1. Introduction

There are a large number of problems today related to management tasks that require the use of modern decision support methods (DSM), part of which are of spatial character. Such problems include the task on selecting the optimum location for a solid waste landfill (SW), which is urgent and requires addressing both at the state and municipal levels.

In Odesa oblast (Ukraine), there are now 608 landfills/solid waste dumps with an area of 1,274.9 hectares, the vast majority of them are operated in violation of the designed indicators for a volume of waste accumulation. According to the Program of SW treatment, prepared by the Odesa oblast council for the period of 2018–2022, it is planned in the territory of Odessa oblast to build four new regional SW landfills of and to eliminate the existing ones. In this regard, it is a relevant task to select and justify the location of landfills in accordance with the construction and sanitary and ecological norms. At the same time, it is necessary to minimize economic and social costs. To this end, a spatial decision support model should maximize the use of existing spatial information and ensure the acceptability of results of the analysis for the majority of stakeholders.

It is clear that solving such a task is impossible without the use of geo-informational technologies. Geographic information systems (GIS) is an ideal basis for the development of fully-fledged spatial decision support systems. They are capable of collecting, analyzing, simulating, and displaying spatial data.

The main limitation of these systems is the lack of analytical capabilities to account for several factors that affect a decision. On the other hand, the discipline of decision theory is being actively developed and enriched with new methods. Therefore, it is an actual approach to integrate these two technologies for solving spatial optimization problems of a multi-criteria character.
making decisions, which is why it is a fundamental tool for solving problems in many areas [2]. Reviews of studies and publications of the last decades, given in papers [3–5], confirm the effectiveness and success of the application of the integration of these technologies in the field of waste management [3–5].

Authors use a variety of classic methods and models of MCDA group and their combination for the selection of a suitable land plot for the construction of a SW landfill. The most popular ones include:

– Boolean Overlay [1] is a simple method which, based on the defined threshold values, converts the criteria in Boolean values, and then combines them, using the overlaying operations, into a final suitability map. The method can be easily implemented in any package of GIS. The disadvantage of the method is the lack of ability of ranking alternatives for suitability;

– Weighted Linear Combination (WLC) [6, 7] is a simple and intuitive method based on the determining of the weight of criteria and the calculation of sum of the weighted estimates. The method can be applied only for the additive attributes and it is compensatory. The disadvantages of the method include the possibility of data loss as a result of the normalization of criteria, as well as the complexity and subjectivity of the process for assigning the weight, especially when a large number of heterogeneous criteria exist;

– Analytic Hierarchy Process (AHP) [7, 8] implements a procedure of pairwise comparison of alternatives based on the judgments of experts and their preferences. The method can serve as an add-in to other methods, such as the methods WLC or TOPSIS. That makes it possible to define normalized weights of criteria; it has tools to identify differences in data. The disadvantages of the method include the ability to handle only clear assessments of experts, which complicates the application of a given method for solving the problems, which are characterized by uncertainty and incomplete information. It can only be used for the mutually independent criteria;

– Ordered Weighted Averaging (OWA) [7, 9] is a family of procedures for multicriteria aggregation, developed in the context of the theory of fuzzy sets. The method includes two sets of weight: the weight of importance of a criterion and the weight of order. By changing weight coefficients of the order, one can create maps for different strategies in decision making. That provides a theoretical link between the Boolean overlay and WLC. It requires adaptation to solve a specific problem; most GIS packages lack the full realization of the method;

– TOPSIS [10] is a method based on the calculation of a distance from the anti-ideal point and the distance to the ideal point. The alternative, which is the closest to the ideal point, is the best solution. The method can take into consideration an unlimited number of alternatives and criteria. Similarly to the WLC method, it requires normalization of criteria and is compensatory in character. This method is somewhat more complicated to implement in GIS;

– methods from the class of ORT (Outranking Relation Theory, or outranking methods) are a family of the methods ELECTRE, PROMETHEE [11]. The methods imply conducting a pairwise comparison of alternatives based on the function of advantage that makes it possible to resolve the issue of compensation, characteristic of the WLC and TOPSIS methods. However, the methods have a higher computational cost, making it impossible to use them for a large number of alternatives. In terms of spatial tasks, they can solve only discrete problems with a limited number of alternatives. There are difficulties of computing character when using methods to estimate each pixel at a map.

In our research, we attempted to choose a simple method for constructing the suitability maps that can be easily implemented in most GIS packages and which has a low computational cost. The task on finding the optimal locations for SW polygons always implies the existence of certain requirements to alternatives that are expressed in clear limits, that is, constraints. Considering restrictive zones when building a suitability map requires the use of the Boolean overlay method. On the other hand, there are always additional factors that are difficult to define in clear limits, and which may be taken into consideration based on expert estimation of their importance. Therefore, we find it appropriate to employ a weighted linear combination method in tandem with the method of analysis of hierarchies. As was already noted above, these methods are characterized by complexity, uncertainty, and subjectivity of the process of making a decision. It is required to take into consideration the existing objectives of the study or the resulting information from experts about objective and subjective nature of the problem. Thus, the initial information is often represented by many different factors that fail to be described in a formalized manner. Subjective information involves expert judgement and advantages of interested persons or parties, including expert estimates on the importance of criteria. Given this, it appears relevant to employ an apparatus of fuzzy logic, which makes it possible to take into consideration the uncertainty of initial information and subjectivity in expert assessments.

Thus, paper [12] applied a fuzzy approach at which the weight of the criteria was described by linguistic variables expressed in triangular fuzzy numbers. The authors focused their attention on the stage of ranking the seven areas that were identified as a result of Boolean overlay of criteria layers. Results of paper [13] showed that a combination of the methods WLC and AHR demonstrates better possibilities for decision making compared with the Boolean logic. In paper [14], the apparatus of fuzzy logic was used at the stage of criteria standardization. Following a fuzzy overlay, the authors obtained a raster map of suitability with cell values in the range [0,1]. To standardize the criteria, they used linear and sigmoid membership functions of a fuzzy set.

An analysis of publications [12–14] shows that authors focused their studies on considering the uncertainty and subjectivity of a decision-making process that occur at the stages of standardization of layers of criteria and when significance of the criteria is estimated by experts. However, the studies tackle only one of the two stages. In our research, the apparatus of fuzzy logic will be applied both at the stage of standardization of layers of criteria and at the stage of estimating the importance of criteria by experts. At the stage of standardization, that will make it possible to apply the fuzzy overlay operations in order to aggregate a suitability map. Thus, it will be possible to run a comparative analysis of modelling results using different operations of Boolean, fuzzy, and weighted overlay with the aim to identify the best
solution. The choice of a suitability model to search for the optimal location of SW landfill, which has a better accuracy and takes into consideration the fuzzy match between criteria in the process of modelling, necessitates further research in this direction.

3. The aim and objectives of the study

The aim of present study was to determine the most suitable GIS-oriented model to support decision-making under conditions of uncertainty in the advantages from experts in order to solve the task on finding the optimal location for SW landfill (using the south of Odessa oblast as an example). A GIS-based model will make it possible to obtain a more informative aggregated suitability map.

To accomplish the aim, the following tasks have been set:
- to create a methodically substantiated hierarchical structure for making a decision on the location of SW landfill with respect to the requirements of the state building standards;
- to justify the choice of a set of criteria and alternatives, methods for the standardization of criteria and aggregation of a suitability map;
- to analyze and compare simulation results of the resulting suitability map based on the application of various methods of aggregation: Boolean overlay, fuzzy overlay, and weighted overlay.

4. Materials and methods for making a multi-criteria decision on the location of solid waste landfills

4.1. Territory of the south of Odessa oblast, considered in the study

Underlying a strategy for handling SW flows in Odessa oblast is a regional approach, which implies creating over a given territory individual clusters (zones) of comprehensive solid waste management and constructing a SW landfill in each of them.

A solution to the task on finding a land plot for a SW landfill is to be found for a a separate cluster, located in the south of Odessa oblast (Fig. 1).

Fig. 1. Geographical location of the examined region

Fig. 1 shows location of the examined region at the territory of Kiliya, Bolghrad, Izmail, and Reny regions. Total area of the examined region is 4,778 km².

4.2. Determining the estimation criteria and construction of a hierarchical decision-making structure

The landfills are designed in accordance with the State Building Standards DBN V.2.4-2 “Solid waste landfills. Basic design provisions”. It is assumed that one polygon with a capacity of 100 thousand t/year must cover a total area of 30 hectares. The basic requirements for the construction of SW landfills are given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Factors</th>
<th>Threshold values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distance from airports and airfields</td>
<td>15 km</td>
</tr>
<tr>
<td>2</td>
<td>Distance from the border of a resort city, open household water reservoirs, facilities for cultural and recreational purposes, natural preserves, recreation of migratory birds, sea coast</td>
<td>3,000 m</td>
</tr>
<tr>
<td>3</td>
<td>Distance from city limits</td>
<td>1,000 m</td>
</tr>
<tr>
<td>4</td>
<td>Distance from residential and public facilities (sanitary-protective zone)</td>
<td>500 m</td>
</tr>
<tr>
<td>5</td>
<td>Distance from farmland, general-purpose automobile and railroads</td>
<td>200 m</td>
</tr>
<tr>
<td>6</td>
<td>Distance from the border of forest and local man-made greenery, not designed for recreational purposes</td>
<td>50 m</td>
</tr>
<tr>
<td>7</td>
<td>Depth of ground water table</td>
<td>not less than 2 m</td>
</tr>
</tbody>
</table>

A digital terrain model (DTM), as well as the maps of inclination and exposure, derived from it, were built based on the ASTER satellite images with a raster cell size of 27 m. A raster map of the ground water table depth was constructed using the method of spatial interpolation (kriging) based on data on monitoring observations carried out by Prychornomorskyy State Regional Geological Enterprise.

Vector maps of land use, water facilities, settlements, rail- and automobile roads were obtained by importing the database Open Street Map. SQL queries to the attributive table of land use map yielded maps of agricultural lands, natural reserves, residential buildings, forests and man-made greenery. We used the method of Euclidian distances as a metric for the proximity of raster cells to appropriate objects, which allowed us to create raster maps of distances from water bodies, rivers, farmland, residential buildings, city limits, bridge, rail- and motorways, airports, natural reserves, forests and man-made greenery.

The research program implied three scenarios for modeling the suitability map: using the Boolean logic, fuzzy logic, and a combination of methods of weighted overlay (WLC) and fuzzy analysis of hierarchies (FAHP).
4.3. Analysis of suitability using Boolean logic (scenario 1)

Analysis of the suitability by using Boolean logic, known as the Boolean Overlay [1], is based on re-categorization of layers of criteria into binary values of suitability taking into consideration constraints (thresholds).

An alternative attribute is assigned with value 1 (true) if the criterion value for it exceeds a specified threshold (for example, the distance from city limits is larger than 1,000 m), and with 0 (false) in the opposite case. Fig. 3 shows criteria maps: Distance from water bodies (E1) and Distance from rail and motorways (S3), which were re-categorized using Boolean logic according to their threshold values given in Table 1.

Combining layers of criteria using the Boolean multiplication (AND) makes it possible to derive a summary suitability map, which determines the plots that simultaneously meet all threshold values. A plot may be suitable or unsuitable, clearly belong or one or another class, which eliminates the possibility of its partial membership in the class.
4.4. Analysis of suitability using fuzzy logic (scenario 2)

The application of clear class limits, which are defined by using threshold values of criteria, can be justified by the need to fulfill strict requirements (constraints) that are given in Table 1. However, when resolving other issues related to the location of SW landfill, such as socio-economic, they may prove to be unrealistic.

For example, building standards imply the distance of 200 m from the rail and automobile roads, however, we cannot assume that any distance larger than 200 m is equally suitable. Cost effective is the proximity of a landfill to the existing transport network. Construction of new roads, especially over long distances, requires huge investments in advance. To cope with the uncertainties and inaccuracies associated with the suitability estimation, where it is difficult to define clear boundaries, it is appropriate to use an apparatus of fuzzy logic.

A fuzzy set of universal set \( X \) is determined as the set of ordered pairs [15]:

\[
\tilde{A} = \{(\mu_x(x), x)\},
\]

where \( \mu_x(x) \) is the membership function that accepts values in a certain ordered set \( M = [0; 1] \).

If \( M = [0; 1] \), the fuzzy subset can then be considered as a distinct set. Membership function indicates the degree of membership between an element \( x \) and unfuzzy subset \( \tilde{A} \). The larger \( \mu_x(x) \), the greater the extent at which an element of the universal set corresponds to properties of the fuzzy subset.

To standardize the criteria, we have chosen piecewise-linear membership functions whose form is shown in Fig. 4. Fuzzy functions and sets with control points for criteria are given in Table 2.

Fig. 4. Piecewise-linear membership functions that were used for the standardization of criteria: \( a \) – trapezoidal; \( b \) – monotonically descending; \( c \) – monotonically ascending

<table>
<thead>
<tr>
<th>No.</th>
<th>Criteria</th>
<th>Control points</th>
<th>Membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Terrain digital model</td>
<td>( a = b = c = 0.5 ) and ( d = 5 ).</td>
<td>Monotonically ascending</td>
</tr>
<tr>
<td>2</td>
<td>Inclination (%)</td>
<td>( a = 1 ), ( b = 10 ), ( c = 20 ), ( d = \max )</td>
<td>Monotonically descending</td>
</tr>
<tr>
<td>3</td>
<td>Exposure</td>
<td>North (N) and North-West (NW)</td>
<td>Trapezoidal</td>
</tr>
<tr>
<td>4</td>
<td>Distance from city limits (km)</td>
<td>10 – 20 – max</td>
<td>Trapezoidal</td>
</tr>
<tr>
<td>5</td>
<td>Distance from residential and public buildings (km)</td>
<td>0.5 – 1.5</td>
<td>Monotonically ascending</td>
</tr>
<tr>
<td>6</td>
<td>Distance from motor and rail roads (km)</td>
<td>0.2 – 0.5 – 1 – 5</td>
<td>Trapezoidal</td>
</tr>
</tbody>
</table>

Out of a general list of criteria for a hierarchical decision-making structure (Fig. 2), we selected the criteria that require fuzzy standardization, namely:

- Digital model of the terrain. According to studies and expert estimates, the range of heights from 5 to 530 m is considered the best range for SW landfill. A height below 5 m is dangerous in terms of flooding, and thus is not recommended for construction. The height of the examined terrain does not exceed 220 m, which is why we have chosen a piecewise-linear monotonically ascending membership function with parameters \( a = b = c = 0 \) and \( d = 5 \).

- Inclination. A steep inclination requires large expenditures related to construction and operation of landfills at steep slopes, while very flat slopes are associated with risk due to poor drainage. We have chosen for the inclination a monotonically descending membership function of the fuzzy set with parameters \( a = b = c = 15 \), \( d = 30 \).

- Exposure. Air pollution, especially with methane, requires consideration of the rose of winds. North (N) and North-West (NW) are the dominant wind directions over the examined territory, which is why we have chosen a trapezoidal membership function; we assigned lower values to these unwanted exposures.

- Distance from city limits. Given that cities are a source of wastes, SW landfills should be located at a logically substantiated distance from them. A distance from 10 to 20 km is considered by experts as optimal, which is why we have chosen a trapezoidal membership function with parameters \( a = 1 \), \( b = 10 \), \( c = 20 \), \( d = \max \).

- Distance from residential and public buildings. To standardize the criteria, we have chosen a piecewise-linear monotonically ascending membership function with parameters \( a = b = c = 0.5 \) and \( d = 1.5 \).

- Distance from motor and railroad routes. Construction of roads to polygons, especially over long distances, requires huge preliminary investments. Therefore, a plot for SW landfill should be placed close to the motorways and main roads. To standardize the criteria, we have chosen trapezoidal membership function with parameters \( a = 0.2 \), \( b = 0.5 \), \( c = 1 \), \( d = 5 \).

- Land use. The criteria for land use were assigned with the following values: meadows – 0, shrubs – 0.9, badland – 1, others – 0.

Fig. 5 shows criteria maps: Distance from city limits (S1) and Distance from rail and motorways (S3), which were re-categorized using fuzzy logic according to their membership functions, given in Table 2.

Fuzzification of criteria, that is conversion of their values into a fuzzy set, based on expert evaluation of a fuzzy membership function of each alternative to the appropriate membership class makes it possible to subsequently combine the criteria using fuzzy output rules. To this end, one may apply the operations of fuzzy logic, for example, intersection or merging.

A standard fuzzy intersection (AND) of sets \( A_1, A_2, ..., A_n \) is determined from membership function:

\[
\mu_{A_1 \land A_2 \land ... \land A_n}(x) = \max \left\{ \min \left( \mu_{A_1}(x), \mu_{A_2}(x), ..., \mu_{A_n}(x) \right) \right\},
\]

for all \( x \in X \).

A standard fuzzy combination (OR) of sets \( A_1, A_2, ..., A_n \) is determined from dependence:
The simulation of scenario 2 of present study implies the aggregation of the resulting map of the suitability using the operations of fuzzy intersection and fuzzy averaging (the geometric mean).

4.5. Analysis of suitability using a weighted linear combination (scenario 3)

Weighted linear combination (9) takes into consideration the relative significance of each factor by using weight coefficients \( w_i \). To calculate the normalized weights of criteria, a method of hierarchy analysis (AHP) is often used [17]. The procedure is based on a pairwise comparison of elements at the assigned level relative to the elements at a higher level of a hierarchical structure for decision making. The comparison employs a fundamental scale of the Saaty absolute numbers, which accepts integer values from 1 to 9 (Table 3).

Let \( C = \{C_j | j = 1, 2, ..., n\} \) be a set of criteria. Based on the results of pairwise comparison of \( n \) criteria, one can build a matrix \( A(n \times n) \), in which each element \( a_{ij}, i, j = 1, 2, ..., n \) is the estimate of pairwise comparison of the \( i \)-th criterion to the \( j \)-th criterion.

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}, \quad a_{ii} = 1, \quad a_{ji} = \frac{1}{a_{ij}}, \quad a_{ij} \neq 0. \quad (7)
\]

Table 3

<table>
<thead>
<tr>
<th>Saaty scale</th>
<th>Definition</th>
<th>Unfuzzy triangular scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no benefit</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>3</td>
<td>weak benefit</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td>5</td>
<td>essential benefit</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>7</td>
<td>clear benefit</td>
<td>(6, 7, 8)</td>
</tr>
<tr>
<td>9</td>
<td>absolute benefit</td>
<td>(9, 9, 9)</td>
</tr>
<tr>
<td>2</td>
<td>intermediate values between adjacent values of the scale</td>
<td>(1, 2, 3)</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>(3, 4, 5)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(5, 6, 7)</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>(7, 8, 9)</td>
</tr>
</tbody>
</table>

For matrix (7), we calculate eigen numbers and eigenvectors and construct a vector of local priorities.

In order to control consistency of expert assessments, we introduce two related characteristics — Consistency Index (C.I.) and Consistency Ratio (C.R.):

\[
C.I. = \frac{\lambda_{\text{max}} - n}{n-1}. \quad (8)
\]

where \( n \) is the number of criteria, and \( \lambda_{\text{max}} \) is the largest eigenvalue of the matrix.

\[
C.R. = \frac{C.I.}{R.I.}. \quad (9)
\]

For matrix (7), we calculate eigen numbers and eigenvectors and construct a vector of local priorities.
Control processes

where R.I. is the index of a random inconsistency, which depends on the rank of the matrix (Table 4). A reasonable level of consistency in paired comparisons C.R.<0.10, while C.R.≥0.10 indicates contradictory opinions of an expert.

| Table 4 Value of random index (R)<sub>I</sub> depending on the rank of the matrix |
|---|---|---|---|---|---|---|---|---|---|---|
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| R.I. | 0.00 | 0.00 | 0.52 | 0.89 | 1.11 | 1.25 | 1.35 | 1.40 | 1.45 | 1.49 |

Because traditional AHP does not account for uncertainty and subjectivity in the judgment of experts, it makes sense then to use the modified fuzzy analysis method of hierarchies (Fuzzy AHP) [18], in which paired comparison of criteria is performed through linguistic variables represented by triangular numbers (Table 3).

At the first stage, an expert transforms a clear matrix of paired comparisons A (after checking the consistency of estimates, C.R.<0.10) into fuzzy matrix A using a scale with triangular fuzzy numbers:

\[
(\tilde{A}^k)^{ij} = \begin{bmatrix}
\tilde{a}_{i1}^k & \tilde{a}_{i2}^k & \cdots & \tilde{a}_{ir}^k \\
\tilde{a}_{s1}^k & \tilde{a}_{s2}^k & \cdots & \tilde{a}_{sr}^k \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{n1}^k & \tilde{a}_{n2}^k & \cdots & \tilde{a}_{nr}^k
\end{bmatrix},
\]

(10)

where \(\tilde{a}_{ij}^k\) is the result of comparison by the \(k\)-th expert of the \(i\)-th criterion to the \(j\)-th criterion expressed by a fuzzy triangular scale.

If the evaluation involves a group of experts, the benefits of each expert (\(\tilde{a}_{ij}^k\)) are averaged:

\[
\tilde{a}_{ij} = \frac{\sum_{k=1}^{K} \tilde{a}_{ij}^k}{K}
\]

(11)

and the matrix of pairwise comparisons, accordingly, takes the following form:

\[
\tilde{A} = \begin{bmatrix}
\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1r} \\
\tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nr}
\end{bmatrix},
\]

(12)

Fuzzy weights of each criterion can be found using the vector summing according to equation:

\[
\tilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 \otimes \tilde{r}_2 \otimes \cdots \otimes \tilde{r}_n)^{-1} = (\tilde{w}_1, m\tilde{w}_2, u\tilde{w}_n),
\]

(13)

where

\[
\tilde{r}_i = \left( \prod_{j=1}^{n} \tilde{a}_{ij}^k \right)^{\frac{1}{n}}, \quad i = 1, 2, \ldots, n
\]

is the geometric mean of the values of fuzzy comparison of each criterion.

Defuzzification of a fuzzy weight is performed using equation:

\[
M_i = \frac{lw_i + mw_i + uw_i}{3},
\]

(14)

where \(M_i\) is a definite number that needs to be normalized:

\[
w_i = \frac{M_i}{\sum_{i=1}^{S} M_i}
\]

(15)

By applying (10)–(15), we calculated weight coefficients of criteria and sub-criteria. The evaluation involved two experts who have experience in the management of waste. For simplicity, this paper presents the already averaged estimates of experts, according to (11).

Table 5 gives a matrix of pairwise comparisons for three criteria (physical, environmental, socio-economic) at the first level of the hierarchical structure (Fig. 2).

| Table 5 Matrix of paired comparisons for criteria |
|---|---|---|---|
| Criteria | F | E | S |
| F: Physical | (1, 1, 1) | (1/3, 1/2, 1) | (1/3, 1/2, 1) |
| E: Environmental | (1, 2, 3) | (1, 1, 1) | (1, 2, 3) |
| S: Socio-economic | (1, 2, 3) | (1/3, 1/2, 1) | (1, 1, 1) |

Relative fuzzy weights of each criterion (13), as well as a definite weight of each criterion \(M_i\) (14) and normalized weights \(w_i\) (15) are given in Table 6.

| Table 6 Relative fuzzy weights and normalized relative weights of each criterion |
|---|---|---|---|---|---|
| Criteria | \(\tilde{w}_i\) | \(M_i\) | \(w_i\) |
| F: Physical | 0.106 | 0.195 | 0.40 | 0.254 | 0.214 |
| E: Environmental | 0.221 | 0.493 | 0.957 | 0.575 | 0.469 |
| S: Socio-economic | 0.153 | 0.311 | 0.663 | 0.376 | 0.317 |

Below are the matrices of paired comparisons: \(\tilde{A}_P\) – for a group of physical criteria (Table 7), \(\tilde{A}_E\) – for a group of environmental criteria (Table 8), \(\tilde{A}_E\) – for a group of socio-economic criteria (Table 9).

| Table 7 Matrix of paired comparisons for the sub-criteria of group of physical criteria |
|---|---|---|---|
| Criteria | F1 | F2 | F3 |
| F1 | (1, 1, 1) | (1/4, 1/3, 1/2) | (6, 7, 8) |
| F2 | (2, 3, 4) | (1, 1, 1) | (9, 9, 9) |
| F3 | (1/8, 1/7, 1/6) | (1/9, 1/9, 1/9) | (1, 1, 1) |

Relative fuzzy weights, as well as definite weight \(M_i\) and normalized weights \(w_i\), calculated for each sub-criterion, are given in Table 10.

We shall subsequently employ the normalized weights according to (6) for the creation of appropriate suitability maps using a weighted overlay (scenario 3).
R. Poprawa, M. Matczak, R. Biedka

5. Results of making a multi-criteria decision on the location of solid waste landfills

The result of the execution of three scenarios is the constructed suitability maps shown in Fig. 6.

Based on scenario 1, which implied the use of Boolean logic, we built a suitability map shown in Fig. 6a, divided into two definite classes where a value of 1 denotes suitable plots, 0 – unsuitable plots. The total area of the found plots suitable for the construction of SW landfill was 12.19 km² (or 0.25% of the total area of the examined territory).

Based on scenario 2, we built suitability maps using fuzzy intersection operations and the geometric mean, shown in Fig. 6. b, c, respectively. The application of a fuzzy model allowed us to build more accurate maps due to the possibility of ranking the raster cells for the degree of suitability. The total area of plots found using a fuzzy intersection, with the index of suitability exceeding 0.9, was 0.81 km². Of these, only three sites have a total area of over 30 hectares. As anticipated, the use of the geometric mean operation produced a more optimistic result. Thus, the total area of plots whose index of suitability has a value exceeding 0.9, totaled 3.97 km². Of these, five sites have a total area of over 30 hectares.

To create a suitability map using a weighted overlay (scenario 3), we originally built combined maps of physical, environmental, and socio-economic criteria according to Eq. (6); the weight of sub-criteria is given in Table 10. Next, these three maps were aggregated into one using weight coefficients from Table 6. To take into consideration requirements to the construction of solid waste landfills, specified in Table 1, we removed from the resulting map, using the overlay operation AND, all the alternatives included in the restrictive zones. We employed, as a map of restrictive zones, a suitability map, which was built based on scenario 1 of our study. Thus, a WLC model was modified by introducing a map of constraints to equation (6). Thus, we obtained the resulting suitability map (Fig. 6d), for which the total area of sites whose suitability index has a value exceeding 0.9 totaled 5.02 km². Of these, five plots have a total area of over 30 hectares.

6. Discussion of results of making a multi-criteria decision on the location of solid waste landfills

Results of present study showed that the model based on the apparatus of Boolean algebra is clear and simple to implement. The model is well suited for solving the problems on decision making in which criteria are represented in the form of definite requirements and constraints with the assigned quantitative threshold values; that makes it possible to obtain a suitability map divided into two distinct classes. However, a downside of this approach is the absence of a possibility to rank alternatives (raster cells), which prevents comparing two plots specified as suitable in order to assess which one is more suitable. A definite approach is not compensatory, that is, it does not make it possible to compensate for a poor suitability of one factor by a good suitability of another factor. These flaws are noticeable when comparing the same site at different suitability maps (Fig. 7).

Note that the use of a fuzzy intersection operation (4) leads to the estimation based only on the lowest ranking, that is, it is a pessimistic approach to decision making. Optimistic approach is the use of operations of fuzzy combination.
Control processes

(5), which takes into consideration only the best estimates of all criteria.

Both the first and second cases may yield errors related to the underestimation or overestimation, respectively. A compromise between these two extremes is the application of a weighted overlay method, which makes it possible to compensate for the low rating of one factor by the highest rating of another factor. That is, a weighted linear combination, applied to fuzzy factors, can be interpreted as the operation of fuzzy averaging.

![Fig. 6. Suitability maps: a – Boolean overlay; b – fuzzy overlay (AND); c – geometric mean (ANDOR); f – weighted linear combination (WLC).](image)

It should be noted that the use of a fuzzy weighted overlay requires the calculation of weight coefficients, which is challenging. Thus, in order to account for the uncertainty of expert opinions, our research employed the modified fuzzy method of hierarchy analysis. In addition, the procedure of a weighted linear combination should be carried out on a set of possible alternatives, which are defined by a set of constraints, that is, a mandatory condition to the fulfillment of the procedure of weighted overlay is the unification of criteria maps with constraints maps. The research has shown that in the case where it is difficult or impossible to determine the weight of criteria, it is possible to use as an alternative variant to the weighted overlay a method of fuzzy geometric averaging.

![Fig. 7. A suitability map site: a – Boolean overlay; b – weighted linear combination (WLC); c – geometric mean (ANDOR); d – fuzzy overlay (AND).](image)

7. Conclusions

1. We have proposed a model of a fuzzy expert system, which makes it possible, based on spatial data, represented in the form of raster criteria maps in the GIS geodatabase and integrated multi-criteria methods for decision making, to search for the sites to construct SW landfill (using the south of Odessa oblast as an example). It is shown that the use of an apparatus of fuzzy logic helps to take into consideration the uncertainty of initial information and subjectivity in expert estimates, as well as obtain a more informative suitability map, by determining a rank of the suitability of alternatives.

2. It was proven that using, in order to aggregate a suitability map, the method of weighted linear combination and a fuzzy method for the analysis of hierarchies is the most effective to account for judgments and estimates of experts. It is shown that in the case where it is difficult or impossible to determine the weight of criteria, an alternative that can be applied is the method of fuzzy geometric averaging whose execution result is close to the result of a weighted linear combination.

3. Results of the study could be used by municipal self-governing bodies to support decision-making on finding suitable sites for the location of SW landfills at the territory of Odessa oblast.

References

