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## SPECTROSCOPY OF MULTICHARGED IONS IN PLASMAS: OSCILLATOR STRENGTHS OF BE-LIKE IONS $G_{\text{A}}\text{XXVIII}$ and $G_{\text{E}}\text{XXIX}$

Oscillator strengths  $gf$  for  $2s^2-[2s1/22p3/2]_1$  transition in Be-like multicharged ions of  $G_{\text{A}}\text{XXVIII}$  and  $G_{\text{E}}\text{XXIX}$  Fe are computed for different values of the electron density and temperature ( $n_e=10^{22}-10^{24}\text{cm}^{-3}$ ,  $T=0.5-2$  keV) of plasmas are presented and compared with available alternative spectroscopic data. The generalized relativistic energy approach and relativistic many-body perturbation theory with the Debye shielding model as zeroth approximation is used for studying spectral parameters of ions in plasmas. An electronic Hamiltonian for  $N$ -electron ion in a plasma is added by the Yukawa-type electron-electron and nuclear interaction potential.

### 1. Introduction

Spectroscopy of multicharged ions in a plasmas is one of the most fast developing branches of modern atomic spectroscopy. The properties of laboratory and astrophysical plasmas have drawn considerable attention over the last decades [1-14]. It is known that multicharged ions play an important role in the diagnostics of a wide variety of plasmas. Similar interest is also stimulated by importance of this information for correct determination of the characteristics for plasma in thermonuclear (tokamak) reactors, searching new mediums for X-ray range lasers. The electron-ion collisions play a major role in the energy balance of plasmas. ([1-6]). Different theoretical methods were employed along with the Debye screening to study plasma medium. Earlier we have developed a new version of a relativistic energy approach combined with the many-body perturbation theory (RMBPT) for multi-quasiparticle (QP) systems for studying spectra of plasma of the multicharged ions and electron-ion collisional parameters. The method is based on the Debye shielding model and energy approach [3-5]. A new element of this paper is in using the effective optimized Dirac-Kohn-Sham method in general relativistic energy approach to collision processes in the Debye plasmas.

In this paper, which goes on our work [3-5], we present the results of computing energy shifts and oscillator strengths  $gf$  for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transitions in the Be-like ions of

$G_{\text{A}}\text{XXVIII}$  and  $G_{\text{E}}\text{XXIX}$ , calculated for different values of the electron density and temperature ( $n_e=10^{22}-10^{24}\text{cm}^{-3}$ ,  $T=0.5-2$  keV) of plasmas and compared with available alternative spectroscopic data.

### 2. Generalized energy approach in scattering theory. Debye shielding model

The detailed description of our approach was earlier presented (see, for example, Refs. [3-5]). Therefore, below we are limited only by the key points.

The generalized relativistic energy approach combined with the RMBPT has been in details described in Refs. [7,13-27]. It generalizes earlier developed energy approach. The key idea is in calculating the energy shifts  $\Delta E$  of degenerate states that is connected with the secular matrix  $M$  diagonalization [6,7,13-16]. To construct  $M$ , one should use the Gell-Mann and Low adiabatic formula for  $\Delta E$ . The secular matrix elements are already complex in the PT second order. The whole calculation is reduced to calculation and diagonalization of the complex matrix  $M$  and definition of matrix of the coefficients with eigen state vectors  $B_{e,i}^K$  [5-8]. To calculate all necessary matrix elements one must use the basis's of the 1QP relativistic functions. Within an energy approach the total energy shift of the state is usually presented as [13-15]:

$$\Delta E = \text{Re}\Delta E + i \Gamma/2 \quad (1)$$

where  $\Gamma$  is interpreted as the level width and decay possibility  $P = \Gamma$ . The imaginary part of electron energy of the system, which is defined in the lowest PT order as [3]:

$$\text{Im}\Delta E(B) = -\frac{e^2}{4\pi} \sum_{\substack{\alpha > n > f \\ [\alpha < n \leq f]}} V_{\alpha n \alpha n}^{|\omega_{\alpha n}|}, \quad (2)$$

where  $\sum$  for electron and  $\sum_{\leq}$  for vacancy. The separated terms of the sum in (3) represent the contributions of different channels. It is known that their adequate description requires using the optimized basis's of wave functions. In [6] it has been proposed "ab initio" optimization principle for construction of cited basis's. It uses a minimization of the gauge dependent multielectron contribution of the lowest QED PT corrections to the radiation widths of atomic levels. This contribution describes collective effects and it is dependent upon the electromagnetic potentials gauge (the gauge non-invariant contribution  $\delta E_{\text{nimv}}$ ). The minimization of  $\text{Im}\delta E_{\text{nimv}}$  leads to integral differential equation, that is numerically solved. In result one can get the optimal one-electron basis of the PT [12-16]. It is worth to note that this approach was used under solving of multiple problems of modern atomic, nuclear and molecular physics (see [17-27]). Further let us firstly consider the Debye shielding model according to Refs. [4,5]. It is known in the classical theory of plasmas developed by Debye-Hückel, the interaction potential between two charged particles is modelled by the Yukawa-type potential, which contains the shielding parameter  $\mu$  [2]. The parameter  $\mu$  is connected with the plasma parameters such as the temperature  $T$  and the charge density  $n$  as follows:  $\mu \sim \sqrt{e^2 n / k_B T}$ . Here, as usually,  $e$  is the electron charge and  $k_B$  is the Boltzman constant. The density  $n$  is given as a sum of the electron density  $N_e$  and ion density  $N_k$  of the  $k$ -th ion species having the nuclear charge

$$q_k: \quad \sum \quad . \quad (3)$$

It is very useful to remind the simple estimates for the shielding parameter. For example, under typical laser plasmas conditions of  $T \sim$

1keV and  $n \sim 10^{22} \text{ cm}^{-3}$  the parameter  $\mu$  is of the order of 0.1 in atomic units; in the EBIT plasmas  $T \sim 0.05 \text{ keV}$ ,  $n \sim 10^{18} \text{ cm}^{-3}$  and  $\mu \sim 10^{-3}$ . We are interested in studying the spectral parameters of ions in plasmas with the temperature  $T \sim 0.1 \text{ keV}$  ( $10^6 \text{--} 10^7 \text{ K}$ ) and  $n \sim 10^{14} \text{--} 10^{26} \text{ cm}^{-3}$  ( $\mu \sim 10^{-5} \text{--} 10^0$ ). It should be noted that indeed the Debye screening for the atomic electrons in the Coulomb field of nuclear charge is well understood due to the presence of the surrounding plasma electrons with high mobility. On the other hand, the contribution due to the Debye screening between electrons would be of smaller magnitude orders. Majority of the previous works on the spectroscopy study have considered the screening effect only in the electron-nucleus potential where the electron-electron interaction potential is truncated at its first term of the standard exponential expansion for its dominant contribution [3-69]. However, it is also important to take into account the screening in the electron-electron interactions for large plasma strengths to achieve more realistic results in the search for stability of the atomic structure in the plasma environment.

By introducing the Yukawa-type e-N and e-e interaction potentials, an electronic Hamiltonian for N-electron ion in a plasma is in atomic units as follows [4]:

$$H = \sum_i [\alpha c p - \beta m c^2 - Z \exp(-\mu r_i) / r_i] + \sum_{i>j} \frac{(1 - \alpha_i \alpha_j)}{r_{ij}} \exp(-\mu r_{ij}) \quad (4)$$

To generate the wave functions basis we use the optimized Dirac-Kohn-Sham potential with one parameter [8], which calibrated within the special ab initio procedure within the relativistic energy approach [6]. The modified PC numerical code "Superatom" is used in all calculations. Other details can be found in Refs. [3-6].

### 3. Results and conclusion

Firstly, we present our results on energy shifts and oscillator strengths for transitions  $2s^2 \text{--} 2s_{1/2} 2p_{1/2,3/2}$  in spectra of the Be-like Fe. The corresponding plasma parameters are as follows:  $n_e = 10^{22} \text{--} 10^{24} \text{ cm}^{-3}$ ,  $T = 0.5 \text{--} 2 \text{ keV}$  (i.e.  $\mu \sim 0.01 \text{--} 0.3$ ).

We studied a behavior of the energy shifts  $\Delta E$  ( $\text{cm}^{-1}$ ) for  $2s^2-[2s_{1/2}2p_{1/2,3/3}]_1$  transitions and oscillator strengths changes for different plasma parameters (the electron density and temperature). In Table 1 there are listed the oscillator strengths  $gf$  for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transition in Be-like GaXXVIII for different values of the  $n_e$  ( $\text{cm}^{-3}$ ) and  $T$  (in eV): the alternative theoretical data by Yongqiang Li et al [1] and our data.

Table 1.  
**Oscillator strengths for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transition in Be-like ion of GaXXVIII different  $n_e$  ( $\text{cm}^{-3}$ ) and  $T$  (eV) ( $gf_0$  –  $gf$  value for free ion)**

$n_e$		$10^{22}$	$10^{24}$
kT	[13]	[13]	[13]
500	0.1416	0.14157	0.14214
1000		0.14157	0.14200
2000		0.14157	0.14190
I-S		0.14157	0.14171
$n_e$		$10^{22}$	$10^{24}$
kT	Our	Our	Our
500	0.1419	0.14185	0.14268
1000		0.14185	0.14255
2000		0.14185	0.14242

There are also listed the available data by Li et al and Saha-Frische: the multiconfiguration Dirac-Fock (DF) calculation results, and ionic sphere (I-S) model simulation data [1, 2] (see refs. therein). In Table 2 we presented our data on the oscillator strengths  $gf$  for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transition in Be-like ion of GeXXIX for different values of the  $n_e$  ( $\text{cm}^{-3}$ ) and  $T$  (in eV).

Table 2.  
**Oscillator strengths for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transition in Be-like ion of GeXXIX, for different  $n_e$  ( $\text{cm}^{-3}$ ) and  $T$  (eV) ( $gf_0$  –  $gf$  value for free ion)**

$n_e$	$10^{22}$	$10^{23}$	$10^{24}$
kT	Our	Our	Our
500	0.14052	0.14068	0.14115
1000	0.14052	0.14067	0.14104
2000	0.14052	0.14065	0.14089

The analysis shows that the presented data are in physically reasonable agreement, however, some difference can be explained by using different relativistic orbital bases and different models for accounting of the plasma screening effect. It is important to note that our computing oscillator strengths within an energy approach with different forms of transition operator (this is corresponding to using the photon propagators in the form of Coulomb, Feynman and Babushkin) gives very close results.

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### **SPECTROSCOPY OF MULTICHARGED IONS IN PLASMAS: OSCILLATOR STRENGTHS OF Be-LIKE IONS GaXXVIII and GeXXIX**

#### **Summary**

Oscillator strengths  $gf$  for  $2s^2-[2s_{1/2}2p_{3/2}]_1$  transition in Be-like multicharged ions of GaXXVIII and GeXXIX are computed for different values of the electron density and temperature ( $n_e=10^{22}$ - $10^{24}\text{cm}^{-3}$ ,  $T=0.5$ - $2$  keV) of plasmas are presented and compared with available alternative spectroscopic data. The generalized relativistic energy approach and relativistic many-body perturbation theory with the Debye shielding model as zeroth approximation is used for studying spectral parameters of ions in plasmas. An electronic Hamiltonian for N-electron ion in a plasma is added by the Yukawa-type electron-electron and nuclear interaction potential.

**Key words:** spectroscopy of ions in plasmas, relativistic energy approach, oscillator strengths

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### **СПЕКТРОСКОПИЯ МНОГОЗАРЯДНЫХ ИОНОВ В ПЛАЗМЕ: СИЛЫ ОСЦИЛЛЯТОРОВ ДЛЯ Be-ПОДОБНЫХ ИОНОВ GaXXVIII и GeXXIX**

#### **Резюме**

Силы осцилляторов  $2s^2-[2s_{1/2}2p_{3/2}]_1$  переходов в Be-подобных многозарядных ионах GaXXVIII, GeXXIX рассчитаны для различных значений электронной плотности и температуры ( $n_e=10^{22}$ - $10^{24}\text{cm}^{-3}$ ,  $T=0.5$ - $2$  keV) плазмы и сравниваются с имеющимися альтернативными спектроскопическими данными. Изучение спектральных параметров ионов в плазме выполнено на основе обобщенного релятивистского энергетического подхода и релятивистской многочастичной теории возмущений с использованием экранировочной модели Дебая. Электронный гамильтониан для N-электронного иона в плазме дополнен потенциалом электрон-электронного и ядерного взаимодействия типа Юкавы.

**Ключевые слова:** спектроскопия ионов в плазме, энергетический подход, силы осцилляторов

## **СПЕКТРОСКОПІЯ БАГАТОЗАРЯДНИХ ІОНІВ В ПЛАЗМІ: СИЛИ ОСЦИЛЯТОРІВ ДЛЯ Ве-ПОДІБНИХ ІОНІВ GaXXVIII і GeXXIX**

### **Резюме**

Сили осциляторів  $2s^2-[2s_{1/2}2p_{3/2}]_1$  переходів в Ве-подібних багатозарядних іонах GaXXVIII, GeXXIX розраховані для різних значень електронної густини і температури ( $n_e=10^{22}-10^{24}\text{cm}^{-3}$ ,  $T=0.5-2\text{ keV}$ ) плазми та порівнюються з наявними альтернативними спектроскопічними даними. Вивчення спектральних параметрів іонів в плазмі виконано на основі узагальненого релятивістського енергетичного підходу і релятивістської багаточастинкової теорії збурень з використанням екраніровочної моделі Дебая. Електронний гамільтоніан для N-електронного іона в плазмі доповнений потенціалом електрон-електронної та ядерного взаємодії типу Юкави.

**Ключові слова:** спектроскопія іонів в плазмі, енергетичний підхід, сили осциляторів