THE KINETICS OF PROCESSES OCCURRING IN GRANULAR MATERIALS IN THE FIELD OF VIBROACCELERATIONS

O.I. GERASYMOV, N.N. KHUDYNTSEV, O.A. KLYMENKOV, A.YA. SPIVAK

Odesa State University of Environment
(15, L'vivs'ka Str., Odesa 65016, Ukraine; e-mail: ogerasimov@paco.net)

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We study the kinetics of compaction and segregation of granular materials in the field of vibroaccelerations or in contact with other external sources of energy. In the vicinity of quasistationary states, the relaxation of the field of the order parameter is described with the use of simple obvious models of inelastically colliding particles, the kinetic model of free volume, and the Landau—Ginzburg and Cahn—Hilliard approaches. The obtained results are in good agreement with experimental data.

1. Introduction

Granular materials are of a great scientific and technological relevance and display the properties intermediate to those of solids, liquids and gases [1—3]. Because granular materials dissipate energy on collision, they exhibit the interesting, sometimes even intriguing dynamical properties under conditions when the collisional energy loss is compensated by a continuous input of energy. In the initial stages of the dynamical evolution of a freely evolving homogeneous system of inelastic granular particles, the system continuously loses energy in a so-called homogeneous cooling state, where the density field is approximately uniform. This state is unstable to density fluctuations, and the system evolves into an inhomogeneous cooling state, where particle-rich clusters are formed and grow. In contrast to a good understanding of the homogeneous cooling state, there is a limited understanding of the nonlinear domain growth processes depending on the granular density and velocity. In particular, the question about the physical reasons of similarities between freely evolving granular materials and the phase-ordering dynamics involving the relevant morphologies in the inhomogeneous cooling state presents a challenge for the statistical physics of complex systems.

In [4], it has been shown that, in terms of the relevantly defined order parameter, the kinetics of clusterization like those of compaction or segregation should obey the Landau—Ginzburg-like scenario of the phase ordering dynamics with a nonconserved order parameter field.

However, it is not clear in advance why the Landau—Ginzburg scenario of the phase-ordering kinetics occurs, but not, for instance, a Cahn—Hilliard dynamics which describes the relaxation of a conserved order parameter field [5].

Consider the nondiffusive dynamics of granular particles which move along straight lines, until they collide with other particles. In this scenario, the density and momentum are conserved quantities. Depending on the density of the fluid, the distance traveled by particles prior to a collision may be considerable. Therefore, the relevant variables (e.g., the order parameter) are conserved on the macroscopic length scale of the mean-free path and not on the microscopic length scale. Typically, the conservation length scale is comparable to the length scale of coarsening clusters which grows with time. Therefore, the density (the order parameter) field are globally conserved, rather than locally conserved.

2. Free Volume Dynamics

If we describe a compaction induced by the weak tapping of a granular system, as the process, where a grain can jump into a hole of the appropriate size under the given distribution of hole sizes, the free volume dynamics obeys the simple kinetic scenario

$$\frac{d\eta}{dt} = kF, \tag{1}$$

where F is the distribution function of hole sizes and k is the kinetic coefficient, $\eta = \rho\omega$ is the compactivity, ρ is the density of grains (the number of grains per unit volume, each with a volume ω).

Taking F to be the Poisson type

$$F = \exp\left(-\frac{\omega}{\Omega}\right), \tag{2}$$

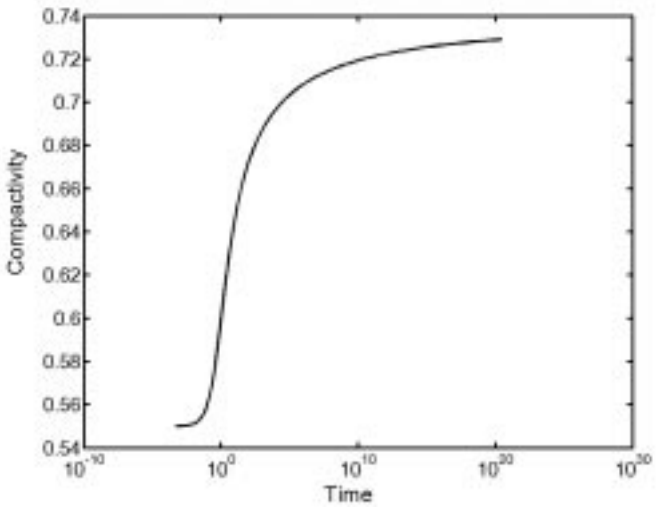

Fig. 1. Density curve for the free-volume kinetics

which describe the probability for any volume to be larger than the volume of a typical grain ω. Here,

$$\Omega = \frac{1}{\rho} - \frac{1}{\rho_m} = \omega \left(\frac{1}{\eta} - \frac{1}{\eta_m} \right) = \frac{\omega}{\eta \eta_m} \left(\eta_m - \eta \right), \qquad (3)$$

ρ_m and η_m are the maximal values of the density and compactivity (which are approached asymptotically), respectively.

Writing the kinetic equation in terms of compactivities, we get

$$\frac{d\eta}{dt} = k \exp \left(-\frac{\eta \eta_m}{\eta_m - \eta} \right). \qquad (4)$$

Integrating (4) from the initial value η_1, we obtain the functional

$$\exp\left(\eta_m\right) kt = \left(\eta_m - \eta_1\right) \exp\left(\frac{\eta_m^2}{\eta_m - \eta_1} \right) -$$

$$- \left(\eta_m - \eta\right) \exp\left(\frac{\eta_m^2}{\eta_m - \eta} \right) +$$

$$+ \eta_m^2 \left[E_1 \left(-\frac{\eta_m^2}{\eta_m - \eta_1} \right) - E_1 \left(-\frac{\eta_m^2}{\eta_m - \eta} \right) \right], \qquad (5)$$

where $E_1\left(z\right)$ is the exponential integral.

Solving (5) with respect to η (because of $\eta_1 < \eta < \eta_m$) in the asymptotic limit and neglecting the vanishing terms, we arrive at

$$\eta = \eta_m \left(1 - \frac{\eta_m \Gamma}{1 + \Gamma \ln\left(1 + t/t_0\right)} \right), \qquad (6)$$

where

$$t_0 = \frac{1}{k} \left(\frac{\eta_m - \eta_1}{\eta_m} \right)^2 \exp\left(\frac{\eta_1 \eta_m}{\eta_m - \eta_1} \right),$$

$$\Gamma = \frac{\eta_m - \eta_1}{\eta_m^2}. \qquad (7)$$

The behavior of η given by (6) as a function of t is plotted in Fig. 1.

Thus, from the simple free volume dynamics, we have obtained the logarithmic law of compaction, which have been revealed in [6]. The time given by (7) tells us roughly how many time we need for compaction.

Note that when the initial state of a granular system is prepared so that η_1 is close to η_m, the slow logarithmic evolution should be observed during a long time.

The model described above can be also expanded on a two-component granular mixture. Introducing the ratio $\zeta = \eta_L / \eta_S$, where η_L and η_S are the compactivities of large and small fractions, respectively, we note that, for such a system, one can expect two types of the dynamics which correspond to two different states of the system: with $\rho > \rho^*$, where the large grains are rather compact, and $\rho < \rho^*$ with raisins floating in the dense matrix of flour. Here, ρ^* is some typical value for ρ on the imaginary phase diagram for a two-component granular system. One can expect along this line that we will get again logarithmic relaxation laws, but with different rates for both states. Also, we expect that the slower mode will dominate the compaction. The cross-over between the two relaxations will give the information about details of the compaction (or segregation).

As follows from the results of the performed analysis and the data of direct physical experiments, the evolution of the density (compactivity) field is analogous to the phase ordering dynamics in the Landau—Ginzburg and Cahn—Hilliard scenarios [7].

3. The Kinetics of Granular Segregation

It is known from numerous experiments [8—13] that, when granular materials are placed in a rotating bed, a different flow dynamics is observed. Generally speaking, many parameters are involved in the process of segregation, such as the size ratio, shape, mass, friction forces, rotating velocity, filling of a drum, etc. The question naturally arises: what is the simplest model representing the essential physics?

One can say that the size segregation mechanism can be seen as a more pronounced influence of one attractor as compared to the other. This is a reminiscence of non-equilibrium phase transitions, the control parameter

being the size ratio (nevertheless, the non-trivial point is the importance of fluctuations which are of the size of the system). If we accept this, we may describe the evolution of such a kind of systems by some displacement field $\xi(\vec{r},t)$. Clearly, we may define deformations $\vec{\nabla}\xi(\vec{r},t)$ inside the frozen phase. To describe the phenomena of segregation more precisely, the order parameter field should be introduced as follows. From experimental data, let us define the region of a drum statistically occupied by the cluster of small particles in the final segregated state. When the process becomes stationary, one can take pictures, and, after developing, the segregated zone will be defined. The kinetic study will be reduced, afterwards, to counting the number of particles which belong to this segregated zone. From these operations, one can deduce the area $\Omega(t)$ occupied by small particles in the segregated zone defined above. The connected mass created in this way is called the reference mass (a mass is said to be "connected" when its particles actually touch each other). Its volume or surface area reached in principle after an infinite time is denoted by $\Omega(\infty)$. Clearly, $\Omega(t) \leq \Omega(\infty)$. At this point, it is natural to introduce the average ordering parameter $P(t)$ that can vary between 0 (for completely random and homogenous mixtures) and 1 (for the fully developed reference mass). This parameter is defined in terms of $\Omega(t)$ by

$$P(t) = \frac{\frac{\Omega(t)}{\Omega(\infty)} - \frac{\Omega(0)}{\Omega(\infty)}}{1 - \frac{\Omega(0)}{\Omega(\infty)}}. \tag{8}$$

As follows from experimental data [13—18], the relevant order parameter $P(t)$ defined by Eq. (8) shows a global trend of increasing in time and saturating asymptotically (on the long run after the cylinder or 2D drum rotation has been started). A typical time evolution of the average order parameter $P(t)$ is well enough approximated by an exponential law of the form

$$P(t) = P(\infty)\left[1 - \exp\left(-\frac{t}{\tau_c}\right)\right] \tag{9}$$

with a characteristic segregation time τ_c. Note that, from the point of view of the general theory, it is not possible to generate an exact steady-state trajectory in the phase space. This is because the measure of any dissipative non-equilibrium steady state within the phase space is zero [19, 20], as well as the probability of selecting the initial phase points that lie exactly in an equilibrium steady state. For such purposes, one can use the equilibrium (relaxation) method of approaching a steady state. This picture can be expressed mathematically in the $(\vec{r},t)$ space by definition of the relevant order parameter which demonstrates the system evolution starting from the initial state, passing the intermediate heterogeneous state, and finishing with the asymptotic non-equilibrium stationary state (local equilibrium or steady state).

Consider the kinetic theory of non-equilibrium phase transitions (which is based on the Landau theory of phase transitions [7]) as an instrument for the investigation of the fragmentation within open dissipative systems (with an example of a driven dry granular system in the segregation state) close to the critical region of a metastable stationary state (steady state).The existence of this state follows definitively from experiments. On that way, considering the final non-equilibrium asymptotic state, when the segregation is reached, as a steady state, one can describe the phenomenon of fragmentation (segregation) as a relaxation with a relevantly determined order parameter field.

Considering the radial segregation as a weakly non-equilibrium relaxation process in terms of a relevant parameter of ordering $\varphi(\vec{r},t)$ we focus on the evolution of $\varphi(\vec{r},t)$ in the vicinity of the steady state. We assume that the evolution of $\varphi(\vec{r},t)$ can be described with a master dynamic equation of the Landau—Ginzburg type for a non-conservative field $\varphi(\vec{r},t)$:

$$\frac{\partial\varphi}{\partial t} = -\Gamma\frac{\delta H}{\delta\varphi} \tag{10}$$

or with the Cahn—Hilliard-type equation in the case of a conservative field of ordering:

$$\frac{\partial\varphi}{\partial t} = -\Delta\left\{-\Gamma\frac{\delta H}{\delta\varphi}\right\}. \tag{11}$$

Here, Γ is a kinetic coefficient, and $H(\varphi)$ is the non-equilibrium potential functional which can be taken as

$$H(\varphi) = \int\left[\frac{c}{2}\left(\vec{\nabla}\varphi\right)^2 - \frac{a}{2}\varphi^2 - \frac{b}{4}\varphi^4\right]d\vec{r}. \tag{12}$$

Substituting functional (12) into (10), (11), we obtain (in dimensionless variables) the following equations of motion for $\tilde{\varphi}(\vec{r},t)$:

$$\frac{\partial\tilde{\varphi}}{\partial\tau} = \Delta'\tilde{\varphi} + \tilde{\varphi} - \tilde{\varphi}^3 \tag{13}$$

and

$$\frac{\partial\tilde{\varphi}}{\partial\tau'} = -\Delta'\left\{\Delta'\tilde{\varphi} + \tilde{\varphi} - \tilde{\varphi}^3\right\}, \tag{14}$$

respectively. Here, we define the dimensionless variables

$$\tau \equiv \Gamma a t \ , \ \tau' \equiv \Gamma c t \ , \ \vec{r}\,' \equiv \sqrt{\frac{a}{c}}\vec{r} \tag{15}$$

and measure the field $\varphi(\vec{r},t)$ in units of $\sqrt{a/b}$:

$$\tilde{\omega}(\vec{r},t) \equiv \sqrt{\frac{a}{b}}\tilde{\varphi}(\vec{r}\,',\tau)\,. \tag{16}$$

The nonlinear partial differential equations (13), (14) cannot be integrated in quadratures in general. But, in the vicinity of the imaginary point of a transition to the steady state, we can use the quasilinearization scheme [21—22]

$$\varphi^3(\vec{r},t) = \left\langle \varphi^2(t)\right\rangle \varphi(\vec{r},t)\,, \tag{17}$$

by neglecting the fluctuations of $\tilde{\varphi}(\vec{r},t)$. Here, the angular brackets denote the procedure of averaging over all initial states. It is possible to show that approximation (17) is more adequate in the limit when the fluctuations of the order parameter field $\tilde{\varphi}(\vec{r},t)$ are negligible in comparison with the quasiequilibrium value of the order parameter. We consider first the field equation of motion in the case of a non-conservative order parameter in more detail. Substituting (17) into (13), we obtain

$$\frac{\partial\tilde{\varphi}}{\partial\tau} = \Delta'\tilde{\varphi} + \left(1 - \left\langle\tilde{\varphi}^2(\tau)\right\rangle\right)\tilde{\varphi}. \tag{18}$$

Equation (18) seems open-circuited because of the presence of the second moment term $\left\langle\tilde{\varphi}^2(\tau)\right\rangle$. But the rigorous solution of Eq.(18) can be obtained, as we are going to show, in terms of $\left\langle\tilde{\varphi}^2(\tau)\right\rangle$, i.e. of the second moment of $\tilde{\varphi}(\vec{r},t)$. In terms of the Fourier transforms of the order parameter denoted as $\tilde{\varphi}_{\vec{k}}(\tau)$, Eq. (18) takes the form

$$\frac{\partial\tilde{\varphi}_{\vec{k}}(\tau)}{\partial\tau} = \left(-k^2 + 1 - \left\langle\tilde{\varphi}^2(\tau)\right\rangle\right)\tilde{\varphi}_{\vec{k}}(\tau)\,, \tag{19}$$

where

$$\tilde{\varphi}_{\vec{k}}(\tau) = \frac{1}{(2\pi)}\int e^{-i\vec{k}\vec{r}\,'}\tilde{\varphi}(\vec{r}\,',\tau)\,d\vec{r}\,'.$$

The solution of (19) has been obtained rigorously in [21, 22]. The different scenario of relaxation for $\left\langle\tilde{\varphi}^2(\tau)\right\rangle$ can be realized asymptotically under certain conditions. Namely, when

$$\hat{\Omega}(A,\alpha,C)\frac{1}{\sqrt{\tau}} > \tilde{\Omega}(A,\alpha,C)\exp(-2\tau)\,, \tag{20}$$

we have

$$\left\langle\tilde{\varphi}^2(\tau)\right\rangle \simeq \frac{1}{1+\frac{\hat{\Omega}}{\sqrt{\tau}}} \approx 1 - \frac{\hat{\Omega}}{\sqrt{\tau}}\,. \tag{21}$$

Respectively, in the case of the inequality opposite to (20), we have

$$\left\langle\tilde{\varphi}^2(\tau)\right\rangle \simeq \frac{1}{1-\tilde{\Omega}\exp(-2\tau)} \approx$$

$$\approx 1 + \tilde{\Omega}\exp(-2\tau)\ . \tag{22}$$

Here, the values of $\hat{\Omega}(A,\alpha,C)$ and $\tilde{\Omega}(A,\alpha,C)$ have been determined in [4, 21, 22].

Thus, within the presented model, the initial conditions influence the character of the asymptotic behavior of the order parameter field, providing long-time memory effects.

Consider now the behavior of the parameter of ordering in the case of conservative fields given by Eq. (14). Using the quasilinearization procedure defined by Eq. (17) and repeating the scheme developed for a non-conservative field, we obtain

$$\left\langle\tilde{\varphi}^2(\tau')\right\rangle = \int d\vec{k}\tilde{g}\left(\vec{k}\right)\exp\left[-2\tau' k^4 + \right.$$

$$\left. + 2k^2\int\limits_0^{\tau'}\left\langle\tilde{\varphi}^2(s)\right\rangle ds\right], \tag{23}$$

where $\tilde{g}\left(\vec{k}\right)$ is the Fourier component of the static structure factor which given in [4], and τ' is the dimensionless time of (15). After some calculations, relation (23) yields

$$\left\langle\tilde{\varphi}^2(\tau')\right\rangle = \frac{2\pi}{\sqrt{2\tau'}}\int\limits_{-\frac{\Xi(\tau')}{\sqrt{2\tau'}}} d\sigma\exp\left(\sigma^2\right)\times$$

$$\times\sqrt{\frac{2\sqrt{2\tau'}\sigma + 2\Xi(\tau')}{4\tau'}}\times\tilde{g}\left[\sqrt{\frac{2\sqrt{2\tau'}\sigma + 2\Xi(\tau')}{4\tau'}}\right], \tag{24}$$

where $\Xi(\tau') \equiv \tau' - \int\limits_0^{\tau'}\left\langle\tilde{\varphi}^2(s)\right\rangle ds$.

The integral in (24) is rapidly (namely exponentially) convergent and furthermore can be well estimated by the expression

$$\left\langle\tilde{\varphi}^2(\tau')\right\rangle = \frac{\pi\sqrt{\pi}}{2}\frac{\sqrt{\Xi(\tau')}}{\tau'}\exp\left[\frac{\Xi^2(\tau')}{2\tau'}\right]\times$$

$$\times \mathrm{erfc}^2\left[-\frac{\Xi(\tau')}{\sqrt{2\tau'}}\right]\tilde{g}\left[\sqrt{\frac{\Xi(\tau')}{2\tau'}}\right]. \quad (25)$$

We are now going to simplify the rigorous functional (25). Since the function $\exp\left[\Xi^2(\tau')/(2\tau')\right] \times \times \mathrm{erfc}^2\left[-\Xi(\tau')/\sqrt{2\tau'}\right]$ behaves exponentially, and the function $\tilde{g}\left[\sqrt{\Xi(\tau')/(2\tau')}\right]$ shows limited variations within the interval $\left[\tilde{g}[0], \tilde{g}\left[\frac{1}{\sqrt{2}}\right]\right]$, without losing the general features, functional (25) can be represented as

$$y(\tau') = \gamma\frac{\Xi(\tau')}{\tau'}\exp(\tau'), \quad (26)$$

where

$$y(\tau') \equiv \left\langle\tilde{\varphi}^2(\tau')\right\rangle, \quad \gamma \equiv \frac{\pi^3}{4}\tilde{g}^2\left(\frac{1}{\sqrt{2}}\right).$$

Expression (26) produces a differential equation for the definition of $y(\tau')$ of the Abelian type, namely

$$\frac{du}{d\tau'} = f_3u^3 + f_2u^2 + f_1u, \quad (27)$$

where

$$f_3 \equiv -2\nu\gamma^2\tau'^2\exp(\tau'), \quad f_2 \equiv \nu\gamma^2\exp(\tau'),$$

$$f_1 \equiv \beta, u(\tau') \equiv 1/2\tau'^2 y(\tau'); \quad (28)$$

$$\nu = \begin{cases} 4, & \tau' \to \infty \\ 1, & \tau' \to 0 \end{cases},$$

$$\beta = \begin{cases} -\frac{1}{\tau'}, & \tau' \to 0 \\ \frac{1}{2} - \frac{1}{\tau'}, & \tau' \to \infty \end{cases}. \quad (29)$$

Equation (27) cannot be integrated in quadratures, except for a few particular cases [23—25]. But, to conclude the qualitative analysis, it is enough to show in our case that the exponentially relaxing functions belong to the class of asymptotic solutions of Eq. (27). In fact, it is simple to show that functions of the type

$$u(\tau') = [1 + \varepsilon(\tau')]/2\tau'^2, \quad (30)$$

where

$$\varepsilon(\tau') = \left| \begin{array}{l} \to 0, \\ \tau' \to \infty, \end{array} \right. \quad \lim_{\tau'\to\infty}\varepsilon(\tau') = 0,$$

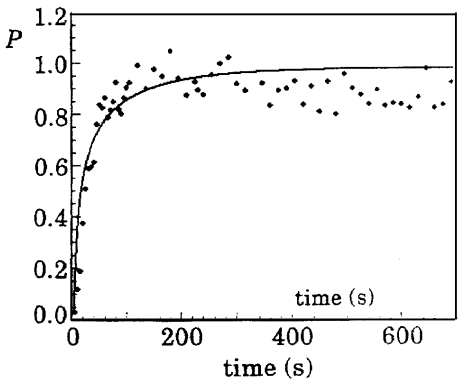

Fig. 2. Granular segregation curve (dots are the data of experiment [10], the solid line is theoretical results)

satisfy Eq. (27) if $\varepsilon(\tau')$ tends to zero more rapidly than any power law (in principle, this strong condition is even not required). Equation (30) perfectly satisfies Eq. (27) for any $\varepsilon(\tau') = \hat{\Delta}(\tau')e^{-\hat{\Gamma}\tau'}$ (where $\hat{\Delta}(\tau')$ belongs to the class of functions of limited variations). Thus, the exponential relaxation law $\left\langle\tilde{\varphi}^2(\tau')\right\rangle = 1 - \exp(-\tau'/\tau_0')$ definitively follows from the scenario given above. Note that, strictly speaking, the observable experimental data on the behavior of the order parameter under segregation show weakly non-monotonic behavior superposed on the general saturation tendency (see Fig. 2) [10].

This behavior can be simply described with the help of the factor $\hat{\Delta}(\tau')$ which could behave also non-monotonically (within a limited variation in the given interval). Note that solutions of the type $\hat{\Delta}(\tau')\exp\left(-\hat{\Gamma}\tau'\right)$ also asymptotically satisfy (27). When the Abelian equation (27) with the enough degree of accuracy is reduced to the Riccati-type differential equation

$$\frac{du}{d\tau'} = \gamma^2u^2 - \frac{u}{\tau'} \quad (31)$$

which has the simple integral [23—25]

$$u(\tau') = \frac{1}{c\tau' - \gamma^2\tau'\ln\tau'}. \quad (32)$$

Expression (32) produces the relevant function

$$y(\tau') = \left\langle\tilde{\varphi}^2(\tau')\right\rangle = \frac{c_1}{2\tau'} - \frac{\gamma^2}{2}\frac{\ln\tau'}{\tau'}, \quad (33)$$

where c_1 is a certain constant. As follows from (33) for the relevant dimensionless times τ', the order parameter can behave nonexponentially slowly (we call this behavior as a metastable "heterogeneous state"). Such a critical dynamic decay belongs, of course, to the

specific character of the considered model. But, at the same time, it is known from the experiments with driven granular materials [10] that a typical relaxation time of a segregation cluster remains almost unchanged as the rotation velocity increases by one order of magnitude. This behavior can reflect the existence of a critical slow dynamic regime (in terms of the order parameter field description — the slow relaxation metastable state regime). It is clear that, in the framework of the considered model, the relaxation picture cannot depend qualitatively on parameters like, e.g., the ratio of the diameters of particles in a (binary) mixture because, by the naked eye, the definition of the order parameter for growing segregated phases is not influenced by such parameters.

Our results demonstrate that, while the proposed model qualitatively behaves in agreement with the observed scenarios, the phenomenon of segregation in terms of order parameter fields still offers surprises. A full and detailed description of this effect (among many other intriguing properties of granular materials) continues to be an interesting challenge to theoretical and experimental physics.

4. Stationary States in a Model 1D System with Dissipative Interactions

The theoretical model is sufficiently simple to allowa rigorous analytic analysis to be performed, but it contains, at the same time, some features relevant to the theory of granular flows. The preliminary developments given above motivate us to proceed the analysis of the effects of dissipative interactions on the structure of stationary states. We will present the description of the coupling to a vibrating base via stochastic boundary conditions or dissipative binary collisions, being the ingredients of the model. In the search for understanding the new features induced by dissipation, it is interesting to compare the situations with and without dissipative coupling. The simplest model allows us to pursue this idea, as well as to examine the details in constructive properties of the asymptotic stationary state (steady state).

Consider a 1D model with dissipative collisions balanced by a single rebound velocity. Our object will be to study the effects of inelastic binary collisions taking place within the one-dimensional N-particle system, first vertically and then horizontally moving and forming a column or a cluster of identical masses falling with acceleration $(-g)$ in an external field and colliding with the base or moving between the hot and rebound walls. Model with horizontal distribution of inelastically colliding particles will be considered elswhere. Note that the contest of maded conclusion also qualitively conserved in this case. The energy losses due to binary collisions between the particles are balanced by encounters with the base or energy input from the hot wall, playing the role of the energy source. The question is to study the possibility and features of the resulting stationary state or the kinetics of a relevant clusterization. Note that the mentioned problem is connected with the famous problem concerning the dynamics of a fluidized granular matter. Such a system with the base represented by a vibrating plate has been the object of intensive studies [26,27].

Choose first the simplest vertical system with $N = 2$ and denote the relevant states of two particles by (z_1, v_1) and (z_2, v_2), respectively. Putting particle 1 being close to the base, we note that the linear ordering $0 \leq z_1 \leq z_2$ is preserved by the dynamics.

Under a binary collision between masses 1 and 2, their initial velocities (v_1, v_2) take instantaneously the post-collisional values

$$v_1' = v_1 - \frac{1+\varepsilon}{2} v_{12},$$

$$v_2' = v_2 + \frac{1+\varepsilon}{2} v_{12}, \tag{34}$$

where $v_{12} = v_1 - v_2$ and ε is the restitution coefficient.

When $\varepsilon = 1$, the particles just exchange their velocities, and the total kinetic energy is conserved. For $\varepsilon < 1$, the dissipation occurs.

By E_1^u, we denote the energy of particle 1 in its ascending motion after a collision with the base. Consider the case where the rebound velocity at the base has a singular distribution represented by the Dirac δ-function. Introducing the rebound probability density $\varphi(v)$, one can write

$$\varphi(v) = \delta(v - v_0), \tag{35}$$

where v_0 is some fixed characteristic value of the rebound velocity.

In accordance with (34), we have

$$E_1^u = \frac{m v_1^2}{2}. \tag{36}$$

Supposing that particle 2 is falling then with energy E_2^d and using the collision law (34), we obtain the dissipated kinetic energy

$$\left(E_1^d + E_2^u\right) - \left(E_1^u + E_2^d\right) = -\frac{1-\varepsilon^2}{4} m v_{12}^2. \tag{37}$$

We try to construct the stationary state of our system along the following line. In case where the energy of particle 2 (upper particle) is conserved and $E_2^u = E_2^d$, the collision with particle 1 just reverses its velocity. From (34) and (37), we have

$$v_1 = v_2 \frac{\varepsilon - 3}{\varepsilon + 1}. \tag{38}$$

Let us introduce the time τ^u, for which particle 1 attains the initial velocity v_1 as

$$\tau^u = \frac{v_0 - v_1}{g}. \tag{39}$$

To fall back to the base, the particle requires the time

$$\tau^d = T - \tau^u, \tag{40}$$

where

$$T = -\frac{2v_2}{g} \tag{41}$$

is the period of the expected collision.

We give v_2 now in terms of v_0 and ε. For that, we introduce the distance z which satisfies the classical dynamics

$$z = v_0 \tau^u - \frac{g}{2} \left[\tau^u\right]^2 \tag{42}$$

or, in terms of $v_1^{'}$ and τ^d,

$$z = -v_1^{'} \tau^d + \frac{g}{2} \left[\tau^d\right]^2. \tag{43}$$

From (42), (43), and (34), we obtain

$$v_2 = -v_0 \frac{\varepsilon + 1}{\varepsilon + 3}. \tag{44}$$

With the help of Eq. (41), (44), in the case of a system which includes two colliding particles, we get the period of oscillations in the considered stationary regime as

$$T_2 = \frac{2v_0}{g} \frac{\varepsilon + 1}{\varepsilon + 3}. \tag{45}$$

In case of a system of three colliding particles, we obtain

$$T_3 = \frac{6v_0}{g} \frac{1 + \varepsilon}{19 - \varepsilon}. \tag{46}$$

The interval Δz_1 where the collision of the lower outmost particle occurs in a stationary regime is

$$\Delta z_1 = \frac{10 v_0^2}{g} \frac{(4\varepsilon - 1)(4 - \varepsilon)}{(19 - \varepsilon)^2}. \tag{47}$$

Because of the obviously positive character of Δz_1, it is follows from (47) that, under the condition

$$\varepsilon < \varepsilon_c = \frac{1}{4}, \tag{48}$$

the colliding regime fails.

The stationary regime can occur also in the case of an N-particle system. The relevant expression for the period of stationary motion has the form

$$T_N = \frac{v_0}{g} \frac{2N(1 + \varepsilon)}{N^2(1 + \varepsilon) + 2(1 - \varepsilon) \sum\limits_{i=1}^{N-1} (N - i)^2}. \tag{49}$$

Note that, under the condition

$$\varepsilon < \varepsilon_c = \frac{N - 2}{N + 1}, \tag{50}$$

the stationary regime will fail. Putting, for instance $N = 99$, we have $\varepsilon_c = 0.97$ which is close to the elastic limit ($\varepsilon = 1$).

Consider now the behavior of the density of particles in the stationary state. For that reason, we introduce the ratio of the intervals of a periodic motion for the higher outmost particle Δz_N and lower outmost particle Δz_1:

$$\frac{\Delta z_1}{\Delta z_N} = (2N - 1) \left[1 - \left(\frac{1 - \varepsilon}{1 + \varepsilon} \frac{2N - 1}{3}\right)^2\right]. \tag{51}$$

The function $\Delta z_1 / \Delta z_N$ is plotted in Fig. 3 as a function of the total number of particles, N.

As follows from the data presented in Fig. 3, the dependence of $\Delta z_1 / \Delta z_N$ on N (contrary to the Boltzmann picture) mimics the nonmonotonic behavior. We also note that Fig. 3 shows the existence of the size of a system, for which the dynamic compaction has a maximal value (under the given restitution coefficient $\varepsilon = 0.97$). When the size of a system approaches the value $N = 99$, the stationarity of the system fails in accordance with criteria (50).

The expression for the total size of a system approaching the stationary state in terms of $\{\Delta z_i\}$ can be written as

$$L = \frac{gT^2}{8} + \sum_{i=1}^{N-1} \Delta z_i, \tag{52}$$

where $\{\Delta z_i\}$ is the set of intervals of the stationary motion of particles included in the system:

$$\Delta z_i = (2(N - i) + 1) \frac{g t_i^+ t_i^-}{2}. \tag{53}$$

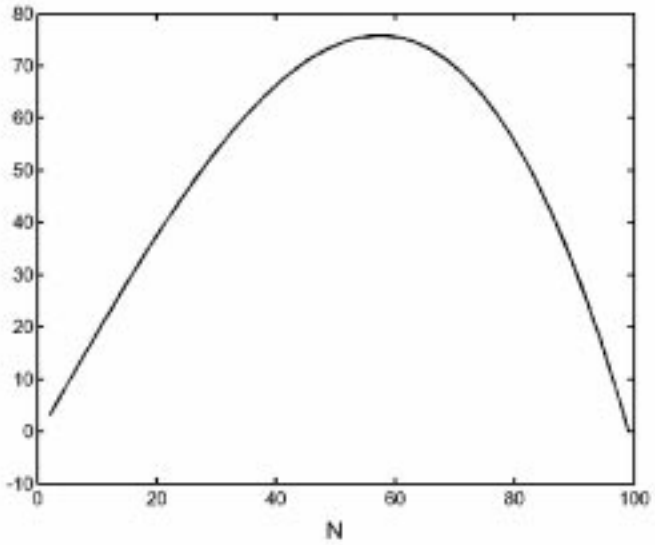

Fig. 3. Distribution $f(\Delta z_i)$ of intervals for the periodic motion of particles in the stationary state

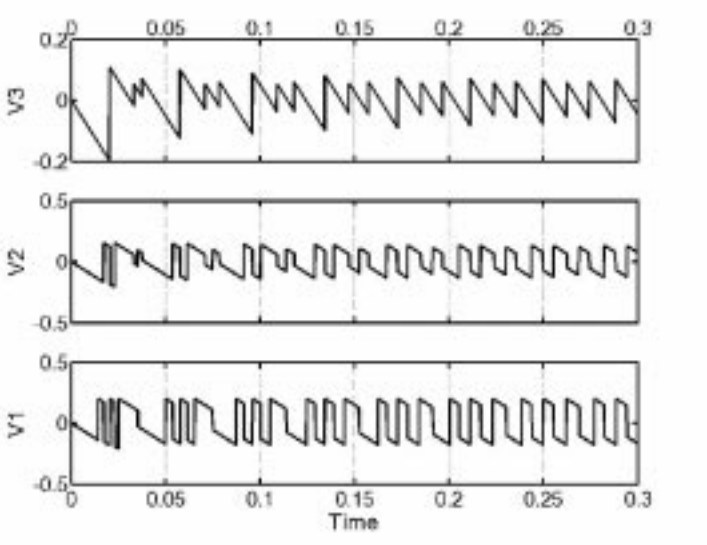

Fig. 4. Velocities of inelastically colliding particles in the steady state

Here, $t_i^{+,-}$ are the times of the free motion of the i-th particle in the stationary state in the directions up and down, respectively.

By induction, it is easy to show that

$$\frac{t_i^+}{t_i^-} = \frac{(N-i+2)\,\varepsilon - (N-i-1)}{(N-i+2) - (N-i-1)\,\varepsilon}. \qquad (54)$$

Respectively, Eq. (52) could be rewritten as

$$L = \frac{gT^2}{2}\left(\frac{1}{4} + \sum_{i=1}^{N-1}\left(2\,(N-i)+1\right)\frac{A}{(1+A)^2}\right), \qquad (55)$$

where $A = t_i^+/t_i^-$.

Fig. 4 shows the behavior of the velocities of inelastically colliding particles (in the case $N = 3$) in the stationary regime, the restitution coefficient being equal to 0.9. Thus, the behavior of the velocity field is influenced by the degree of thermalization.

Thus, we have considered an approach within statistical mechanics to the problems of compaction and segregation for granular materials. The possibility to describe some general features in the physics of a granular matter with the concepts of statistical mechanics requires a more precise investigation of the criteria and properties of asymptotic quasistationary states, where statistical mechanics should be valid. The obtained results prove that the simple kinetic models can be effectively used along this line in some particular cases, where such steady states are artificially constructed or clearly observed experimentally.

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КІНЕТИКА ПРОЦЕСІВ У ГРАНУЛЬОВАНИХ МАТЕРІАЛАХ У ПОЛІ ВІБРОПРИСКОРЮВАНЬ

О.І. Герасимов, М.М. Худинцев, О.А. Клименков, А.Я. Співак

Р е з ю м е

Модельний аналіз процесів компактизації та сегрегації гранульованих матеріалів у полі віброприскорювань або у контакті з зовнішним резервуаром енергії здійснено за допомогою моделей статистичної механіки поблизу квазістаціонарних станів, можливість існування яких витікає з експерименту та обгрунтовується теоретично за допомогою простих наочних моделей непружних частинок у контакті з термостатом, що припускають можливість термалізації. Отримані теоретичні висновки, зроблені на основі кінетичної моделі вільного об'єму та підходу, який застосовується за сценарієм Ландау—Гінзбурга та Кана—Хілліарда для опису релаксації поля відповідно визначеного параметра порядку, добре узгоджуються з результатами експериментальних досліджень.