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RELATIVISTIC THEORY OF SPECTRA OF PIONIC ATOMIC SYSTEMS ^{208}Pb WITH ACCOUNT OF STRONG PION-NUCLEAR INTERACTION EFFECTS

It is presented a consistent relativistic theory of spectra of the pionic atoms on the basis of the Klein-Gordon-Fock with a generalized radiation and strong pion-nuclear potentials. It is applied to calculation of the energy and spectral parameters for pionic atoms of the 208Pb with accounting for the radiation (vacuum polarization), nuclear (finite size of a nucleus) and the strong pion-nuclear interaction corrections. The measured values of the Berkley, CERN and Virginia laboratories and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uehling-Serber radiation correction and optical atomic theory are listed too. There are listed new data on shift and broadening of the 4f level in 208Pb due to the strong pion-nuclear interaction.

1. Introduction

In papers [1-3] we have presented a new relativistic method of the Klein-Gordon-Fock equation with an generalized pion-nuclear potential to determine transition energies in spectroscopy of light, middle and heavy pionic atoms with accounting for the strong interaction effects. In this paper, which goes on our studying on spectroscopy of pionic atoms, we firstly applied method [1-3] to calculating calculation of the energy and spectral parameters for pionic atom of the ^{208}Pb with accounting for the radiation (vacuum polarization), nuclear (finite size of a nucleus) and the strong pion-nuclear interaction corrections. There are listed new data on shift and broadening of the 4f level in ^{208}Pb due to the strong pion-nuclear interaction.

Following [1-3], let us remind that spectroscopy of hadron atoms has been used as a tool for the study of particles and fundamental properties for a long time. Exotic atoms are also interesting objects as they enable to probe aspects of atomic and nuclear structure that are quantitatively different from what can be studied in electronic or "normal" atoms. At present time one of the most sensitive tests for the chiral symmetry breaking scenario in the modern hadron's physics is provided by studying the exotic hadron-atomic systems. Nowadays the transition energies in pionic (kaonic, muonic etc.) atoms are measured with an unprecedented precision and from studying spectra of the hadronic atoms it is possible to investigate the strong interaction at low energies measuring the energy and natural width of the ground level with a precision of few meV [1-20]. The strong interaction is the reason for a shift in the energies of the low-lying levels from the purely electromagnetic values and the finite lifetime of the state corresponds to an increase in the observed level width. For a long time the similar experimental investigations have been carried out in the laboratories of Berkley, Virginia (USA), CERN (Switzerland). The most known theoretical models to treating the hadronic (pionic, kaonic, muonic, antiprotonic etc.) atomic systems are presented in refs. [21-48]. The most difficult aspects of the theoretical modeling are reduced to the correct description of pion-nuclear strong interaction [1-3] as the electromagnetic part of the problem is reasonably accounted for.

2. Relativistic approach to pionic atoms spectra

As the basis's of a new method has been published, here we present only the key topics of an approach [1-3]. All available theoretical models to treating the hadronic (kaonic, pionic) atoms are naturally based on the using the Klein-Gordon-Fock equation [2,5], which can be written as follows :

$$m^2c^2\Psi(x) = \{\frac{1}{c^2}[i\hbar\partial_t + eV_0(r)]^2 + \hbar^2\nabla^2\}\Psi(x) \quad (1)$$

where c is a speed of the light, *h* is the Planck constant, and Ψ_0(x) is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential [V_0(r), 0] approximation with the stationary solution:

$$\Psi(x) = \exp(-iEt/\hbar)\,\varphi(x), \quad (2)$$

where $\varphi(x)$ is the solution of the stationary equation:

$$\{\frac{1}{c^2}[E + eV_0(r)]^2 + \hbar^2\nabla^2 - m^2c^2\}\varphi(x) = 0 \quad (3)$$

Here E is the total energy of the system (sum of the mass energy mc^2 and binding energy ε_0). In principle, the central potential V_0 naturally includes the central Coulomb potential, the vacuum-polarization potential, the strong interaction potential. The most direct approach to treating the strong interaction is provided by the well known optical potential model (c.g. [2]). The nuclear potential for the spherically symmetric density $\rho(r|R)$ is [13-15]:

$$V_{nucl}(r|R) = -((1/r)\int_0^r dr' r'^2 \rho(r'|R) + \int_r^\infty dr' r' \rho(r'|R) \quad (4)$$

The most popular Fermi-model pproximation the charge distribution in the nucleus $\rho(r)$ is:

$$\rho(r) = \rho_0 / \{1 + \exp[(r-c)/a)]\}, \quad (5)$$

where the parameter *a=0*.523 fm, the parameter *c* is chosen by such a way that it is true the following condition for average-squared radius:

$$<r^2>^{1/2} = (0.836 \cdot A^{1/3} + 0.5700)\text{fm}.$$

The effective algorithm for its definition is used in refs. [12] and reduced to solution of the following system of the differential equations with the corresponding boundary conditions. Another, probably, more consistent approach is in using the relativistic mean-field (RMF) model, which been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [21]. The detailed presentation of our method for construction of the many-body relativistic perturbation theory with accounting for relativistic, exchange-correlation, nuclqar and radiative (QED) effects is presented in Refs. [41-77]. Here we note that to account QED effect, namely, the vacuum polarization one we have used the generalized Ueling-Serber potential with modification to take into account the high-order corrections.

The most difficult aspect is an adequate account for the strong interaction. On order to describe the strong π^-N interaction we have used the optical potential model in which the generalized Ericson-Ericson potential is as follows:

$$V_{\pi^-N} = V_{opt}(r) = -\frac{4\pi}{2\bar{m}}\left\{q(r)\nabla\frac{\alpha(r)}{1+4/3\pi\xi\alpha(r)}\nabla\right\} \quad (9)$$

$$q(r) = \left(1+\frac{m_\pi}{m_N}\right)\{b_0\rho(r) + b_1[\rho_n(r) - \rho_p(r)]\} + \quad (10)$$

$$+\left(1+\frac{m_\pi}{2m_N}\right)\{B_0\rho^2(r) + B_1\rho(r)\delta\rho(r)\}, \quad (10)$$

$$\alpha(r) = \left(1+\frac{m_\pi}{m_N}\right)^{-1}\{c_0\rho(r) + c_1[\rho_n(r) - \rho_p(r)]\} + \quad (11)$$

Here $\rho_{p,n}(r)$ – distribution of a density of the protons and neutrons, respectively, ξ – parameter ($\xi = 0$ corresponds to case of "no correlation", $\xi = 1$, if anticorrelations between nucleons); respectively isoscalar and isovector parameters b_0, c_0, B_0, b_1, c_1, C_0, B_1, C_1 –are corresponding to the s-wave and p-wave (repulsive and attracting potential member) scattering length in the combined spin-isospin space with taking into account the absorption of pions (with different channels at p-p pair $B_{0(pp)}$ and *p-n* pair $B_{0(pn)}$), and isospin and spin dependence of an amplitude π^-N scattering

$$(b_0\rho(r) \rightarrow b_0\rho(r) + b_1\{\rho_p(r) - \rho_n(r)\},$$

the Lorentz-Lorentz effect in the p-wave interaction. For the pionic atom with remained electron shells the total wave-function is a product of the product Slater determinant of the electrons subsystem (Dirac equation) and the pionic wave function. In whole the energy of the hadronic atom is represented as the sum:

$$E \approx E_{KG} + E_{FS} + E_{VP} + E_N; \tag{12}$$

Here E_{KG} -is the energy of a pion in a nucleus (Z, A) with the point-like charge (dominative contribution in (12)), E_{FS} is the contribution due to the nucleus finite size effect, E_{VP} is the radiation correction due to the vacuum-polarization effect, E_N is the energy shift due to the strong interaction V_N.

The strong pion-nucleus interaction contribution can be found from the solution of the Klein-Gordon-Fock equation with the corresponding pion-nucleon potential. The detailed description and analysis of different aspects of the computational procedure can be found in Refs. [1-4,48-75].

the shift and broadening (keV) of the 4f level due to the strong pion-nuclear interaction [2-8].

Table 1

Transition energies (keV) in the spectra of heavy pionic atom ^{208}Pb (see text)

Trans.	CERN E_{EXP}	E_N [14, 18]	E_N, Our data
4f-3d	1282 ± 2.2	1261.23	1281.78
5g-4f	575.46 ± 0.04	-	575.78

Here we use the short designation of the $V_{\pi N}$: potential parameter sets: Tauscher, -Tau1; Tauscher, -Tau2; Batty etal-Bat .; Seki etal- Sek; Laat-Konijin etal - Laat, Our set – our.

Our parameterization $V_{\pi N}$ upheld options that are the most reliably determined (B_0,c_0,c_1,C_0). The potential parameters whose values differ greatly in different sets, in particular, b_1 (b_1=–0.094) plus not included still to the $V_{\pi N}$ parameter set (ImB_1, ImC_1) were optimized by calculating the strong dependencies shifts for the pionic π^- ^{20}Ne, ^{24}Mg, ^{93}Nb, ^{133}Cs, ^{175}Lu, ^{181}Ta, ^{197}Au, ^{208}Pb atoms upon the values of b_1, ImB_1, ImC_1; further the selected these values meet the standard deviation of the least reliable experimental values.

Table 2

Shift and broadening (keV) of the 4f level due to the strong pion-nuclear interaction

ε_{4f}, Γ_{4f}	Exp	H-like Func.	Tau1 $\xi = 0$	Tau2 $\xi = 1$	Bat $\xi = 1$	Sek $\xi = 1$	Laat $\xi = 1$	Our $\xi = 1$
^{208}Pb: ε	1.68±0.04	-	1.76	1.62	1.58	1.39	1.68	1.65
^{208}Pb: Γ	0.98±0.05	-	1.18	1.04	1.03	0.86	0.98	0.97

3. Results and conclusions

In table 1 our data on the *4f-3d, 5g-4f* transition energies for pionic atom of ^{208}Pb are presented. The measured values of the CERN and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uehling-Serber radiation correction and optical atomic theory are listed too [2-10]. In table 2 we present data on

The analysis of the presented data indicate on the importance of the correct accounting for the radiation (vacuum polarization) and the strong pion-nuclear interaction corrections. Obviously, it is clear that that the contributions provided by the finite size effect should be accounted in a precise theory. Besides, taking into account the increasing accuracy of the X-ray pionic atom spectroscopy experiments, it can be noted that knowledge of the exact electromagnetic theory data will make more clear the true values for parameters of the pion-nuclear potentials and correct the disadvantage of widely used parameterization of the potentials (9)-(11).

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RELATIVISTIC THEORY OF SPECTRA OF PIONIC ATOMIC SYSTEM ^{208}Pb WITH ACCOUNT OF STRONG PION-NUCLEAR INTERACTION EFFECTS

Abstract. It is presented a consistent relativistic theory of spectra of the pionic atoms on the basis of the Klein-Gordon-Fock with a generalized radiation and strong pion-nuclear potentials. It is applied to calculation of the energy and spectral parameters for pionic atoms of the ^{208}Pb with accounting for the radiation (vacuum polarization), nuclear (finite size of a nucleus) and the strong pion-nuclear interaction corrections. The measured values of the Berkley, CERN and Virginia laboratories and alternative data based on other versions of the Klein-Gordon-Fock theories with taking into account for a finite size of the nucleus in the model uniformly charged sphere and the standard Uehling-Serber radiation correction and optical atomic theory are listed too. There are listed new data on shift and broadening of the 4f level in ^{208}Pb due to the strong pion-nuclear intetraction.
Key words: strong interaction, pionic atom ^{208}Pb, relativistic theory

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РЕЛЯТИВИСТСКАЯ ТЕОРИЯ СПЕКТРОВ ПИОННЫХ АТОМНЫХ СИСТЕМ ^{208}Pb С УЧЕТОМ ЭФФЕКТОВ СИЛЬНОГО ПИОН-ЯДЕРНОГО ВЗАИМОДЕЙСТВИЯ

Резюме. Представлена последовательная релятивистская теория спектров пионных атомов на основе уравнения Клейна-Гордона-Фока с обобщенными радиационным и сильным пион-ядерным потенциалом. Выполнен расчет энергетических и спектральных параметров для пионного атома ^{208}Pb, с учетом радиационных (поляризация вакуума), ядерных (конечный размер ядра) эффектов и поправки на сильное пион-нуклонное взаимодействие. Также для сравнения

представлены данные измерений в лабораториях Berkley, ЦЕРН и Вирджиния и теоретические результаты, полученные на основе альтернативных теорий Клейна-Гордона-Фока с учетом конечного размера ядра в модели равномерно заряженной сферы и стандартной Юлинг-Сербер поправки. Представлены новые данные по сдвигу и уширению 4f уровня в атоме ^{208}Pb благодаря сильному пион-ядерному взаимодействию.
Ключевые слова: сильное взаимодействие, пионный атом ^{208}Pb, релятивистская теория

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РЕЛЯТИВІСТСЬКА ТЕОРІЯ СПЕКТРІВ ПІОННИХ АТОМНИХ СИСТЕМ ^{208}Pb З УРАХУВАННЯМ ЕФЕКТІВ СИЛЬНОЇ ПІОН-ЯДЕРНОЇ ВЗАЄМОДІЇ

Резюме. Представлена послідовна релятивістська теорія спектрів півоній атомів на основі рівняння Клейна-Гордона-Фока з узагальненими радіаційним і сильним півонія-ядерним потенціалом. Виконано розрахунок енергетичних і спектральних параметрів для піоного атома ^{208}Pb з урахуванням радіаційних (поляризація вакууму), ядерних (кінцевий розмір ядра) ефектів та поправки на сильну піон-нуклонну взаємодію. Також для порівняння представлені дані вимірювань в лабораторіях Berkley, ЦЕРН і Вірджинія і теоретичні результати, отримані на основі альтернативних теорій Клейна-Гордона-Фока з урахуванням кінцевого розміру ядра в моделі рівномірно зарядженої сфери і стандартної Юлінг-Сербер поправки. Представлені нові данні щодо зсуву та уширення 4f рівня в атомі ^{208}Pb завдяки сильній піон-ядерній взаємодії
Ключові слова: сильна взаємодія, піонний атом ^{208}Pb, релятивістська теорія