UDC 539.192

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BREMSSTRAHLUNG AND X-RAY SPECTRA FOR KAONIC AND PIONIC HYDROGEN AND NITROGEN

The level energies, energy shifts and transition rates are estimated for pionic and kaonic atoms of hydrogen and nitrogen on the basis of the relativistic perturbation theory with an account of nuclear and radiative effects. New data about spectra of the exotic atomic systems can be considered as a new tool for sensing the nuclear structure and creation of new X-ray sources too.

1. INTRODUCTION

At present time, the light hadronic (pionic, kaonic etc.) atomic systems are intensively studied and can be considered as a candidate to create the new low-energy X-ray standards [1-12]. In the last few years transition energies in pionic [1] and kaonic atoms [2] have been measured with an unprecedented precision. Besides, an important aim is to evaluate the pion mass using high accuracy X-ray spectroscopy [1-10]. Similar endeavour are in progress with kaonic atoms. It is easily to understand that the spectroscopy of pionic and kaonic hydrogen gives unprecedented possibilities to study the strong (nuclear) interaction at low energies [5-8] by measuring the energy and natural width of the ground level with a precision of few meV. Naturally, studying the hadronic atomic systems is of a great interest for further development of atomic and nuclear theories as well as new tools for sensing the nuclear structure and fundamental interactions, including the Standard model [1-15]. The collaborators of the E570 experiment [7,8] measured X-ray energy of a kaonic hydrogen atom, which is an atom consisting of a kaon (a negatively charged heavy particle) and a hydrogen nucleus (proton). The kaonic hydrogen X-rays were detected by large-area Silicon Drift Detectors, which readout system was developed by SMI (see [1,8]). It is known that the shifts and widths due to the strong interaction can be systematically understood using phenomenological optical potential models. Nevertheless, one could mention a large discrepancy between the theories and experiments on the kaonic atoms states. (for example, well known puzzle with helium 2p state). A large repulsive shift (about -40 eV) has been measured by three experimental groups in the 1970's and 80's, while a very small shift (< 1 eV) was obtained by the optical models calculated from the kaonic atom X-ray data with Z>2 (look [1]). This significant disagreement (a difference of over 5 standard deviations) between the experimental results and the theoretical calculations is known as the "kaonic helium puzzle". A possible large shift has been predicted using the model assuming the existence of the deeply bound kaonic nuclear states. However, even using this model, the large shift of 40 eV measured in the experiments cannot be explained. A re-measurement of the shift of the kaonic helium X-rays is one of the top priorities in the experimental research activities. In the last papers (look, for example, [5,6] and [10] too) this problem is physically reasonably solved. In the theory of the kaonic and pionic atoms there is an important task, connected with a direct calculation of the radiative transition energies within consistent relativistic quantum mechanical and QED methods (c.f.[13-15]). The multi-configuration Dirac-Fock (MCDF) approximation is the most reliable approach for multi-electron systems with a large nuclear charge; in this approach one- and two-particle relativistic effects are taken into account practically precisely. The next important step is an adequate inclusion of the radiative corrections. This topic has been a subject of intensive theoretical and experimental interest (see [13]). Nevertheless, the problem remains quite far from its final solution. It is of a great interest to study and treat these effects in the pionic and kaonic systems (for example, hydrogen, nitrogen, oxygen etc.). In this paper the hyperfine structure (HFS) level energies, energy shifts, transition rates are estimated for the pionic and kaonic atoms of hydrogen and nitrogen. New data about spectra of the hadronic systems can be considered as a new tool for sensing nuclear structure and creation of new X-ray sources. Our method is based on the relativistic perturbation theory (PT) [10,15,16,] with an accurate account of the nuclear and radiative effects. The Lamb shift polarization part is described in the Uehling-Serber approximation; the Lamb shift self-energy part is considered effectively within the advanced scheme.

2. METHOD OF RELATIVISTIC PERTURBATION THEORY

Let us describe the key moments of our scheme to relativistic calculation of the spectra for exotic atomic systems with an account of relativistic, correlation, nuclear, radiative effects (more details can be found for example, in ref. [15]; see also [10]). In general, the one-particle wave functions are found from solution of the Klein-Gordon equation with potential, which includes the self-consistent V_0 potential (including electric, polarization potentials of nucleus):

$$\{1/c^2[E_0+eV_0(r)]^2+\hbar\nabla^2 -m^2c^2\}\Psi(r)=0$$

where E_0 is the total energy of the system (sum of the mass energy mc^2 and the binding energy ε_0). To describe the nuclear finite size effect the smooth Gaussian function of the charge distribution in a nucleus is used. With regard to normalization we have:

$$\rho(r|R) = \left(4\gamma^{3/2}/\sqrt{\pi}\right)\exp\left(-\gamma r^2\right) \qquad (1)$$

where $\gamma = 4/\pi R^2$, R is the effective nucleus radius. The Coulomb potential for the spherically symmetric density $\rho(r)$ is:

$$V_{nucl}(r|R) = -\left((1/r)\int_0^r dr' r'^2 \rho(r'|R) + \int_r^\infty dr' r' \rho(r'|R)\right) \qquad (2)$$

It is determined by the following system of differential equations:

$$V'nucl(r,R) = \left(1/r^2\right)\int_0^r dr' r'^2 \rho(r',R) \equiv \left(1/r^2\right) y(r,R) \qquad (3)$$

$$y'(r,R) = r^2 \rho(r,R) \qquad (4)$$

$$\rho'(r,R) = -8\gamma^{5/2}\, r/\sqrt{\pi}\exp\left(-\gamma r^2\right) =$$

$$= -2\gamma r \rho(r,R) = -\frac{8r}{\pi r^2}\rho(r,R) \qquad (5)$$

with the corresponding boundary conditions. The pion (kaon) charge distribution is also included in the strict accordance to the scheme [2,7]. Further, one can write the Klein-Gordon type equations for one- or multi-particle system. In general, formally they fall into one-particle equations with potential, which includes the self-consistent potential, electric, polarization potentials of a nucleus. Procedure for an account of the radiative corrections is given in detail in refs. [15]. Regarding the vacuum polarization (Vac.Pol.) effect let us note that this effect is usually taken into account in the first PT order by means of the Uehling potential:

$$U(r) = -\frac{2\alpha}{3\pi r}\int_1^\infty dt \exp\left(-2rt/\alpha Z\right) \times$$

$$\times\left(1 + 1/2t^2\right)\frac{\sqrt{t^2-1}}{t^2} \equiv -\frac{2\alpha}{3\pi r}C(g), \qquad (6)$$

where $g = r/\alpha Z$. In our calculation we usually use more exact approach. The Uehling potential, determined as a quadrature (6), is approximated with high precision by a simple analytical function. The use of new approximation of the Uehling potential [15] permits one to decrease the calculation errors for this term down to ~0.5%. Besides, using such a simple function form for the Uehling potential allows its easy inclusion to the general system of differential equations. It is very important to underline that the scheme used includes automatically high-order vacuum polarization contributions, including, the well known Wichman-Kroll and Kдllen-Sabry ones. A scheme for estimating the self-energy part of the Lamb shift is based on the method [22-24]. In an atomic system the radiative shift and relativistic part of the energy are, in principle, defined by one and the same physical field. It may be supposed that there exists some universal function that connects a self-energy correction and relativistic energy. The self-energy correction for states of a hydrogen-like ion is presented by Mohr [18,19]. In ref. [22-24] this result is modified for the corresponding states of the multiparticle atomic system. Further let us note that so called relativistic recoil contribution is not calculated by us and its value is taken from refs [4,7]. The transition probabilities between the HFS sublevels are defined by the standard energy approach formula. Other details of the method used (and Superatom code) can be found in refs. [15,16].

3. DATA FOR HADRONIC ATOMS AND DISCUSION

We studied the X-ray spectra for the hadronic hydrogen and nitrogen. In figure 1 the experimental kaonic hydrogen X-ray energy spectra is presented [7,8]. In table 1 we present the measured and theoretical X-ray energies of kaonic hydrogen atom for the 2-1 transition (in keV). In figure 1 this transition is clearly identified. The notations are related to the initial (n_i) and final (n_f) quantum numbers. The calculated value of transition energy is compared with available measured (E_m) and other calculated (E_c) values [1,3,9,10].

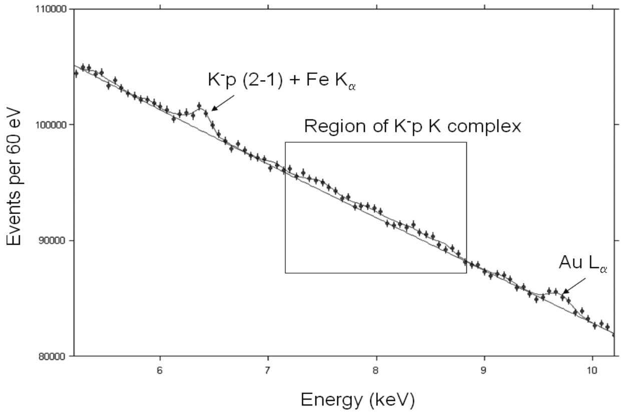

Fig. 1. The experimental kaonic hydrogen X-ray energy spectrum [7,8].

Table 1

Calculated (E_c) and measured (E_m) kaonic H atom X-ray energies (in keV)

Transition	E_c, this work	E_c [10]	E_c [3]	E_c [9]	E_m [1,7]	E_m [1,8]
2-1	6.420	6.65	6.48	6.480 6,482	6,44(8)	6.675(60) 6,96 (9)

In tables 2-4 we present the data on energy (in eV) contribution for selected levels (transitions 8k-7i and 8i-7h), hyperfine transition energies and transition rates in kaonic nitrogen. The radiative effects contributions are indicated separately. For comparison, the estimating data from [3,9] are given too.

In tables 5-7 we present theoretical data on energy (in eV) contribution for the selected levels (transitions 5g-4f and 5f-4d), hyperfine transition energies and transition rates in the pionic nitrogen. The radiative corrections are separately indicated. For comparison, the estimating data from refs. [4,9] are given too. The detailed analysis of theoretical and separated experimental data shows that indeed there is a physically reasonable agreement between the cited data. But, obviously, there may take a place the exception too as it is shown on example of the kaonic uranium in ref. [10].

Table 2

Energy (in eV) contribution for the selected levels in kaonic nitrogen. The first error takes into account neglected next order radiative corrections. The second is due to the accuracy of the kaon mass (±32 ppm)

Contributions	8k-7i [9]	8k-7i This work	8i-7h This work
Coulomb	2968.4565	2968.4492	2968.5344
Vac. Pol.	1.1789	1.1778	1.8758
Relativistic Recoil	0.0025	0.0025	0.0025
HFS Shift	-0.0006	-0.0007	-0.0009
Total	2969.6373	2969.6288	2970.4118
Error	0.0005	0.0004	0.0004
Error due to the kaon mass	0.096	0.096	0.096

Table 3

Hyperfine transition (8k-7i) energies in kaonic nitrogen

Transition	F-F’	Trans. E (eV) [3]	Trans. E (eV) This work	Trans. rate (s^{-1}) [3]	Trans. rate (s^{-1}) This work
8k-7i	8-7	2969.6365	2969.6289	1.54×10^{13}	1.51×10^{13}
	7-6	2969.6383	2969.6298	1.33×10^{13}	1.32×10^{13}
	7-7	2969.6347	2969.6264	1.31×10^{13}	1.29×10^{13}
	6-5	2969.6398	2969.6345	1.15×10^{13}	1.12×10^{13}
	6-6	2969.6367	2969.6284	0.03×10^{13}	0.02×10^{13}
	6-7	2969.6332	2969.6248	0.00×10^{13}	0.00×10^{13}

Table 4

Hyperfine transition (8i-7h) energies in kaonic nitrogen (this work)

Transition	F-F’	Trans. E (eV)	Trans. rate (s^{-1})
8i — 7h	7-6	2970.4107	1.16×10^{13}
	6-5	2970.4135	0.99×10^{13}
	6-6	2970.4086	0.96×10^{13}
	5-4	2970.4193	0.81×10^{13}
	5-5	2970.4114	0.02×10^{13}
	5-6	2970.4073	0.00×10^{13}

Table 5

Energy (in eV) contribution for the selected levels in pionic nitrogen. The first error takes into account neglected next order radiative corrections. The second is due to the accuracy of the pion mass (±2.5 ppm)

Contributions	5g-4f [4]	5g-4f This work	5f-4d This work
Coulomb	4054.1180	4054.1146	4054.7152
Self Energy	-0.0001	-0.0002	-0.0004
Vac. Pol.	1.2602	1.2599	2.9711
Relativistic Recoil	0.0028	0.0028	0.0028
HFS Shift	-0.0008	-0.0009	-0.0030
Total	4055.3801	4055.3762	4057.6857
Error	±0.0011	±0.0007	±0.0009
Error due to the pion mass	±0.010	±0.010	±0.010

We mean the agreement between theoretical estimating data and experimental results. One should keep in mind the following important moment. In a case of good agreement between theoretical and experimental data, the corresponding levels are less sensitive to strong nuclear interaction. In the opposite case one could point to a strong-interaction effect in the exception cited (as example, some transitions in the hadronic U) [10]. In a whole, to understand further information on the low-energy kaon-nuclear (pion-nuclear) interaction, new experiments to define the shift and width of kaonic H/deuterium are now in preparation in J-Parc and in LNF (c.f. [2,6,8]). Finally, let us turn attention at the new possibilities, which are opened with X-ray, γ -lasers (raser, graser) action on hadronic system. Namely, speech is about a set of the possible new nuclear quantum-optical effects in the kaonic and pionic systems [25-27].

Table 6

Hyperfine transition (5g -4f) energies and transition rates in pionic nitrogen

Transition	F-F’	Trans. E (eV) [3]	Trans. E (eV) This work	Trans. rate (s^{-1}) [3]	Trans. rate (s^{-1}) This work
5g -4f	5-4	4055.3779	4055.3744	7.13×10^{13}	7.10×10^{13}
	4-3	4055.3821	4055.3784	5.47×10^{13}	5.42×10^{13}
	4-4	4055.3762	4055.3735	5.27×10^{13}	5.23×10^{13}
	3-2	4055.3852	4055.3828	4.17×10^{13}	4.12×10^{13}
	3-3	4055.3807	4055.3769	0.36×10^{13}	0.33×10^{13}
	3-4	4055.3747	4055.3712	0.01×10^{13}	0.01×10^{13}

Table 7

Hyperfine transition (5f — 4d) energies and transition rates in pionic N (this work)

Transition	F-F’	Trans. E (eV)	Trans. rate (s^{-1})
5f — 4d	4-3	4057.6821	4.52×10^{13}
	3-2	4057.6914	3.11×10^{13}
	3-3	4057.6793	2.93×10^{13}
	2-1	4057.6954	2.09×10^{13}
	2-2	4057.6892	2.21×10^{13}
	2-3	4057.6768	0.01×10^{13}

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UDC 539.192

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Abstract

The level energies, energy shifts and transition rates are estimated for pionic and kaonic atoms of hydrogen and nitrogen on the basis of the relativistic perturbation theory with an account of nuclear and radiative effects. New data about spectra of the hadronic atomic systems can be considered as a new tool for sensing the nuclear structure and creation of new X-ray sources too.

Key words: relativistic perturbation theory, kaonic and pionic atoms, X-ray spectrum.

УДК539.192

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РЕНТГЕНОВСКИЕ СПЕКТРЫ ИЗЛУЧЕНИЯ КАОННОГО И ПИОННОГО АТОМОВ ВОДОРОДА И АЗОТА

Аннотация

На основе релятивистской теории возмущений с учетом ядерных и радиационных эффектов выполнена оценка энергий уровней, энергетических сдвигов, скоростей переходов для пионных и каонных атомов водорода и азота. Новые данные о спектрах искомых адронных атомных систем могут быть использованы для изучения структуры ядра и также создания новых типов источников рентгеновского излучения.

Ключевые слова: релятивистская теория возмущений, каонные и пионные атомы, рентгеновский спектр.

УДК539.192

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РЕНТГЕНІВСЬКІ СПЕКТРИ ВИПРОМІНЮВАННЯ КАОННОГО ТА ПІОННОГО АТОМІВ ВОДНЮ ТА АЗОТУ

Анотація

На підставі релятивістської теорії збурень з урахуванням ядерних і ефектів виконано оцінку енергії рівнів, енергетичних зсувів, швидкостей переходів для піонних і каонних атомів водню і азоту. Нові данні про спектри шуканих адронних атомних систем можуть бути використані для вивчення структури ядра і також побудови нових типів джерел рентгенівського випромінювання.

Ключові слова: релятивістська теорія збурень, каонні та піонні атоми, рентгенівський спектр.