 1. Introduction

This issue considers a group of topics such as mathematical and physical models of quantum neural networks, the system of quantum dots in a semiconductor heterostructure, the excitonic system in semiconductor and atom in electric field as an example of a quantum neural network. In fact the known Stark effect is used as the fundamental one in mathematical and physical modeling quantum neural networks.

The remarkable Stark effect has a long history and until recently it was believed that the Stark effect is fully understood and fundamental problems remained. However, an observation of the Stark effect in a constant (DC) electric field near threshold in hydrogen and alkali atoms led to the discovery of resonances extending into the ionization continuum by Glab et al and Freeman et al (c.f.[1]). Calculation of the characteristics of these resonances as well as the Stark resonances in the strong electric field remains very important problem of modern atomic physics.

It should be noted that the same class of problems has been arisen in a physics of semiconductors (c.f.[14-17]). It is well known that the availability of excitons in semiconductors resulted experimentally in the special form of the main absorption band edge and appearance of discrete levels structure (f.e. hydrogen-like spectrum in  $\text{Cu}_2\text{O}$ ). Beginning from known papers of Gross-Zaharchenya, Thomas and Hopfield et al (c.f.[14-17]), a calculation procedure of the Stark effect for exciton spectrum attracts a deep interest permanently.

Very interesting physics occurs in a case of the excitons in quantum dots, wires etc, where the other geometry and energetics in comparison with the bulk semiconductor makes the field effect more intriguing. The exciton states in the quantum dots have been studied in a number of papers and have been observed by photoluminescence experiments (c.f. [14-17]). Naturally, the electronic states in quantum dots (wires) depends on either the confining potential and the interacting force between the particles. Now the electric field effect on the electron-hole states and on the confined excitonic states is often referred to the quantum confined Stark effect. Now it is represented an interest to study an influence of quantum well concentration profile and width on Stark shifts in case of such a system as system GaAs-AlGaAs etc. In this paper we study the Stark effect for non-H atom of rubidium and for excitons in an external uniform electric field within the operator perturbation theory method. The Stark resonance energies in rubidium and the Stark shift for the  $n=2$  state of the Wannier-Mott excitons in the  $\text{Cu}_2\text{O}$  semiconductor and excitons in the parabolic quantum dot (GaAs) in the electric field are listed.

### **Method for calculating energies and widths of Stark resonances**

The Schrödinger equation for the electron function taking into account the uniform electric field and field of the nucleus (Coulomb units are used: for length, 1 unit is  $\hbar^2/Ze^2m$ ; for energy 1 unit is  $mZ^2e^4/\hbar^2$ ) is [6,57]:

$$[-(1 - N/Z) / r + V_m(r) + \varepsilon z - 1/2\Delta - E ] \psi = 0, \quad (1)$$

where  $E$  is the electron energy,  $Z$  is the nucleus charge,  $N$  is the number of electrons in the atomic core (for the hydrogen atom:  $Z=1, N=0$ ),  $V_m$  is an model potential (for the hydrogen atom  $V_m=0$ ). Firstly, we only deal with the Coulomb part of the electron- atomic residue interaction. The non-Coulomb part, as well as relativistic effects, can be approximately accounted for next step. The separation of variables in the parabolic coordinates ( $\xi = r + z, \eta = r - z, \varphi = \tan^{-1}(y/x)$ ) :

$$\psi(\zeta, \eta, \varphi) = f(\zeta) g(\eta) (\zeta \cdot \eta)^{|m|/2} \exp(im\varphi) / (2\pi)^{1/2} \quad (2)$$

transforms it to the system of two equations for the functions  $f, g$ :

$$f'' + \frac{|m|+1}{t} f' + [1/2E + (\beta_1 - N/Z) / t - 1/4\varepsilon(t) t] f = 0, \quad (3)$$

$$g'' + \frac{|m|+1}{t} g' + [1/2E + \beta_2 / t + 1/4\varepsilon(t) t] g = 0, \quad (4)$$

coupled through the constraint on the separation constants:

$$\beta_1 + \beta_2 = 1 \quad (5)$$

For the uniform electric field

$$\varepsilon(t) = \varepsilon.$$

In principle, the more realistic models can be considered in the framework of our approach. Potential energy in equation (4) has the barrier. Two turning points for the classical motion along the  $\eta$  axis,  $t_1$  and  $t_2$ , at a given energy  $E$  are the solutions of the quadratic equation ( $\beta = \beta_1, E = E_0$ ):

$$t_2 = \{ [E_0^2 - 4\varepsilon(1-\beta)]^{1/2} - E_0 \} / \varepsilon, \quad (6)$$

$$t_1 = \{ -[E_0^2 - 4\varepsilon(1-\beta)]^{1/2} - E_0 \} / \varepsilon, \quad t_1 < t_2 \quad (7)$$

Here and below  $t$  denotes the argument common for the whole equation system. To simplify the calculational procedure, the uniform electric field  $\varepsilon$  in (3) and (4) should be substituted by the function [57,58]:

$$\varepsilon(t) = \frac{1}{t} \varepsilon \left[ (t - \tau) \frac{\tau^4}{\tau^4 + t^4} + \tau \right] \quad (8)$$

with sufficiently large  $\tau$  ( $\tau = 1.5t_2$ ).

The function  $\varepsilon(t)$  practically coincides with the constant  $\varepsilon$  in the inner barrier motion region ( $t < t_2$ ) and disappears at  $t \gg t_2$ .

The minimal acceptable value of  $\tau$  introduced in the spatial dependence of the electric field, which does not influence the final results, can be established experimentally. Thus, the final results do not depend on the parameter  $\tau$  (the further calculation has entirely confirmed this fact). Besides the pure technical

convenience, the case of an asymptotically disappearing electric field is more realistic from the physical point of view. Now we deal with the asymptotically free (without electric field) motion of the ejected electron along the  $\eta$ -axis. The corresponding effective wavenumber is:

$$k = (E/2 + \varepsilon\tau/4)^{1/2}. \quad (9)$$

The scattering states energy spectrum now spreads over the range  $(-\varepsilon\tau/2, +\infty)$ , compared with  $(-\infty, +\infty)$  in the uniform field. In contrast to the case of a free atom in scattering states in the presence of the uniform electric field remain quantified at any energy  $E$ , i.e. only definite values of  $\beta_1$  are possible.

The total Hamiltonian  $H(\zeta, \nu, \varphi)$  does not possess the bound stationary states. According to OPT [6, 56-58]), one has to define the zero order Hamiltonian  $H_0$ , so that its spectrum reproduces qualitatively that of the initial one. In contrast to  $H$ , it must have only stationary states. To calculate the width  $\Gamma$  of the concrete quasistationary state in the lowest PT order one needs only two zeroth-order EF of  $H_0$ : bound state function  $\Psi_{Eb}(\varepsilon, \eta, \varphi)$  and scattering state function  $\Psi_{Es}(\varepsilon, \eta, \varphi)$  with the same EE. We solve a more general problem: a construction of the bound state function along with its complete orthogonal complementary of scattering functions  $\Psi_E$  with  $E \in (-\frac{1}{2}\varepsilon\tau, +\infty)$ . First, one has to define the EE of the expected bound state. It is the well known problem of states quantification in the case of the penetrable barrier [65,66]. Following [57], we solve the system (3) and (4) with the total Hamiltonian  $H$  under the conditions:

$$f(t) \rightarrow 0 \text{ at } t \Rightarrow \infty, \quad (10a)$$

$$\partial x(\beta, E) / \partial E = 0 \quad (10b)$$

with 
$$x(\beta, E) = \lim_{t \rightarrow \infty} [g^2(t) + \{g'(t)/k\}^2] t^{|\mathbf{m}|+1}. \quad (11)$$

The first condition ensures the finiteness of motion along the  $\zeta$ -axis, the second condition minimizes the asymptotic oscillation amplitude for the function describing the motion along the  $\eta$ -axis. These two conditions quantify the bound energy  $E$  and separation constant  $\beta_1$ . We elaborated a special numerical procedure for this two-dimensional eigenvalue problem. Our procedure deals



repeatedly with the solving of the system of the ordinary differential equations (3) and (4) with probe pairs of  $E, \beta_1$ . The corresponding EF:

$$\psi_{Eb}(\zeta, \eta, \varphi) = f_{Eb}(\zeta) g_{Eb}(\eta) (\zeta \eta)^{|m|/2} \exp(im\varphi) (2\pi)^{-1/2}. \quad (12)$$

Here  $f_{Eb}(t)$  is the solution of (3) (with the just determined  $E, \beta_1$ ) at  $t \in (0, \infty)$  and  $g_{Eb}(t)$  is the solution of (4) (with the same  $E, \beta_1$ ) at  $t < t_2$  (inside barrier) and  $g(t) = 0$  otherwise. These bound state EE, eigenvalue  $\beta_1$  and EF for the zero-order Hamiltonian  $H_0$  coincide with those for the total Hamiltonian  $H$  at  $\varepsilon \Rightarrow 0$ , where all the states can be classified due to the quantum numbers  $n, n_1, n_2, m$  (principal, parabolic, azimuthal) connected with  $E, \beta_1, m$  by the well known expressions. We preserve the  $n, n_1, m$  states classification in the non-zero  $\varepsilon$  case. The scattering state functions:

$$\psi_{E's}(\zeta, \eta, \varphi) = f_{E's}(\zeta) g_{E's}(\eta) (\zeta \eta)^{|m|/2} \exp(im\varphi) (2\pi)^{-1/2} \quad (13)$$

must be orthogonal to the above defined bound state function and to each other. In addition, these functions must describe the motion of the ejected electron, i.e.  $g_{E's}$  must satisfy the equation (4) asymptotically. Following the OPT ideology [57], we choose the next form of  $g_{E's}$ :

$$g_{E's}(t) = g_1(t) - z_2' g_2(t) \quad (14)$$

with  $f_{E's}$  and  $g_1(t)$  satisfying the differential equations (3) and (4). The function  $g_2(t)$  satisfies the non-homogeneous differential equation, which differs from (4) only by the right-hand term, disappearing at  $t \Rightarrow \infty$ . The total equation system, determining the scattering function, reads

$$\begin{aligned} f''_{E's} + \frac{|m|+1}{t} f'_{E's} + [1/2E' + (\beta_1' - N/Z) / t - 1/4 \varepsilon(t)t] f_{E's} &= 0, \\ g_1'' + \frac{|m|+1}{t} g_1' + [1/2E' + \beta_2' / t + 1/4 \varepsilon(t)t] g_1 &= 0, \\ g_2'' + \frac{|m|+1}{t} g_2' + [1/2E + \beta_2' / t + 1/4 \varepsilon(t)t] g_2 &= 2g_{Eb}, \end{aligned} \quad (15)$$

$(\beta'_1 + \beta'_2 = 1)$ . As mentioned above there remains motion quantification for  $E' \in \left(-\frac{1}{2}\varepsilon\tau, +\frac{1}{2}\varepsilon\tau\right)$ . At the given  $E'$ , the only quantum parameter  $\beta'_1$  is determined

by the natural boundary condition:  $f_{E's} \Rightarrow 0$  at  $t \Rightarrow \infty$ .

Of course:  $\beta'_1 = \beta_1$ ,  $f_{E's} = f_{Eb}$  at  $E' = E$ ; only this case is needed in the particular problem we deal with here. The coefficient  $z'_2$  ensures the orthogonality condition  $\langle \Psi_{Eb} | \Psi_{E's} \rangle = 0$ :

$$z'_2 = \{ \iint d\zeta d\eta (\zeta + \eta) f^2_{Eb}(\zeta) g_{Eb}(\eta) g_1(\eta) \} / \{ \iint d\zeta d\eta (\zeta + \eta) f^2_{Eb}(\zeta) g_{Eb}(\eta) g_2(\eta) \}. \quad (16)$$

One can check that

$$\langle \Psi_{E's'} | \Psi_{E''s} \rangle = 0 \text{ for } E' \neq E''$$

The imaginary part of state energy in the lowest PT order is

$$\text{Im}E = \Gamma/2 = \pi |\langle \Psi_{Eb} | H | \Psi_{E's} \rangle|^2 \quad (17)$$

with the total Hamiltonian  $H$ . The state functions  $\Psi_{Eb}$  and  $\Psi_{E's}$  are assumed to be normalized to 1 and by the  $\delta(k - k')$  condition, accordingly. The action of  $H$  on  $\Psi_{Eb}$  is defined unambiguously by (15):

$$(H - E')\Psi_{E's} = 2|m|(\zeta \cdot \eta^2) \cdot f_{E's}(\zeta) g_{Eb}(\eta) z'_2 \exp(im\varphi) / [(2\pi)^{1/2} (\zeta + \eta)],$$

$$\langle \Psi_{Eb} | H | \Psi_{E's} \rangle = \iint d\zeta d\eta (\zeta \eta)^{|m|} \eta f^2_{Eb}(\zeta) f^2_{E's}(\zeta) g_{Eb}(\eta) z'_2. \quad (18)$$

The matrix elements  $\langle \Psi_{Eb} | H | \Psi_{E's} \rangle$  entering the high-order PT corrections can be determined in the same way. All the two-dimensional integrals in (16)-(18) and the normalization coefficients can be expressed through the next set of one-dimensional integrals:

$$\begin{aligned} I_1 &= \int dt f_b^2(t) t^{|m|}, & I_2 &= \int dt f_b^2(t) t^{|m|+1}, \\ I_3 &= \int dt g_b(t) g_1(t) t^{|m|}, & I_4 &= \int dt g_b(t) g_1(t) t^{|m|+1}, \\ I_5 &= \int dt g_b(t) g_2(t) t^{|m|}, & I_6 &= \int dt g_b(t) g_2(t) t^{|m|+1}, \end{aligned}$$

$$I_7 = \int dt g_b^2(t) t^{|m|}, \quad I_8 = \int dt g_b^2(t) t^{|m|+1}, \quad (19)$$

calculated with the arbitrary normalized functions  $f_{Eb}, g_{Eb}, f_2, g_2$ , and  $f_1 = f_{Eb}, g_1 = g_{Eb}$ . In this notation

$$\Gamma = 32\pi\alpha_2^2 N_s^2 I_1^2 I_8^2 / [I_2 I_7 + I_1 I_8],$$

$$z_2 = [I_1 I_4 + I_2 I_3] / [I_1 I_6 + I_2 I_5] \quad (20)$$

with

$$N_s^2 = \lim_{t \rightarrow \infty} X(t) / \{ 2\pi\eta^{2|m|+1} [g_s^2(\eta)X^2(t) + g_s'^2(\eta)] \},$$

$$X(t) = \{ E/2 + (\beta - N/Z)/t - E t/4 \}^{1/2} \quad (21)$$

Remember that arbitrary normalized state functions are assumed in (20) and (21). The whole calculational procedure at known resonance energy  $E$  and separation parameter  $\beta_1$  has been reduced to the solution of one system of the ordinary differential equations. This master system includes the differential equations for the state functions  $f_{Eb}, g_{Eb}, f_{Es}, g_{Es}$ , as well as the equations for the integrals  $I_1 - I_8$ .

### Stark effect for excitonic system in semiconductors

The above analogous method can be formulated for description of the Stark effect for the Wannier-Mott excitons in the bulk semiconductors [4]. Really, the Schrödinger equation for the Wannier-Mott exciton looks as follows:

$$[-\hbar^2 \nabla_e^2 / 2m_e^* - \hbar^2 \nabla_h^2 / 2m_h^* - e^2 / \epsilon r_{eh} + eFr_e - eFr_h] \Psi = E\Psi \quad (22)$$

where  $m_e^*$  ( $m_h^*$ ) are the effective-mass for the electron (hole),  $\epsilon$  is the background dielectric constant. Introducing the relative coordinates:  $r = r_e - r_h$  and the corresponding momenta  $p$  with reduced mass  $p = m_e^* m_h^* / M$  (the momenta  $P$  with the total-mass  $M = m_e^* + m_h^*$ ) and center-of-mass coordinate

$$\rho = (m_e^* r_e + m_h^* r_h) / (m_e^* + m_h^*),$$

one could rewrite (22) as:

$$[-\hbar^2 \nabla^2 / 2\mu - e^2 / \epsilon r - \hbar / 2 \cdot (1/m_h^* - 1/m_e^*) K \cdot p - eFr] \Psi = [E - \hbar^2 K^2 / 8\mu] \Psi \quad (23)$$

This equation then could be solved by the method, described above. The other details can be found in Refs. [1,4].

A problem of the Stark effect for quantum dots requires more detailed consideration. For definiteness, below we study the Stark effect in the parabolic quantum dot.

Within the effective-mass approximation and neglecting the band-structure effects, the Hamiltonian of an exciton in a parabolic quantum dot with the same quantization energy  $\hbar\Omega$  (for the electron and hole), and subjected to an external electric field, can be expressed as :

$$H = -\hbar^2\nabla_e^2 / 2m_e^* + (1/2)(m_e^*)\Omega^2 r_e^2 - \hbar^2\nabla_h^2 / 2m_h^* + (1/2)(m_h^*)\Omega^2 r_h^2 - e^2 / \epsilon r_{eh} + eFr_e - eFr_h \quad (24)$$

where all notations are defined above.

Further, as above, using the relative coordinate, the momenta with reduced mass and center-of-mass coordinate and the momenta with the total-mass  $M$ , the Hamiltonian  $H$  (7) can be represented as :

$$H = P^2 / 2M + (1/2)M\Omega^2 \rho^2 + p^2 / 2\mu + (1/2)\mu\Omega^2 r^2 - e^2 / \epsilon r + eFz \quad (25)$$

Further let us note that the part which depends only on the center-of-mass coordinate in Eq.(10) is corresponding to the Hamiltonian of a well-known 3D harmonic oscillator and the exciton properties is essentially determined by the relative Hamiltonian  $H_r$ .

The field term added to the z-direction confinement describes a displaced harmonic oscillator centred in  $z_0 = eF / \mu\Omega^2$  with the frequency  $\Omega_r$  inferior to  $\Omega$ . Besides, as usually [17], in order to solve the Hamiltonian  $H_r$ , one should introduce an interaction potential which obeys to the known Hooke's force with the parameter  $\lambda$  by adding and subtracting the potential:

$$V(r) = \lambda[(1/2)\mu\Omega^2 r^2 - \hbar\Omega].$$

Surely then the Hamiltonian  $H$  is splitted into two terms with the one term being exactly solvable while the other can be treated as a perturbation.

Such a scheme is corresponding to method by Jaziri-Bastard-Bennaceur [17]. Our approach is in the direct numerical solving the problem. Let us remind that the introduced potential is similar to the interaction potential between electron-electron used by Johnson-Payne and it can hardly be considered as the correct potential for the all electron-hole separation. Nevertheless, here one

could adjust the interaction parameter  $\lambda$  so in order to provide the best fit of the true interaction which is the Coulomb interaction, and for the dominant range of separation  $r$  [17].

The attraction potential  $V(r)$  will have negative value with positive  $\lambda$ , and it yields a physically reasonable fit to the exact interaction for electron-hole separation  $r < (2)^{1/2} R_0$  (here  $R_0$  is the quantum dot radius defined as  $\sqrt{\hbar / \mu \Omega}$ ). As usually, the total energy corresponding to exciton ground state is obtained as :

$$E_T = (3/2)\hbar\Omega + (3/2)\hbar\Omega_r - \lambda\hbar\Omega - (e^2 F^2)/(2\mu\Omega^2)$$

where  $\Omega_r = \Omega\sqrt{1+\lambda}$ . The field-induced energy shifts can be expressed as:  $\Delta E = E_T(F=0) - E_T(F)$ , where  $E(F=0)$  is the corresponding energy in the free (i.e. zero-field) state. The main aim is to determine the exciton binding energy defined by  $E_B = E_e + E_h - E_T$ , where  $E_e, E_h$  are the energies corresponding to the one-particle Hamiltonian.

The calculation results for Stark resonances energies in the rubidium atom for the electric field strength  $\varepsilon=2.189$  kV/cm are presented in Table 1. For comparison we have also presented the experimental data [13], the results of calculation within the  $1/n$ -expansion method by Popov et al [12]. For the most long-living Stark resonances with quantum numbers  $n_2 = 0$ ,  $m = 0$ , a width of energy level is significantly less than a distance between them. These states are mostly effectively populated by  $\pi$ -polarized light under transitions from states with  $(n_1 - n_2) = \max$ ,  $m = 0$ .

As a result, the sharp isolated resonances (their positions under  $E > 0$  are determined by energies of quasi-stationary states with  $n_2=0$ ,  $m=0$ ) are appeared under photo ionization from these states in a case of  $\pi$ -polarization

In ref. [4] there are listed the preliminary estimates of the Stark shifts of the  $n=2$  state of excitons in the  $\text{Cu}_2\text{O}$  semiconductor (yellow series) at the electric field strength 600 V/cm results and indicated on the physically reasonable agreement with the known results by Thomas and Hopfield (TH) [14]. Our final value for the Stark shift of the  $n=2$  state excitons in the  $\text{Cu}_2\text{O}$  semiconductor (yellow series) at the electric field strength 600 V/cm results in  $-0,308$  meV in a good agreement with experimental data of Gross et al.[28]. Under increasing the electric field strength changing a potential on a small enough distance (the orbits diameter) will become comparable with the bond energy of particle on this orbit

Table 1. The energies ( $\text{cm}^{-1}$ ) of the Stark resonances for the Rb atom ( $\epsilon=3,59$   $\text{kV/cm}$ ):

A-experimental data ; B- Popov et al; C- OPT approach.

$n_1n_2m$	$\delta$	A	B		C
23,0,0	0,656	133,1	132,8	132,9	133,0
22,0,0	0,681	157,0	157,1	157,2	157,1
21,1,0	0,517	161,1	159,5	160,6	160,9
20,2,0	0,400	163,9	163,2	163,7	163,9
21,0,0	0,708	185,2	184,2	184,8	185,1
20,1,0	0,531	186,3	185,4	185,8	186,2
20,0,0	0,737	217,2	214,6	214,9	216,9
18,1,0	0,561	248,4	247,2	247,3	248,2
16,2,0	0,428	-	-	-	285,5
18,0,0	0,802	-	-	-	289,3

. According to our data and data by Gross et al., the corresponding electric field is  $\sim 9 \cdot 10^3$  V/cm. we have tried to discover the chaotic behavior of the exciton dynamics in an electric field, however near ionization boundary exciton does not demonstrate behaviour of quantum chaotic system, which is similar to hydrogen or on-H atom dynamics in a strong field and manifested as unusual features in a photoionization spectra (alkali atoms) [5,14]. Further we list some data on the Stark shifts excitons in a GaAs semiconductor quantum dot (table 2).

Table 2. The Stark shifts (meV) for exciton in the GaAS quantum dot: A- Jaziri-Bastard-Bennaceur method; B- OPT approach

$F$ (kV/cm)	$R$ (Å)	A	B
50	50	3.9	4.0
50	80	14	14.1
50	120	45	45.2
100	50	13	13.1
100	80	56	56.6
100	120	158	159.8

Comparison of the presented preliminary data shows that the different results are in the physically reasonable agreement. The corresponding accuracy

is of the order of 1%, however, it should be noted [17] that though the method [17] is much simpler in comparison with the direct variational approach, but it cannot envisaged for any strength electric field and/nor any quantum dot size. The important feature of the operator perturbation theory formalism is that it can be applied for any strength electric field.

## 2. Task options

### Task Option 1.

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor InP** and atom in electric field **Fr**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).

### Task Option 2.

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor GaN** and atom in electric field **Rb**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).



### **Task Option 3.**

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor GaAs** and atom in electric field **K**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).

### **Task Option 4.**

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor AlAs** and atom in electric field **Li**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).

### Task Option 5.

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor CdTe** and atom in electric field **Cs**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).

### Task Option 6.

- 1). Give the key definitions in mathematical and physical models of quantum neural networks. An array of quantum dots in a semiconductor heterostructure as an example of a quantum neural network and a way to construct adiabatic quantum computers. Excitonic system in semiconductor and atom in electric field and Stark effect. Computing the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov:
- 2). Mathematical and physical essence of quantum-mechanical approach to treatment of the excitonic system in semiconductor and atom in electric field and Stark effect.
- 3). To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of the excitonic system in **semiconductor InGa** and atom in electric field **Na**. Construct the simple mathematical and physical models of quantum NNW. To perform its practical realization (using Fortran Power Station, Version 4.0; PC Code: "Superatom-Stark" (all necessary numerical parameters should be self-taken).

## References

1. Lisitsa, V.S. New results on the Stark and Zeeman effects in the hydrogen atom. *Sov. Phys. Usp.* **1987**, *30*, 927-960.
2. Mendelson, L.B. High-Order Perturbation Theory for a One-Electron Ion in a Uniform Electric Field. *Phys. Rev.* **1968**, *176*, 90-98.
3. Dolgov, A.D.; Turbiner, A.V. A new approach to the Stark effect in hydrogen. *Phys. Lett. A.* **1977**, *77*, 15-18.
4. Alexander, M.H. Exact Treatment of the Stark Effect in Atomic Hydrogen. *Phys. Rev.* **1969**, *178(1)*, 34-40.
5. Rao, J.; Liu, W.; Li, B. Theoretical complex Stark energies of hydrogen by a complex-scaling plus B-spline approach. *Phys. Rev. A.* **1994**, *50*, 1916-1919 (1994).
6. Rao, J.; Li, B. Resonances of the hydrogen atom in strong parallel magnetic and electric fields. *Phys. Rev. A.* **1995**, *51*, 4526-4530.
7. Meng, H.-Y.; Zhang, Y.-X.; Kang, S.; Shi, T.-Y.; Zhan, M.-S. Theoretical complex Stark energies of lithium by a complex scaling plus the B-spline approach. *J. Phys. B: At. Mol. Opt. Phys.* **2008**, *41*, 155003.
8. Hehenberger, M.; McIntosh, H.V.; Brändas, E. Weyl's theory applied to the Stark effect in the hydrogen atom. *Phys. Rev. A* **1974**, *10 (5)*, 1494-1506.
9. Hehenberger, M.; McIntosh, H.V.; Brändas, E. Weyl's theory applied to the Stark effect in the hydrogen atom. *Phys. Rev. A* **1974**, *10 (5)*, 1494-1506.
10. Brändas, E.; Froelich, P. Continuum orbitals, complex scaling problem, and the extended virial theorem. *Phys. Rev. A* **1977**, *16, No. 6*, 2207.
11. Brändas, E.; Hehenberger, M.; McIntosh, H.V. Dispersion Relations and Spectral Densities. *Int. J. Quant. Chem.* **1975**, *9*, 103-117.
12. Rittby, M.; Elander, N.; Brändas, E. Weyl's theory and the complex-rotation method applied to phenomena associated with a continuous spectrum. *Phys. Rev. A* **1981**, *24, No. 3*, 1636.
13. Froelich, P.; Davidson, E.R.; Brändas, E. Error estimates for complex eigenvalues of dilated Schrödinger operators. *Phys. Rev. A* **1983**, *28, No. 5*, 2641.
14. Lipkin, N.; Moiseyev, N.; Brändas, E. Resonances by the exterior-scaling method within the framework of the finite-basis-set approximation. *Phys. Rev. A* **1989**, *40, No. 2*, 549.
15. Rittby, M.; Elander, N.; Brändas, E. Scattering in view of the Titchmarsh-Weyl theory. *Int. J. Quant. Chem.* **1983**, *23 (3)*, 865-874.
16. Simon, B. The definition of molecular resonance curves by the method of

- exterior complex scaling. *Phys. Lett. A* **1979**, 71 (2-3), 211-214.
17. Brändas, E.; Froelich, P.; Obcemea, C.H., Elander, N.; Rittby, M. Note on the complex stabilization method. *Phys. Rev. A* **1982**, 26, No. 6, 3656.
  18. Engdahl, E.; Brändas, E.; Rittby, M.; Elander, N. Generalized Green's functions and spectral densities in the complex energy plane. *J. Math. Phys.* **1986**, 27, 2629.
  19. Engdahl, E.; Brändas, E.; Rittby, M.; Elander, N. Resonances and background: A decomposition of scattering information. *Phys. Rev. A* **1988**, 37(10), 3777-3789.
  20. *Resonances. The Unifying Route Towards the Formulation of Dynamical Processes - Foundations and Applications in Nuclear, Atomic and Molecular Physics, Series: Lecture Notes in Physics*; Brändas, E.; Elander, N. Eds.; Springer: Berlin, **1989**, vol. 325, pp 1-564.
  21. Scrinzi, A.; Elander, N. A finite element implementation of exterior complex scaling for the accurate determination of resonance energies. *J. Chem. Phys.* **1993**, 98(5), 3866-3875.
  22. Ostrovsky, V.N.; Elander, N. Scattering resonances and background associated with an asymmetric potential barrier via Siegert pseudostates. *Phys. Rev. A* **2005**, 71, 052707.
  23. Sigal, I.M. Geometric theory of Stark resonances in multielectron systems. *Commun. Math. Phys.* **1988**, 119 (2), 287-314.
  24. Herbst, I.W.; Simon, B. Stark Effect Revisited. *Phys. Rev. Lett.* **1978**, 41(2), 67-69.
  25. Silverstone, H.J.; Adams, B.G.; Cizek, J.; Otto, P. Stark Effect in Hydrogen: Dispersion Relation, Asymptotic Formulas, and Calculation of the Ionization Rate via High-Order Perturbation Theory. *Phys. Rev. Lett.* **1979**, 43, No. 20, 1498-1501.
  26. Cerjan, C.; Hedges, R.; Holt, C.; Reinhardt, W.P.; Scheibner, K.; Wendoloski, J.J. Complex coordinates and the Stark effect. *Int. J. Quant. Chem.* **1978**, 14 (4), 393-418.
  27. Luc-Koenig, E.; Bachelier, A. Systematic theoretical study of the Stark spectrum of atomic hydrogen. I. Density of continuum states. *J. Phys. B: At. Mol. Phys.* **1980**, 13, 1743-1756
  28. Luc-Koenig, E.; Bachelier, A. Systematic theoretical study of the Stark spectrum of atomic hydrogen. I. Density of continuum states. *J. Phys. B: At. Mol. Phys.* **1980**, 13, No. 9, 1743.
  29. Damburg, R.J.; Kolosov, V.V. A hydrogen atom in a uniform electric field.

- J. Phys. B: At. Mol. Phys.* **1976**, 9, No. 18, 3149.
30. Maquet, A.; Chu, S.I.; Reinhardt, W.P. Stark ionization in dc and ac fields: An L2 complex-coordinate approach. *Phys. Rev. A* **1983**, 27, No. 6, 2946-2970.
  31. Reinhardt, W.P. Padé summations for the real and imaginary parts of atomic stark eigenvalues. *Int. J. Quant. Chem.* **1982**, 21(1), 133-146.
  32. Franceschini, V.; Grecchi, V.; Silverstone, H. Complex energies from real perturbation series for the LoSurdo-Stark effect in hydrogen by Borel-Padé approximants. *J. Phys. Rev. A* **1985**, 32 (3), 1338.
  33. Benassi, L.; Grecchi, V. Resonances in the Stark effect and strongly asymptotic approximants. *J. Phys. B: At. Mol. Phys.* **1980**, 13, No. 5, 911.
  34. Farrelly, D.; Reinhardt, W.P. Uniform semiclassical and accurate quantum calculations of complex energy eigenvalues for the hydrogen atom in a uniform electric field. *J. Phys. B: At. Mol. Phys.* **1983**, 16, No. 12, 2103.
  35. Kolosov, V.V. A hydrogen atom in a strong electric field. *J. Phys. B: At. Mol. Phys.* **1987**, 20, No. 11, 2359.
  36. Filho, O.; Fonseca, A.; Nazareno, H.; Guimarães, P. Different approach to the Stark effect: Application to the hydrogen ground state. *Phys. Rev. A* **1990**, 42, No. 7, 4008-4014.
  37. Kondratovich, V.D.; Ostrovsky, V.N. Resonance and interference phenomena in the photoionisation of a hydrogen atom in a uniform electric field. II. Overlapping resonances and interference. *J. Phys. B: At. Mol. Phys.* **1984**, 17, No. 10, 2011.
  38. Anokhin, S.B.; Ivanov, M.V. The hydrogen and lithium atom in a strong electric field. *Opt. Spectr.* **1984**, 59, 499-502.
  39. Telnov, D.A. DC Stark effect in a hydrogen atom via Sturmian expansions. *J. Phys. B: At. Mol. Opt. Phys.* **1989**, 22, No. 14, L399-403.
  40. Ho, Y.-K. The method of complex coordinate rotation and its applications to atomic collision processes. *Phys. Rev.* **1983**, 99 (1), 1-68.
  41. Ivanov, I.A.; Ho, Y.-K. Complex rotation method for the Dirac Hamiltonian. *Phys. Rev. A* **2004**, 69, 023407.
  42. González-Férez, R.; Schweizer, W. *Quantum Systems in Chemistry and Physics, Series: Progress in Theoretical Chemistry and Physics*; Hernández-Laguna, A.; Maruani, J.; McWeeny, R.; Wilson, S. Eds.; Springer: Berlin, **2000**, vol. 23, p 17.
  43. Sahoo, S.; Ho, Y.-K. Stark effect on the low-lying excited states of the hydrogen and the lithium atoms. *J. Phys. B: At. Mol. Opt. Phys.* **2000**, 33,

44. Sahoo, S.; Ho, Y-K. The complex absorbing potential method (CAP) to study the Stark effect in hydrogen and lithium. *J. Phys. B: At. Mol. Opt. Phys.* **2000**, *33*, 2195-2206.
45. Mur, V. D. ; Popov, V.S.; Sergeev, A.V.; Shcheblykin, A.V. Stark resonances and scaling in Rydberg atoms. *JETP*. **1989**, *96*, 91-106.
46. Zimmerman, M.L.; Littman, M.G.; Kash, M.M.; Kleppner, D. Stark structure of the Rydberg states of alkali-metal atoms. *Phys. Rev. A* **1979**, *20*, 2251.
47. Richard R. Freeman, R.R.; Economou, N.P. Electric field dependence of the photoionization cross section of Rb. *Phys. Rev. A*. 1979, *20*, 2356-2364.
48. Harmin, D.A. Theory of the Stark effect. *Phys. Rev. A* **1982**, *26*, 2656.
49. Glushkov, A.V.; Ambrosov, S.V.; Ignatenko, A.V. Non-hydrogenic atoms and Wannier-Mott excitons in a DC electric field: Photoionization, Stark effect, Resonances in ionization continuum and stochasticity. *Photoelectronics*, **2001**, *10*, 103-106.
50. Ignatenko, A.V. Probabilities of the radiative transitions between Stark sublevels in spectrum of atom in an DC electric field: New approach. *Photoelectronics*, **2007**, *16*, 71-74.
51. Popov, V.; Mur, V.; Sergeev A.; Weinberg, V. Strong-field Stark effect: perturbation theory and  $1/n$  expansion. *Phys. Lett. A* **1990**, *149(9)*, 418-424.
52. Benvenuto, F.; Casati, G.; Shepelyansky, D.L. Rydberg Stabilization of atoms in strong fields: “magic” mountain in chaotic sea. *Z.Phys.B.* **1994**, *94*, 481-486.
53. Buchleitner, A.; Delande, D. Secular motion of three-dimensional Rydberg states in a microwave field. *Phys. Rev. A* **1997**, *55*, R1585(R).
54. Gallagher, T.F.; Noel, M.; Griffith, M.W. Classical subharmonic resonances in microwave ionization of lithium Rydberg atoms. *Phys. Rev. A* **2000**, *62*, 063401.
55. Grutter, M.; Zehnder, O.; Softley, T.; Merkt, F. Spectroscopic study and multichannel quantum defect theory analysis of the Stark effect in Rydberg states of neon. *J. Phys. B: At. Mol. Opt. Phys.* **2008**, *41*, 115001.
56. Stambulchik, E.; Maron, Y. Stark effect of high- $n$  hydrogen-like transitions: quasi-contiguous approximation. *J. Phys. B: At. Mol. Opt. Phys.* **2008**, *41*, 095703.
57. Dunning, F.B.; Mestayer, J.J.; Reinhold, C.O.; Yoshida S.; Burgdörfer, J. Engineering atomic Rydberg states with pulsed electric fields. *J. Phys. B: At.*

- Mol. Opt. Phys.* **2009**, 42, 022001.
58. Bryant, H.C.; Clark, D.A.; Butterfield K.B. et al. Effects of strong electric fields on resonant structures in H- photodetachment. *Phys. Rev. A* **1983**, 27, No. 6, 2889-2912.
  59. Letokhov, V.S., Principles of nonlinear optical spectroscopy. *Sov. Phys. Uspekhi*. **1988**, 41, 523-533.
  60. Letokhov, V.S., Problems of nanooptics. *Sov.Phys.Uspekhi*.**1999**, 42, 241-282.
  61. Letokhov, V.S.; Balykin, V.I. Information cooling of neutral atoms. *Phys. Rev. A*. **2001**, 64, 063410.
  62. Letokhov, V.S., Minogin, V.G., Pavlik, B.D., Cooling and trapping of atoms and molecules by resonant laser-field. *Optics Comm*. Vol.19 (1976) p.72-76.
  63. Letokhov, V.S., Balykin, V.I., Mishin, V.I., Bekov, G.I., Laser detection of single atoms. *Sov.Phys.-Uspekhi*, Vol.132,N10 (1980) p.79-85
  64. Letokhov, V.S. ; Hurst, G.S. Resonance Ionization Spectroscopy. *Phys. Today*. **1994**, Oct., 38-45.
  65. Letokhov, V.S., Laser induced processes in spectroscopy, isotope separation and photochemistry. *Sov.Phys.-Uspekhi* Vol.29 (1986) p.70-81.
  66. Glushkov, A.V.; Ivanov, L.N.; Ivanova, E.P. Autoionization Phenomena in Atoms. *Moscow University Press, Moscow*, **1986**, 58-160.
  67. Ivanov, L.N. Shift and deformation of radiation atomic lines in a laser field. Multiphoton processes. *Izv. AN USSR. Ser. Phys.* **1984**, 48, 1273-1280.
  68. Glushkov, A.V. Shift and deformation of radiation atomic lines in a laser field. *In Proceedings of 1<sup>st</sup> International Seminar on Optical Orientation of Atoms and Molecules*; Ioffe Phys.-Tech. Inst.: Leningrad, **1986**, 203.
  69. Glushkov, A.V.; Ivanov, L.N. Spectroscopy of atom in a laser field: AS Stark effect and Multiphoton resonances. *Preprint of Institute for Spectroscopy of the USSR Academy of Sciences*; ISAN: Moscow-Troitsk, **1991**, AS N3, 10p.
  70. Glushkov, A.V.; Ivanov, L.N. QED theory of shift and deformation of the radiation atomic lines in a laser field. *Proc. of 3<sup>rd</sup> USSR Seminar on Atomic Spectroscopy* Nauka: Chernogolovka, **1992**, pp 113-125.
  71. Glushkov, A.V.; Ivanov, L.N. Shift and deformation of radiation atomic lines in the laser emission field. Multiphoton processes. *Preprint of Institute for Spectroscopy of the USSR Academy of Sciences*; ISAN: Moscow-Troitsk, **1992**, AS N3, 12p.
  72. Glushkov, A.V.; Ivanov, L.N. Radiation decay of atomic states: atomic

- residue polarization and gauge noninvariant contributions. *Phys. Lett. A* **1992**, *170*, 33-36.
73. Glushkov, A.V. Spectroscopy of atom and nucleus in a strong laser field: Stark effect and multiphoton resonances. *J. Phys.: Conf. Ser.* **2014**, *548*, 012021
  74. Glushkov, A.V.; Ivanov, L.N. A broadening of the thulium atom autoionization resonances in a weak electric field *Preprint of Institute for Preprint of Institute for Spectroscopy of the USSR Academy of Sciences*; ISAN: Moscow-Troitsk, **1992**, AS N1, 10p.
  75. Glushkov A.V.; Ivanov, L.N. DC strong-field Stark effect: consistent quantum-mechanical approach. *J. Phys. B: At. Mol. Opt. Phys.* **1993**, *26*, L379-386.
  76. Glushkov, A.V.; Ivanov, L.N. DC Strong Field Stark effect: consistent quantum mechanical approach. *Preprint of Institute for Spectroscopy of the USSR Academy of Sciences*; ISAN: Moscow-Troitsk, **1992**, AS N1, 13p.
  77. Glushkov, A.V.; Ambrosov, S.V.; Ignatenko, A.V.; Korchevsky, D.A. DC strong field stark effect for nonhydrogenic atoms: Consistent quantum mechanical approach. *Int. Journ. Quant. Chem.* **2004**, *99*, 936-939.
  78. Glushkov, A.V. QED theory of radiation emission and absorption lines for atoms and ions in a strong laser field. *AIP Conf. Proceedings.* **2008**, *1058*, 134-136.
  79. Glushkov, A.V. QED energy approach to atoms and nuclei in a strong laser field: Radiation lines. *AIP Conf. Proceedings.* **2010**, *1290(1)*, 258-262. <http://doi.org/10.1063/1.3517569>
  80. Glushkov, A.V.; Loboda, A.V. Calculation of the characteristics of radiative multiphoton absorption and emission lines when an atom interacts with pulsed laser radiation. *J. Appl.Spectr.(Springer).* **2007**, *74*, 305-309
  81. Glushkov, A.V. Operator Perturbation Theory for Atomic Systems in a Strong DC Electric Field. *In Advances in Quantum Methods and Applications in Chemistry, Physics, and Biology, Series: Progress in Theoretical Chemistry and Physics*; Hotokka, M., Brändas, E., Maruani, J., Delgado-Barrio, G., Eds.; Springer: Cham, **2013**; Vol. 27, pp 161–177.
  82. Glushkov, A.V.; Ambrosov, S.V.; Borik, S.A.; Shpinareva, I.M.; Antonenko, T.N. Resonances in quantum systems in strong external fields: Nonlinear effect autoionization resonances in Tm. *J. Techn. Phys.* **1997**, *38(2)*, 211-213.
  83. Glushkov, A.V.; Malinovskaya, S.V.; Ambrosov, S.V.; Shpinareva, I.M.;



- Troitskaya, O.V. Resonances in quantum systems in strong external fields consistent quantum approach. *J. Techn. Phys.* **1997**, 38(2), 215-218.
84. Glushkov, A.V.; Dan'kov, S.V.; Prepelitsa, G.P.; Polischuk, V.N.; Efimov, A.E. Qed theory of nonlinear interaction of the complex atomic systems with laser field multi-photon resonances. *J. Techn. Phys.* **1997**, 38(2), 219-222.
85. Glushkov, A.V.; Malinovskaya, S.V.; Filatov, V.V. S-Matrix formalism calculation of atomic transition probabilities with inclusion of polarization effects. *Sov. Phys. Journal.* **1989**, 32(12), 1010-1014.
86. Ivanov, L.N.; Ivanova, E.P. Atomic ion energies for Na-like ions by a model potential method  $Z = 25-80$ . *Atom. Data Nucl. Data Tabl.* **1979**, 24, 95-109
87. Ivanov, L.N.; Ivanova, E.P. First order function for semi-relativistic atomic equation. *Theor. and Mathem. Phys.* **1974**, 20, 282-287.
88. Ivanov, L.N.; Ivanova, E.P. Method of Sturm orbitals in calculation of physical characteristics of radiation from atoms and ions. *JETP.* **1996**, 83, 258-266.
89. Driker, M.N.; Ivanova, E.P.; Ivanov, L.N.; Shestakov, A.F. Relativistic calculation of spectra of 2-2 transitions in O-and F-like atomic ions. *J.Quant.Spectr. Rad. Transfer.* **1982**, 28, 531-535.
90. Glushkov, A.V.; Ivanova, E.P. A new method for determining the ionization potential of atoms and ions with spectroscopic accuracy. *Optics and Spectroscopy.* **1985**, 58, 585-586.
91. Vidolova-Angelova, E.; Ivanov, L.N.; Ivanova, E.P.; Angelov, D.A. Relativistic perturbation theory method for investigating the radiation decay of highly excited many electron atomic states. Application to the Tm atom. *J. Phys. B: At. Mol. Opt. Phys.* **1986**, 19, 2053-2069.
92. Ivanova, E.P.; Zinoviev, N.A. The possibility of X-ray lasers based on inner-shell transitions of Ne-like ions. *Phys. Lett. A.* **2001**, 274(2), 239-246.
93. Ivanova, E.P.; Ivanov, L.N.; Glushkov, A.V.; Kramida, A.E. High order corrections in the relativistic perturbation theory with the model zeroth approximation, Mg-Like and Ne-Like Ions. *Phys. Scripta* 1985, 32, 513-522.
94. Ivanova, E.P.; Glushkov, A.V. Theoretical investigation of spectra of multicharged ions of F-like and Ne-like isoelectronic sequences. *J. Quant. Spectr. Rad. Transfer.* 1986, 36, 127-145.
95. Glushkov, A.V. Advanced Relativistic Energy Approach to Radiative Decay Processes in Multielectron Atoms and Multicharged Ions. *In Quantum Systems in Chemistry and Physics: Progress in Methods and Applications*,

- Series: Progress in Theoretical Chemistry and Physics*; Nishikawa, K., Maruani, J., Brandas, E., Delgado-Barrio, G., Piecuch, P., Eds.; Springer: Dordrecht, **2012**; Vol. 26, pp 231–252.
96. Glushkov, A.V. Negative ions of inert gases. *JETP Lett.* **1992**, *55*, 97-100.
  97. Glushkov, A.V. Relativistic polarization potential of a many-electron atom. *Sov. Phys. Journal.* **1990**, *33(1)*, 1-4.
  98. Glushkov, A.V.; Malinovskaya, S.V.; Loboda, A.V.; Shpinareva, I.M.; Gurnitskaya, E.P.; Korchevsky, D.A. Diagnostics of the collisionally pumped plasma and search of the optimal plasma parameters of x-ray lasing: calculation of electron-collision strengths and rate coefficients for Ne-like plasma. *J. Phys.: Conf. Ser.* **2005**, *11*, 188-198.
  99. Glushkov, A.V.; Malinovskaya, S.V.; Prepelitsa, G.P.; Ignatenko, V. Manifestation of the new laser-electron nuclear spectral effects in the thermalized plasma: QED theory of co-operative laser-electron-nuclear processes. *J. Phys.: Conf. Ser.* **2005**, *11*, 199-206.
  100. Glushkov, A.V.; Ambrosov, S.V.; Loboda, A.V.; Gurnitskaya, E.P.; Prepelitsa, G.P. Consistent QED approach to calculation of electron-collision excitation cross sections and strengths: Ne-like ions. *Int. J. Quantum Chem.* **2005**, *104*, 562-569.
  101. Glushkov, A.V.; Malinovskaya, S.V.; Loboda, A.V.; Shpinareva, I.M.; Prepelitsa, G.P. Consistent quantum approach to new laser-electron-nuclear effects in diatomic molecules. *J. Phys.: Conf. Ser.* **2006**, *35*, 420-424.
  102. Glushkov, A.V. *Relativistic and Correlation Effects in Spectra of Atomic Systems*; Astroprint: Odessa, **2006**.
  103. Khetselius, O.Yu. Spectroscopy of cooperative electron-gamma-nuclear processes in heavy atoms: NEET effect. *J. Phys.: Conf. Ser.* **2012**, *397*, 012012
  104. Khetselius, O.Yu. Relativistic Energy Approach to Cooperative Electron- $\gamma$ -Nuclear Processes: NEET Effect *In Quantum Systems in Chemistry and Physics, Series: Progress in Theoretical Chemistry and Physics*; Nishikawa, K., Maruani, J., Brändas, E., Delgado-Barrio, G., Piecuch, P., Eds.; Springer: Dordrecht, **2012**; Vol. 26, pp 217-229.
  105. Khetselius, O.Yu. Atomic parity non-conservation effect in heavy atoms and observing P and PT violation using NMR shift in a laser beam: To precise theory. *J. Phys.: Conf. Ser.* **2009**, *194*, 022009.
  106. Electron-Hole Drops in Semiconductors. Eds. Jefferiss K.D., Keldysh L.V. (Moscow, 1988).

107. Singh S. Physics of Semiconductors and Heterostructures.-Singapore: McGraw-Hill Book Co., 1993.
108. Nguyen Ba A., Hartmut Haug, Theory of the excitonic optical Stark effect in quasi-one-dimensional semiconductor quantum wires//Phys.Rev.A.-1992.-Vol.46(15).-P.9569-9576.
109. Jaziri S., Bastard G., Bennaceur R., Stark effect in parabolic quantum dot// J. de Physique.IV. Colloque C.5.-1993.-Vol.11(3).-P.367-374.

Methodical instructions  
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in the discipline “MATHEMATICAL AND PHYSICAL MODELS OF  
QUANTUM AND NEURAL NETWORKS”, **Part 3.**  
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