ГАЗОДИНАМІКА

UDK 539.328

Khetselius O.Yu., Glushkov A.V., Sofronkov A.N., Stepanenko S.N., Svinarenko A.A.

Odessa State Environmental University, Odessa
E.mail: glushkovav@gmail.com

New energy, angle momentum and entropy balance approach to modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer at macroscale. III. Low-frequency approximation and singularities in fields of meteoelements

An generalized low-frequency approximation of energy, angle momentum and entropy balance relationships to modelling climate and macro-turbulent atmospheric dynamics, heat and mass transfer at macroscale is introduced and allow significantly to simplify the main fundamental equations. A new equilibrium approach to modelling the global mechanisms of climatic and macroturbulent atmospheric low-frequency processes, including heat and mass transfer processes, teleconnection effects, etc., is based on the use of equilibrium relations for entropy, energy, angular momentum, spectral theory of atmospheric macroturbulence and moisture turnover in connection with the continuity of forms of atmospheric circulation (teleconnection, genesis of fronts). The physical features of singularities in the fields of meteorological elements and the balance of the angular momentum as well as a generalized Arakawa-Schubert model are introduced and discussed.

***Kew words**: macro turbulent atmospheric processes, heat-mass transfer, low-frequency approximation, energy, angle momentum and entropy balance*

Introduction. This paper goes on our work on quantitative studying energy, angle momentum and entropy balance relationships in modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer in atmospheric environment at macroscale. Really, one could remind [1, 2] that understanding global mechanisms in atmospheric macroturbulent processes, teleconnection effects etc attracts a fundamental interest in a modern physics of climate and heat-and mass transfer in complex aerodispersed (atmospheric) systems [1-20]. As it is earlier indicated [1, 2], nowdays there are different, quite consistent approaches to modelling global atmospheric macroprocesses in atmosphere and other geospheres (look for example, review in [1, 2]). Such known methods as MLDP0 (Modèle Lagrangien de Dispersion de Particules d'ordre 0), HYSPLIT (Hybrid Single-Particle Lagrangian Integrated Trajectory Model), NAME (Numerical Atmospheric-dispersion Modelling Environment), RATM (Regional Atmospheric Transport Model), FLEXPART (Lagrangian Particle Dispersion Model), model of the European Center for Mdium-Range Weather Forecasts (ECMWF) and others are usually mentioned (e.g. [13-22]). . From the other side, correct quantitative description of the global atmospheric processes, the heat- and mass transfer processes in an atmosphere, macromodelling dispersion of the pol-

DOI: 10.18524/0367-1631.2021.59.227306

lutants in an atmosphere remains very actual and hitherto unsolved problem. In ref. [1,2] there are considered the elements of a generalized approach, which is based on the entropy, energy, moment balance relationships for the global atmospheric low-frequency processes, theory of atmospheric macroturbulence and circulation over the position of the front sections. It has been stressed that the main process-forming factor is a triplet of interacting solitons: "the planetary soliton of Hadley cells - the entire complex of atmospheric fronts - the Rossby soliton wave packet". In Ref. [2] it has been presented the detailed description of an generalized effective computational algorithm or the advanced non-stationary balance approach with accounting for the macro turbulent, circulation low-frequency processes.

In this paper we consider the energy, angle momentum and entropy balance relationships to modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer at macroscale in so called low-frequency approximation. It allows to simplify the fundamental energy (entropy) and atmospheric angle momentum balance relationships. Besides, some singularities in fields of meteoelements are discussed.

1. Qualitative features of the low-frequency vibrations in geospheres. In modern climate physics, the need to develop special methods for observing low-frequency oscillations of non-equilibrium thermodynamic processes in the geosphere has been determined [1, 2, 20-25]. To date, methods of physical and statistical analysis and processing of data sets of the network of ground or satellite measurements are used to indicate such phenomena. However, these techniques are far from standardized and to some extent unique for each of these long-term processes. Therefore, the development of methods for monitoring the most low-frequency processes on a planetary scale to observe some geophysical factors, summarize the contributions of low-frequency fluctuations, especially relevant in the long-term physics of climate forecasts [1, 2, 20-23]. Currently, this problem is far from being solved, although a number of indirect steps in this direction have been made in a number of works (see, for example, [1-20, 26-32]). The information base of modern long- and ultra-long-term forecasts can be both satellite information and observation materials on the characteristics of radio waveguides, especially in the lower trophospheric layers, which are performed on the basis of radio analysis of radio transmission in the ultra-short wave (USW) range. Both methods are based on the main criterion of the concentration of hydrometeors in the clouds for satellite sounding and water vapor together with hydrometeors for USW waveguides. Since any foci of planetary scale water accumulation in the atmosphere in three phase states (steam, water, ice) are formed on the basis of the mechanics of cyclo- and frontogenesis or in the lines of convective instability, which form the basis of the process of synoptic changes mainly in tropical latitudes and in anticyclonic formations, it is possible to enter some physical and mathematical model on the basis of thermodynamics and hydromechanics of the processes forming these accumulations. For example, the physics of these processes may coincide with the mechanics of soliton, which has a long-term basis of energy supply [20-25]. The mechanics of action of such a soliton determines the main thermo-hydrodynamic parameters of the USW waveguide. It can be assumed that the soliton of the atmospheric front is more characteristic of the elevated tropospheric

USW waveguide [1, 2, 20, 22]. The soliton of the front is based on the long-term existence and on the independent dynamism of the frontal section of the polar front of the temperate latitudes, encircling the globe. Similar sections of the Arctic and tropical fronts have a slightly less stable structure, because they are in the zone of active anticyclogenesis of the Arctic anticyclone and the high-pressure subtropical belt, in which Rossby solitons are active ([1, 2, 13-16, 20-22]). Therefore, the soliton of the polar front is a characteristic planetary ensemble of low-frequency wave and vortex process associated with the sub-carried tropospheric USW waveguide.

It should also be noted that the polar front is an active reflector of the teleconnection process between the Hadley cells and the El Niño-based Southern Process and the Arctic anticyclone, which have crest spurs in the Siberian and Canadian-Greenlandic anticyclones, which are likely to have a structure solitons Rossby. The effect of teleconnection is set out in [20-23]. The main attention is paid to the balance of the angular momentum in planetary dynamic movements of air masses and, in particular, on the basis of radiosonde measurements the zonal distribution of the relative angular momentum in the atmosphere is estimated [1, 2]. The observed balance (imbalance) of the angular momentum should in principle be calculated from direct measurements of wind in the atmosphere and averaged over the year. The angular momentum is transmitted from the Earth's surface (mainly over the oceans) in the tropics and is transferred up to the Hadley cell, then moves in the upper atmosphere to the pole and is given back to the Earth in mid-latitudes. One of the important modern geophysical puzzles is connected with the elucidation of not only atmospheric but also hydro and lithosphere contributions to the balance of the Earth's angular momentum.

2. Balance of energy and angular momentum for the atmospheric system. According to ref. [1,2] master equation for the balance of the angular momentum is an equation of integral form [20,22]:

$$\frac{\partial}{\partial t}\int \rho M dV = \int_{\lambda_1}^{\lambda_2}\int_0^H\int_0^{2\pi} \rho v M d\lambda dz d\varphi + \int_0^H\int_{\lambda_1}^{\lambda_2}\int_0^{2\pi}\left(p_E^i - p_W^i\right)\cos\lambda dz d\lambda d\varphi +$$

$$+\int_{\lambda_1}^{\lambda_2}\int_0^{2\pi}\int_0^H \tau_0 a \cos\lambda d\lambda d\varphi dz\,, \tag{1}$$

where $M = \Omega a^2 \cos^2\lambda + u\, a \cos\lambda$ – angle momentum, Ω is angle velocity of rotation of the Earth; λ – latitude ($\lambda_1 - \lambda_2$ determine the latitudinal belt between the Arctic and polar fronts); ρ – air density; V – the entire volume of the atmosphere in the specified latitudinal zone from sea level to the average height of the elevated tropospheric USW waveguide (H) [4, 20] (note that Oort uses $H = \infty$); $p_E^i - p_W^i$ – the pressure difference on the eastern and western slopes of the i-th mountain; z - altitude; τ_0 – surface friction stress.

Equation (1) is an integral equation with respect to the angular momentum M with the nucleus ρV (in the stationary version, the left part (1) is 0 [1,2]). The function of the meridional component v directly depends on the type of function ρ. The function u is directly introduced into the unknown integral equation (1). The left part of equation (1) does not include the component v, which means the problem of a priori closed loop angular momentum along the meridian. Thus, we can introduce a cycle of angular momentum in the form of a complicated Hadley cell of temperate latitudes, in which the closure of Hadley's circulation by the magnitude of the angular momentum does not occur in the atmosphere, but passes into the ocean and further into the lithosphere, and in the southern direction angular momentum occurs through the lithosphere until the beginning of the cycle of rising air masses in subtropical latitudes.

The hydrosphere in the oceans is usually determined only by the zonal direction of transmission of the angular momentum, because the ocean is not able to match its frequencies with atmospheric frequencies in the circular balance cycle of the angular velocity component v, but only possible frequency matching of the u component. At the time of collision with the lithosphere, the circulating center of Hadley at an angular momentum in the north is within the range of the Arctic Front, and at the time of exit from the lithosphere is within the range of the polar front. The convergence of these atmospheric fronts could then close the atmospheric cycle of balance at angular momentum (or reduce the imbalance) without activating the ocean and lithosphere and in the same frequency range of atmospheric oscillations.

Naturally, the convergence of the Arctic and polar fronts occurs through a complex of interconnected cyclonic circulations, tele-connecting the southern circulations with the northern ones through the Ferrell cell of temperate latitudes (see qualitative picture of macroturbulent processes in [1-3,13-12]).

The tropical Hadley cell teleconnects the polar front with the southern process by a similar mechanism of communication between the tropical and polar fronts or the tropical Hadley cell with the Hadley cell of temperate latitudes. Since the refractive index is uniquely associated with the density field, it can be complex, measured by USW indicator of the course of the whole process of teleconnection. Tropospheric USW radio waveguides determine the value of H in equation (1), although the upper part of the circulating ring of the Hadley cell does not always coincide with the level of the elevated tropospheric USW waveguide. However, the determination of the position of the level of the upper part of the Hadley cell by the velocity field or by the criterion of basic mass transfer can be specified by an effective density criterion or, also, by the refractive index. From the point of view of physics, the cycle of angular momentum balance in the zones of collision with the hydrosphere and with the lithosphere becomes singular. This singularity can be detected due to the appearance of zones of frontal sections and in solitons of the front type.

Then the core of equation (1) can be given in the density field by the functional ensemble of the complex velocity potential (see [20]):

$$w=\overline{v_\infty} z+\frac{1}{2\pi}\sum_{k=1}^{n} q_k \ln(z-a_k)+\frac{1}{2\pi}\sum_{k=1}^{p}\frac{M_k e^{\alpha_k i}}{z-c_k}-\frac{i}{2\pi}\sum_{k=1}^{m}\Gamma_k \ln(z-b_k) \qquad (2)$$

the complex velocity will be accordingly:

$$v=\frac{dw}{dz}=\overline{v_\infty}+\frac{1}{2\pi}\sum_{k=1}^{n}\frac{q_k}{z-a_k}-\frac{1}{2\pi}\sum_{k=1}^{p}\frac{M_k e^{\alpha_k i}}{(z-c_k)^2}-\frac{i}{2\pi}\sum_{k=1}^{m}\Gamma_k /(z-b_k), \qquad (3)$$

where w – complex potential; v_∞ – complex velocity of the general circulatory background (mainly zonal circulation); b_k – coordinates of vortex sources in the singularity zone; c_k – coordinates of dipoles in the zone of singularity; a_k – coordinates of vortex points in the zones of singularity; M_k – the magnitudes of the moments of these dipoles; α_k – orientation of dipole axes; Γ_k, q_k – values of circulation in vortex sources and vortex points, respectively. The kernel of integral equation (3) becomes singular of Cauchy and Hilbert type. The relationship of the density field or refractive index with the field of complex potential or with the field of complex velocity is trivial using the equations of the theory of "shallow water", according to the model described, for example, in [22].

Methods for solving such equations are in principle well known. Focus on the basic ideas of the solution, without detailing the calculations. Since the nucleus through the functional ensemble of the complex velocity potential contains features of the form $1/(\zeta - t)$ etc., it is convenient to use the connection of Hilbert and Cauchy kernel [31,32]:

$$\frac{d\zeta}{\zeta - t} = \frac{1}{2}\operatorname{ctg}\frac{\sigma - s}{2}\, d\zeta + p(s,\sigma)d\sigma. \qquad (4)$$

The function $p(s, \sigma)$ corresponds to the condition: $\zeta = t(\sigma)$, де $t(s) = x(s) + iy(s)$ and determines the zoning factor by the weights of the dipoles in formula (3). In the stationary variant, formula (3) is only a kind of self-similar approximation. Then, in general, the singular integral equation can be reduced to the equation:

$$a(s)\varphi(s) + \frac{b(s)}{2\pi}\int_0^{2\pi}\varphi(s)\operatorname{ctg}\frac{\sigma - s}{2}\, d\sigma + \int_0^{2\pi} K(s,\sigma)\varphi(\sigma)d\sigma = f(s). \qquad (5)$$

The frontline is given by the formula [20]:

$$v_x - iv_y = \frac{\Gamma}{2\pi i}\, ctg\frac{\pi}{l}\left(z - z_0\right), \qquad (6)$$

and the kernel $K(s,\sigma)$ together with the functions $a(s)$, $b(s)$ and $f(t)$ specify the weight contributions of the source in the frontal lime of the typical front, which corresponds to the form of circulation. In (5), (6) the operation of conformal transformation of a rectilinear front to a real front line is omitted. But in the model experiment, it is permissible to replace the curved sections of the fronts with straight lines, without particularly distorting the essence of the process. Moreover, in the current situation it is important to have even a rectilinear position of the frontal section from the central point in the middle. Equation (5) can be rewritten using, according to [20,22], the operator:

$$M\omega = a(s)\omega(s) - \frac{b(s)}{2\pi}\int_0^{2\pi}\omega(s)\operatorname{ctg}\frac{\sigma - s}{2}\, d\sigma, \qquad (7)$$

Then Eq. (7) passes into the Fredholm equation. Operation (7) is performed further numerically using the previously mentioned method of decomposition into a Laurent series and the application of the theory of subtractions. For equations with Cauchy kernel used to describe purely dipole situations rather than vortices (see the relevant terms in (3)):

$$a(t)\varphi(s)+\frac{b(t)}{2\pi}\oint_L\frac{\varphi(\zeta)}{\zeta-t}dt+\oint_L K(t,\zeta)\varphi(\zeta)d\zeta=f(t) \qquad (8)$$

we apply integration on a contour by means of deductions at once. Therefore, equation (4) can be reduced to any of these two types, or solve both for different situations. It all depends on the actual convergence of the Laurent series and the number of required analytical extensions. The transition to the Fredholm equation for (9) is performed by the operator:

$$M\omega=a(t)\omega(t)-\frac{b(t)}{2\pi}\oint_L\frac{\omega(\zeta)}{\zeta-t}d\zeta \qquad (9)$$

The solution of Fredholm's equation is performed according to the scheme:

$$\varphi(x)-\lambda\int_a^b K(x,s)\varphi(x)ds=f(x), \qquad (10)$$

$$\varphi_1(x)=f(x)+\lambda\int_a^b K(x,s)f(s)ds, \qquad (11)$$

$$\begin{aligned}\phi_2(x)&=f(x)+\lambda\int_a^b K(x,s)\phi_1(x)ds=\\&=f(x)+\lambda\int_a^b K(x,s)f(s)ds+\lambda^2\int_a^b K(x,s)dt\int_a^b K(t,s)f(s)ds.\end{aligned} \qquad (12)$$

The resolvent, which is the solution of Fredholm's equation, will be:

$$\Gamma(x,s,\beta)=\sum_{m=1}^{\infty}\beta^{m-1}K_m(\lambda,s). \qquad (13)$$

For further details, see, e.g., [14,25].

3. Singularity in the fields of meteorological elements and the balance of the angular momentum. The generalized Arakawa-Schubert model. The solution of the obtained singular integral equation with respect to the angular momentum, already given by a regular function, makes it possible both to estimate the weight of the singularity in the angular momentum field and to estimate the atmospheric contribution to the angular momentum balance [1,2,20-22]. The gaps in the fields of meteorological elements that accompany the phenomenon of the atmospheric front form singular features of these fields in narrow zones of frontal sections, which are usually parametrized by regular functions of vortex sources in vortex structures and functions of dipoles that reflect the dynamics of convective

$$\underbrace{\sum_{k=1}^{p}\frac{M_k e^{\alpha_k i}}{(z-c_k)^2}}_{\text{Dipole}}\;\underbrace{\frac{i}{2\pi}\sum_{k=1}^{m}\Gamma_k\ln(z-b_k)}_{\text{Vortex sources}} \qquad (14)$$

The balance of the angular momentum at the close location of the Arctic and polar fronts over the oceans (which is almost always all-season), and over the continents in summer and in the transition seasons, is mainly maintained by centrifugal "traction" of moisture along the line frontal section of the polar front south of the center of the cyclonic depression. The desired mechanism for the atmospheric front is adequately described by the model [20,22].

The system of Arakawa equations can be used to calculate the height of the elevated tropospheric USW waveguide The total mass flow in a single cloud, as well as in the cloud system, by the Arakawa-Schubert model, is expressed by the formula:

$$M(z) = \int m(z,\lambda)d\lambda = \int m_B(\lambda)\eta(z,\lambda)d\lambda\,, \tag{15}$$

where η – is a function that characterizes the cumulative effect of inflow; the effect of inflow itself occurs in much less time than somehow noticeable changes in the horizontally oriented process; z – height above the base of the cloud; m – mass of air; m_B – mass flow at the base of the clouds, which is determined by the value of the rate of attraction λ. Next, the Arakawa-Schubert model records the power ratios (introducing the function of mechanical interaction of cloud ensembles through the mechanisms of attraction $K(\lambda, \lambda')$):

$$\begin{gathered} \frac{dA}{dt}_{up} + \frac{dA}{dt}_{down} = 0\,, \\ \frac{dA}{dt}_{down} = F(\lambda)\,, \\ \frac{dA}{dt}_{up} = \int_0^{\lambda_{\max}} K(\lambda,\lambda')m_B(\lambda')d\lambda''\,, \end{gathered} \tag{16}$$

where the first term on the left is the change in the operation of the ascending currents in the convective cloud, the second, respectively, which descend to the edge of the cloud. From Eq. (16) it follows:

$$\int_0^{\lambda_{\max}} K(\lambda,\lambda')m_B(\lambda')d\lambda' + F(\lambda) = 0\,. \tag{17}$$

The integral equation with respect to $m_B(\lambda)$ is solved for given functions of the kernel K and F. The Boltzmann equation is used to calculate the type of the kernel K:

$$\frac{\partial K}{\partial t} + \xi_i \frac{\partial K}{\partial x_i} + \frac{\partial \xi_i}{\partial t}\frac{\partial K}{\partial \xi_i} = J(t,x,\xi)\,, \tag{18}$$

where (x,ξ) – 6-timer phase space coordinates (x_1, x_2, x_3) and rates of involvement (ξ_1, ξ_2, ξ_3); J – is the integral of the interaction of cloud systems. The solution of equation (17) is the integration of the equation from the initial condition in the form of the Maxwell distribution.

The solution of equation (17) is trivially reduced to the solution of a system of algebraic equations that defines m over the entire interval λ. Note that when the inflow is absent, the value of λ is zero. Further:

$$\frac{d\eta}{dz} = \lambda\eta$$

$$\ln\eta = \int \lambda(z)dz + C; \quad \eta = Ce^{\int \lambda(z)dz}\,. \tag{19}$$

At different λ the value of η increases in different ways and at the level of z_D (upper limit of the cloud) the value of λ again vanishes. That is, λ is by no means a constant, but completely determines the value of η over the entire interval ($z_D - z_B$), and at some level z within this interval the value of η reaches a maximum. This corresponds to the average height of the elevated radio waveguide [22]. The values of λ and z_D are to be determined from the solution of the system [13, 20, 22]:

$$\begin{gathered} E - D - \frac{\partial M_c}{\partial z} = 0, \\ \widetilde{E}_s - \widetilde{D}_{S_C} \frac{\partial M_c S_c}{\partial z} + \rho L c = 0, \\ \widetilde{E}_q - \widetilde{D}_{q_C} \frac{\partial M_c q_c}{\partial z} + \rho c = 0, \end{gathered} \tag{20}$$

where E is the inflow, D is the outflow, $M_c = \sum \rho w_i \sigma_i = \rho w_c \sigma$ – is the vertical flow of air mass in the cloud (w_i is the average vertical velocity in the i-th cloud, σ – is the horizontal cross-sectional area of the i-th cloud); w_c, $S_c = c_p T$ and q_c – weighted average values of vertical velocity, static energy and the ratio of water vapor mixture; S, q – average values of static energy and the ratio of the mixture of water vapor in the surrounding cloud air, ρ – air density; c is the amount of condensed moisture.

The criterion of angular momentum is complex, as it closes a series of physical mechanisms, and in the long run. Violation of the balance of the angular momentum requires the immediate intervention of all environments to eliminate the imbalance. Each form of circulation must have its own cycle of imbalance, which involves cells of Hadley, Ferrell and wet-rotation, directly related to the frontal activity of the Arctic, polar and tropical fronts. Due to the imbalance of the angular momentum there is a dynamism of climatic fronts, which are the main mechanism of wet rotation. One of the most reasonable mechanisms for eliminating imbalance, introduced by orth, is that the imbalance of the angular momentum is eliminated by its transmission through the lithosphere, most likely through the transpiration of moisture in the underground hydrology layer. It is worth mentioning the prophecy of huge water bodies in underground hydrology, in particular, in desert areas [22]. A more correct mechanism has been worked out [1, 2, 22].The problem, however, remains the combination of frequency scans of the mechanisms of transmission of the angular momentum through the atmosphere and lithosphere. Perhaps another contribution may be due to the existence in the Earth's core of a natural nuclear georeactor [33].

On the other hand, similarly, the process of teleconnection associated with El Niño through the southern process must often coincide with the process of restoring the balance of the angular momentum. It is obvious that the restoration of the balance of the angular momentum requires a tangible reaction of the atmosphere, which is expressed in the movement of the main fronts relative to each other. The process of teleconnection is also then directly related to the movement of circumpolar vortices, and thus the fronts. Fronts, which are transporters of moisture over long distances, create a total current of attraction across the front. Thus, the balance of the angular momentum can be closed. Once in the hydrosphere, moisture fields change the frequency spectrum from atmospheric high-frequency to low-frequency hydrosphere.

Forms of circulation (see details in [1, 2, 20-22]): W3, WM1, WM2, E3, EM1, EM2, C3, CM1, CM2, but mainly C, M1, M2, summarize these processes. We can then assume that the change in the forms of circulation is determined by the cycles of coordination of the balances of the angular momentum through the system of frontal atmospheric rotation. More precisely, this hypothesis, which follows from the logic of the processes occurring in the system of atmospheric moisture rotation and the balance of the angular momentum, is uniquely related to the processes of teleconnection, El Niño and the southern process. Is the process of closing the balance of the angular momentum due to underground moisture rotation and tectonic movements (nuclear georeactor? [33]) And to what extent it is consistent with the above processes is an equivalent hypothesis that is currently being tested. Being on the side of the first hypothesis, we still leave part of the energy load on the second hypothesis.

According to the equation of kinetic energy (see [1,2]), which is a spectral analogue of equation (1), the transport of energy in the direction of the wave vector in the presence of a predominant pseudovector with increasing modulus of the wave vector will meet resistance in the form of increasing tensor density. Moreover, if at the end of the spectral interval the motion is almost completely determined by the axial vectors, then we should expect advection (transport, transformation) of the kinetic energy in the direction opposite to the direction of the wave vector. Therefore, the double integration of the third-order tensor leads to the "advection of determinism" toward the mean motion and to the degeneracy of turbulence. The time interval of energy transport in the opposite direction depends on the truncation of the forecast model. From the point of view of physics, the degeneracy of turbulence is due to the fact that at the end of the spectral interval the motion is quite definite, namely, these are fronts that have a clear structure, as well as convective cells and orographic winds. If turbulence developed stochastically, its transformation along the wave vector could not lead to clear structures, for example, in the form of fronts.

Therefore, the development of turbulence must be tied to these structures, and it gradually degenerates. In practice, turbulence can persist on a scale smaller than for these structures, and then at this scale it is possible to apply the closure formulas 9e.g. [1, 2]). However, even after carrying out operations to degenerate turbulence, there are still physically justified fourth points, which have collapsed into the second and their weight will determine the problem of predictability. Further, convolution operations for closure should lead only to axial vectors or, what is the same, to elements of the theory of a flat field. The degeneracy of the turbulent regime should lead to the degeneracy of the singularity. Highlighting the singularities in the effective part obtained in the theory of a flat field, we practically perform the operation of closure in turbulent mode [14]. One should eave from the equation of kinetic energy (see [1,2]) only two operators:

$$\frac{\partial \overline{V'U'}}{\partial t} = \frac{i}{a} \overline{V' L_6 \left(\Phi' \right)}, \tag{21}$$

expressing Φ' through φ the complex velocity potential w, and the velocity components V' through the functions ψ of the same velocity potential, one could get the opportunity to economically solve the corresponding system of equations with sufficient accuracy for our purposes. In equations of the type (21), the spectral representations of the pulsation part are replaced by elements of Laurent series, and more specifically, by pole subtractions. Then in physically intelligible elements of complex fields equations are easily solvable. In the Laurent decomposition, we include the poles above the first order in the spectrum of pulsation motion, because:

$$\bar{R} = X - iY = i\rho\Gamma\bar{v}_{\infty}; \qquad \operatorname{res} f(\infty) = -C_{-1}, \tag{22}$$

$$f(z) = -\frac{1}{2\pi i}\oint_{G'}\frac{f(\zeta)d\zeta}{\zeta - z} = \frac{C_{-1}}{z-a} + \left[\frac{C_{-2}}{(z-a)^2} + \ldots + \frac{C_{-n}}{(z-a)^{\mathrm{n}}} + \ldots\right].$$

At the same time, the perturbations of the frontal type belong to the zone of medium motion, because they include poles of only the first order:

$$\begin{gathered} V_x - iV_y = \frac{df}{d\zeta} = \frac{\Gamma}{2\pi i}\left\{\frac{1}{\zeta-\zeta_0} + \sum_{k=1}^{\infty}\left[\frac{1}{\zeta-\zeta_0-k_1} + \frac{1}{\zeta-\zeta_0+k_1}\right]\right\} + \\ + \frac{d}{d\zeta}\left[\sum_{k=1}^{n}\Gamma_k \ln(\zeta - b_k)\right] + \frac{C_m}{(z-a_m)^m} + \frac{C_{m-1}}{(z-a_{m-1})^{m-1}} + \frac{C_1}{z-a_1}, \end{gathered} \tag{23}$$

and to the zone of pulsating motion include the expression in parentheses.

The balance of the angular momentum is determined by the well-known Blasius-Chaplygin theorem through the forces of external pressure arising in places of violation of the desired balance:

$$\begin{gathered} L = -\oint_C p\left[x\cos(n,^\wedge y) - y\cos(n,^\wedge x)\right]dz = \\ = \oint_C p[x\cos\theta + y\sin\theta]dS = \oint_C p\,[xdx + ydy]. \end{gathered} \tag{24}$$

Given that the angular momentum can be defined as:

$$L = \operatorname{Re}\left[-\rho\overline{v_{\infty}}\sum_{k=1}^{m}\Gamma_k b_k - i\rho M\overline{v_{\infty}}\right], \tag{25}$$

It can be naturally written as: $\operatorname{res} v(z_1) + \operatorname{res} v(z_2) + \ldots.. + \operatorname{res} v(z_p) = -\operatorname{res}\bar{v}(\infty)$.

That is, in places of disturbance of the balance of the angular momentum there are singularities of orders of magnitude higher than the first, which is taken into account when solving equation (21). Following the solution, we get the opportunity to track the relationship of singularities in the balance of the angular momentum with the turbulent regime by equation (21). Equation (23) allows to determine the weight of the unstable mode of motion and to calculate the frequency scan of a typical process. Equation (23) is substantially abbreviated in the right part, however, the main weight term is left, which allows to trace the logical scheme of the process quite correctly, and with the help of operations [22]

$$\frac{\partial\overline{u_i'u_j'}}{\partial\overline{u_k'u_l'u_n'u_m'}}, \qquad \frac{\partial\overline{u_i'u_j'u_k'}}{\partial\overline{u_r'u_l'u_n'u_m'}} \tag{26}$$

one can monitor the closure.

To conclude, let us underline that the key points here are, firstly, the connection of the tropospheric radio waveguide with atmospheric moisture circulation and, accordingly, with the form of atmospheric circulation through the position of the frontal sections (atmospheric fronts as the main accumulators of moisture). Secondly, atmospheric moisture turnover is associated with such a purely low-frequency process as the fulfilment of the angular momentum balance. At the same time, the dynamics and characteristics of the atmospheric radio waveguide are associated with teleconnection and, thus, with the forms of circulation, more precisely, with the processes of continuity of these forms. Naturally, this seems to be extremely important in the long-term forecast. The theories outlined for the first time definitely and clearly show that the dynamics of tropospheric radio waveguides, atmospheric moisture circulation, the fulfilment of the balance of the angular momentum of the atmosphere and the change in circulation forms, their continuity (as well as frontogenesis and teleconnection) are directly and inversely closely related physical characteristics of the atmosphere.

References:

1. *Buyadzhi V., Sofronkov A.N., Glushkov A.V.,Khetselius O., Dubrovskaya Yu., Svinarenko A.* New energy, angle momentum and entropy balance approach to modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer at macroscale. I. General Formalism// Phys. Aerodisp.Syst. 2018. Vol.55. P. 94-103.
2. *Glushkov A.V., Khetselius O., Stepanenko S.N., Sofronkov A., Svinarenko A., Buyadzhi V.* New energy, angle momentum and entropy balance approach to modelling climate and macroturbulent atmospheric dynamics, heat and mass transfer at macroscale. II. Computational algorithm// Phys. Aerodisp.Syst 2019. Vol.57. P. 104-113
3. *Glushkov A., Khetselius O., Agayar E., Buyadzhi V., Romanova A., Mansarliysky V.* Modelling dynamics of atmosphere ventilation and industrial city's air pollution analysis: New approach//IOP Conf.Ser: Earth Env. Sci. 2017.Vol.92.012014.
4. *Peixoto J.P.,* Oort A.H. Physics of Climate – N.-Y.: AIP.-1992.–520p.;
5. *Matveev V.,* Physics of Atmosphere.-Moscow: Nauka, 1982.
6. *Khokhlov V. N., Glushkov A. V., Tsenenko I. A.* Atmospheric teleconnection patterns and eddy kinetic energy content: wavelet analysis. Nonlinear Processes in Geophysics. 2004, 11 (3), pp.295-301.
7. *Ghil M., Kimoto M., Neelin J.D.* Nonlinear dynamics and predictability in the atmospheric sciences//Rev.Geophys.-1991.-Vol.29.-P.46–55.
8. *Wang C.*, ENSO, climate variability, and the Walker and Hadley circulations// In: The Hadley Circulation: Present, Past, and Future.- Eds Diaz H.F. and Bradley R.S.-Berlin: Springer.- 2004.-P.131-164.
9. *Khetselius O.Yu.,* Forecasting evolutionary dynamics of chaotic systems using advanced non-linear prediction method// Dynamical Systems – Theory and Applications, Eds. J. Awrejcewicz et al.-2013.-Vol.1.-P.145-152.
10. *Kurbatskii A. F.,* Computational modeling of the turbulent penetrative convection above the urban heat island in stably stratified environment // Journal of Applied Meteorology. 2001. V. 40. P.1748 -1761
11. *Lu J., Araya S.P., Snyder W.H., Jr. Lawson R.E.,* A Laboratory Study of the Urban Heat Island in a Calm and Stably Stratified Environment. Part I: Temperature

Field; Part II: Velocity Field // J. Appl. Meteor.1997. V.36. P.1377.
12. *Glushkov A., Safranov T., Khetselius O., Ignatenko A., Buyadzhi V., Svinarenko A.,* Analysis and forecast of the environmental radioactivity dynamics based on methods of chaos theory: General conceptions//Env. Prob. 2016. Vol.1(2_. P.115.
13. *Lin W., Chen L., Yu W., Ma H., Zeng Z., Lin J., Zeng S.* Radioactivity impacts of Fukushima Nuclear Accident on atmosphere//Atm.Env. 2015.Vol.102. P.311-322
14. *Korsakissok I., Mathieu A., Didier D.,* Atmospheric dispersion and ground deposition induced by the Fukushima Nuclear Power Plant accident: A local-scale simulation and sensitivity study// Atm.Env.-2013.-Vol.70.-P.267-279
15. *Kang I., Lau K.,* Principal modes of atmospheric circulation anomalies associated with global angular momentum fluctuations//J.Atm.Sci.-1994.-Vol.51.-P.1194.
16. *Wang, C.* ENSO, climate variability, and the Walker and Hadley circulations. The Hadley Circulation: Present, Past, and Future (Springer).-2004.
17. *Boer G.J., Sargent N.E.* Vertically integrated budgets of mass and energy for the globe // J. Atmos. Sci.-1985.-Vol. 42.-P.1592-1613.
18. *Trenberth K.E., Stepaniak D.P., Caron J.M.* Interannual variations in the atmospheric heat budget // J. Geophys. Res.-2002.-Vol.107.-P.4.1-4.15.
19. *Fyfe J., Boer G., Flato G.,* Predictable winter climate in the North Atlantic sector during 1997–1999 ENSO cycle// Geophys.Res.Lett.- 1999.-Vol.26.- P.1601-1604
20. Glushkov A.V., Ambrosov S. Atmospheric moisture circulation and singularities in the angular momentum balance//Meteor., Climat., Hydr. 1999. N38. P.54-58.
21. *Glushkov A.V., Efimov V.A., Kivganov A.F.,* Modelling climate as a problem of interaction of the solitons triplete//Meteor., Climat., Hyd.-1999.-Issue 38.-P.3-8.
22. *Glushkov A., Khetselius O., Ambrosov S., Bunyakova Yu., Mansarliysky V.,* The use of microsystems technology "Geomath" to modeling the balance of the angular momentum of the Earth, atmospheric processes and parameters of radio waveguides: III. Nonstationary theory//Sensor Electr. and Microsyst. Techn.-2013.- Vol.10, N1-P.22-28.
23. *Glushkov A.V.,* Renorm-group and fractal approach to turbulence spectrum in planetary atmosphere system, “cosmic plasma – galactic cosmic rays”//Ukrainian Hydrometeorology Journal-2013.-№12.-C.25-30.
24. .*Glushkov A.V., Buyadzhi V.V., Ponomarenko E.L.,* Geometry of Chaos: Advanced approach to treating chaotic dynamics in some nature systems// Proc. Intern. Geom. Center.-2014.-Vol.7(1).-P.24-30.
25. *Sofronkov A., Khetselius O., Glushkov A., Buyadzhi V., Romanova A., Ignatenko A.,* New geophysical complex-field approach to modelling dynamics of heat-mass-transfer and ventilation in atmosphere of the industrial region// Phys. Aerodisp. Syst.–2018.–Vol.55.–P.104-111
26. *Gubanova E., Glushkov A., Khetselius O., Bunyakova Yu., Buyadzhi V., Pavlenko E.,* New methods in analysis and project management of environmental activity: Electronic and radioactive waste.– Kharkiv: FOP.-2017.-120P.
27. *Wittman, M. A. H., Charlton, A. J. and Polvani, L. M.,* On the meridional structure of annular modes// J. Climate,-2005.-Vol. 18.-P.2119–2122.

28. *Voss R. and Mikolajewicz U.,* Long-term climate changes due to increased CO2 concentration in the coupled atmosphere-ocean general circulation model ECHAM3/LSG// Climate Dynamics.-2001.-Vol.17.-P.45–60
29. *Arakava A., Shubert W.H.,* Interaction of cumulus cloud ensemble with the lerge-scale environment//Journ. of Atm. Sci.-1974.-Vol.31.-P.674-701.
30. *Rusov V.D., Glushkov A.V., Vaschenko V.N., Myhalus O.T., Bondartchuk Yu.A., Smolyar V.P., Linnik E.P., Mavrodiev S.C., Vachev B.I.,* Galactic cosmic rays – clouds effect and bifurcation model of the Earth global climate. Part 1. Theory // J. Atm.c and Solar-Terrestrial Phys.-2010.-Vol.72.-P.498-508.
31. *Khetselius O.Yu.,* Hyperfine structure of atomic spectra.-Odessa: Astroprint, 2008.
32. *Glushkov A.V., Khetselius O.Yu., Svinarenko A.A., Buyadzhi V.V.,* Methods of computational mathematics and mathematical physics. P.1. TES: Odessa, 2015.
33. *Rusov V., Pavlovich V., Vaschenko V., Tarasov V.A., Zelentsova T.N.,* Geoantineutrino spectrum and slow nuclear burning on the boundary of the liquid and solid phases of the Earth's core// J. Geophys. Res.: Solid Earth 2007, Vol. 112 (B9).

Хецеліус О.Ю., Глушков О.В., Софронков О.Н., Степаненко С.М., Свинаренко А.А.

Новий балансовий по ентропії, енергії і кутовому моменту підхід до моделювання клімату та макротурбулентної динаміки атмосфери, тепломасопереносу в макромасштабі. ІІІ. Низькочастотна апроксимація та сингулярності в полях метеоелементів

АНОТАЦІЯ

Введено узагальнене низькочастотне наближення при записі співвідношень балансу енергії, кутового імпульсу і ентропії в задачах моделювання клімату і макротурбулентної динаміки атмосфери, тепло-масо-переносу на макрорівні, що дозволяє значно спростити основні фундаментальні рівняння. Новий нестаціонарний підхід до моделювання глобальних механізмів кліматичних і макротурбулентних атмосферних низькочастотних процесів, включаючи процеси тепло-масо-переносу, ефекти телеконекції і т.і. викладено і базується на використанні балансових співвідношень для ентропії, енергії, кутового моменту, спектральної теорії атмосферної макротурбулентності і кругообігу вологи в зв'язку з безперервністю форм атмосферної циркуляції (телеконнекція та генезис фронтів). Визначені і проаналізіровані фізичні сингулярності в полях метеорологічних елементів і балансі кутового моменту, а також узагальнена модель Аракави-Шуберта.

Ключові слова: *макротурбулентні атмосферні процеси, тепло-масо-обмін, низькочастотне наближення, баланс енергії, кутового моменту та ентропії*

Хецелиус О.Ю., Глушков А.В., Софронков А.Н.,
Степаненко С.Н., Свинаренко А.А.

Новый балансовый по энтропии, энергии и угловому моменту подход к моделированию климата и макротурбулентной динамики атмосферы, тепломассопереноса в макромасштабе. III. Низкочастотная аппроксимация и сингулярности в полях метеоэлементов

АННОТАЦИЯ

Введено обобщенное низкочастотное приближение при записи соотношений баланса энергии, углового импульса и энтропии в задаче моделирования климата и макротурбулентной динамики атмосферы, тепло-массо-переноса на макроуровне, что позволяет значительно упростить основные фундаментальные уравнения. Новый равновесный подход к моделированию глобальных механизмов климатических и макротурбулентных атмосферных низкочастотных процессов, включая процессы тепломассопереноса, эффекты телесвязи и т.д. изложен и базируется на использовании балансовых соотношений для энтропии, энергии, углового момента, спектральной теории атмосферной макротурбулентности и круговороте влаги в связи с непрерывностью форм атмосферной циркуляции (телеконнекция, генезис фронтов). Определены и проанализированы физические сингулярности в полях метеорологических элементов и балансе углового момента, а также обобщенная модель Аракавы-Шуберта.

***Ключевые слова**: макротурбулентные атмосферные процессы, тепломассоперенос, низкочастотное приближение, баланс энергии, углового момента и энтропии.*