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# Spectroscopy of cooperative muon-gamma-nuclear processes: Energy and spectral parameters

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**Abstract.** We present a consistent relativistic theory of discharge of a metastable nucleus with emission of  $\gamma$  quantum and further  $\mu$  conversion. The numerical calculation is carried out for the scandium nucleus ( $A=49$ ,  $N=21$ ) with using the Dirac-Woods-Saxon model.

## 1. Introduction

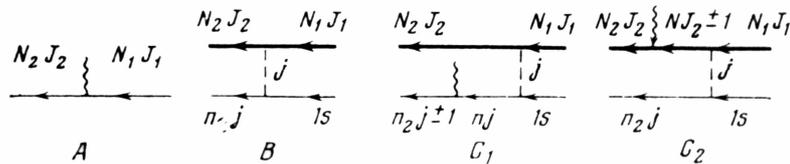
Muonic atoms have always been useful tools for nuclear spectroscopy employing atomic-physics techniques [1-12]. Muonic atoms also play an important role as catalysts for nuclear fusion [1,2]. It should be also mentioned the growing importance of muon spectroscopy in molecular physics. Electrons, muons (other particles such as kaons, pions etc) originally in the ground state of the target atom can be excited reversibly either to the bound or continuum states. With appearance of the intensive neutron pencils and laser sources studying the gamma-muon-nuclear interactions is of a great importance [4-16]. The rapid progress in laser technology even opens prospects for nuclear quantum optics via direct laser-nucleus coupling [7-12]. It is known that a negative muon captured by a metastable nucleus may accelerate a discharge of the latter by many orders of magnitude [3,4,6].

A principal possibility of storage of the significant quantities of the metastable nuclei in the nuclear technology processes and their concentrating by chemical and laser methods leads to problem of governing their decay velocity [3,4]. The  $\mu$ -atom system differs advantageously of the usual atom; the relation  $r_n/r_a$  ( $r_n$  is a radius of a nucleus and  $r_a$  is a radius of an atom) can vary in the wide limits in dependence upon the nuclear charge. Because of the large muon mass and the correspondingly small Bohr radius, the muonic wave function has a large overlap with the nucleus and thus effectively probes its structural features, such as finite size, deformation, polarization etc. For a certain relation between the energy range of the nuclear and muonic levels a discharge of the metastable nucleus may be followed by the ejection of a muon, which may then participate in the discharge of other nuclei. The estimates of probabilities for discharge of a nucleus with emission of gamma quantum and further muon or electron conversion are presented in ref. [4,6,11,12]. Despite the relatively long history, studying processes of the muon-atom and muon-nucleus interactions hitherto remains very actual and complicated problem. Here we present a consistent relativistic theory of discharge of a metastable nucleus with emission of  $\gamma$  quantum and further  $\mu$  conversion. The accurate numerical calculation is carried out for scandium nucleus ( $A=49$ ,  $N=21$ ) with using the Dirac-Woods-Saxon model.

## 2. Relativistic theory of metastable nucleus discharge during negative muon capture

For simplicity, we consider the model of a nucleus as the 1-QP system [6]. Further we suppose that a nucleus consists off a twice-magic core and a single proton and single muon, which move in the

nuclear core field. The proton and muon interact through the Coulomb potential. This interaction will be accounted for in the first order of the atomic PT or in the second order of the QED PT [13-16]. Surely a majority of the known excited nuclear states have the multi-body character and it is hardly possible to describe their structure within the one-QP model [17]. Nevertheless, the studied effects of the muon-proton interaction are not covered by the one-particle model. It is possible to consider a dynamical interaction of two particles through the core too. It accounts for finite core mass. However, this interaction may decrease a multipolarity of the nuclear transition only by unity. Strongly forbidden transitions of high multipolarity are of a great interest. We calculate the decay probabilities to different channels of the system, which consists of the proton (in an excited state  $\Phi_{N_1 J_1}$ ) and muon (in the ground state  $\Psi_{1s}^{\mu}$ ). Three channels should be taken into account [4,6,12]: i). a radiative purely nuclear  $2^j$ -poled transition (probability  $P_1$ ); ii). a non-radiative decay, when proton transits to the ground state and the muon leaves a nucleus with energy:  $E = \Delta E_{N_1 J_1}^p - E_{\mu}^i$ ;  $\Delta E_{N_1 J_1}^p$  is the energy of nuclear transition;  $E_{\mu}^i$  is a bond energy of muon in the  $1s$  state (probability  $P_2$ ); iii). transition of a proton to the ground state with excitation of muon and emission of  $\gamma$ -quantum with energy  $h\omega = \Delta E_{N_1 J_1}^p - \Delta E_{nl}^{\mu}$  (probability  $P_3$ ). Feynman diagrams, corresponding to different muon-nuclear decay channels, are shown in figure 1. Diagram A (fig.1) corresponds to the first channel (i), diagram B - to the second channel (ii) and diagrams  $C_1$  and  $C_2$  - to the third channel (iii).



**Figure 1.** Feynman diagrams corresponding to different channels of a decay (see text).

The thin line on the diagrams (figure 5) corresponds to the muon state, the bold line – to the proton state. The initial and final states of proton and muon are designated by indices on the lines. The dashed line with the index  $j$  means the Coulomb interaction between muon and proton with an exchange of the  $2^j$ -pole quanta. The wavy line corresponds to operator of the radiative dipole transition. This effect is due to the muon-proton interaction. The diagram A (fig.1) has the zeroth order on the muon-proton interaction; other diagrams (fig.1) are first order. The probability of purely radiative nuclear  $2^j$ -pole transition is defined by convention ( $r_n=5 \cdot 10^{-13}$  cm) [17]:

$$P_1 = 2 \cdot 10^{20} \cdot \frac{j+1}{j!(2j+1)!!^2} \left(\frac{3}{j+3}\right)^2 \left(\frac{\Delta E [MeV]}{40}\right)^{2j+1} . \tag{1}$$

(standard notation). Diagrams  $C_1$  and  $C_2$  (figure 1) account for an effect of the QP interaction on the initial state. Surely there are other versions of these diagrams, but their contribution to probabilities of the decay processes is significantly less than contributions of the diagrams  $C_1$  and  $C_2$ .

Within the relativistic energy approach [6,12-16] the total probability is divided into the sum of the partial contributions, connected with decay to the definite final states of a system. These contributions are equal to the corresponding transition probabilities ( $P_i$ ). If  $\Delta E_{N_1 J_1}^p > E_{\mu}^i$  the probability determination reduces to relativistic calculation of probability for two-QP system autoionization decay. An imaginary excited state energy in the lowest QED PT order is written as follows:

$$\begin{aligned} \text{Im} \Delta E = e^2 \text{Im} i \lim \iint d^4 x_1 d^4 x_2 \exp[\gamma(t_1 + t_2)] \{ & D(r_{c t_1}, r_{c t_2}) \cdot \\ & \langle \Phi_I | (j_{cv}(x_1) j_{cv}(x_2)) | \Phi_I \rangle + D(r_{p t_1}, r_{p t_2}) \langle \Phi_I | (j_{pv}(x_1) j_{pv}(x_2)) | \Phi_I \rangle + \\ & + D(r_{\mu t_1}, r_{\mu t_2}) \langle \Phi_I | (j_{\mu v}(x_1) j_{\mu v}(x_2)) | \Phi_I \rangle \} \end{aligned} \quad (2)$$

Here  $D(r_1 t_1, r_2 t_2)$  is the photon propagator;  $j_{cv}$ ,  $j_{pv}$ ,  $j_{\mu v}$  are the 4-dimensional components of a current operator for the particles: core, proton, muon;  $x=(r_c, r_p, r_\mu, t)$  is the four-dimensional space-time coordinate of the particles, respectively;  $\gamma$  is an adiabatic parameter. Further one should use the exact electrodynamic expression for the photon propagator. Below we are limited by the lowest order of the QED PT, i.e. the next QED corrections to  $\text{Im} \Delta E$  will not be considered. Finally, the imaginary part of energy of the excited state can be represented as a sum of the corresponding QP contributions [12]:

$$\begin{aligned} \text{Im} \Delta E = \text{Im} E_c + \text{Im} E_p + \text{Im} E_\mu, \\ \text{Im} E_a = -Z_a^2 / 4\pi \sum_F \iint dr_{c1} dr_{c2} \iint dr_{p1} dr_{p2} \iint dr_{\mu1} dr_{\mu2} \cdot \\ \cdot \Phi_I^*(1) \Phi_F^*(2) \cdot T_a(1,2) \Phi_F(1) \Phi_I(2), \end{aligned} \quad (3)$$

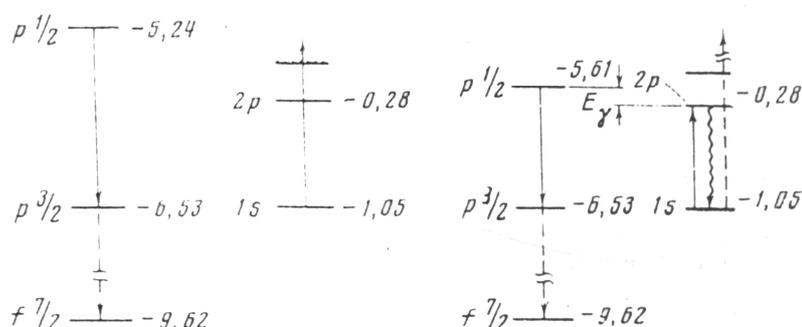
$$T_a(1,2) = \frac{\exp(-w_{IF} r_{a12})}{r_{a12}} (1 - \alpha_1 \alpha_2),$$

Here  $r_{a12} = |r_{a1} - r_{a2}|$ ;  $\Phi_c, \Phi_p, \Phi_e$  are the second quantization operators of field of the core particles, proton and muon. Sum on  $F$  means the summation on the final states of a system. The technique of calculating the matrix elements in Eq. (3) is very well carried out and in details presented in Refs. [12-16,18,19].

### 3. Results and conclusions

The nucleus of  ${}^{49}_{21}\text{Sc}_{28}$  contains a single proton above the twice magic core  ${}^{48}_{20}\text{Ca}_{28}$ . The scheme of the corresponding energy levels for this nucleus is presented in figure 2. The transitions of proton and muon on the first and second stages are noted by the solid and dotted lines. The level  $p_{1/2}$  is connected with the ground level  $f_{7/2}$  by the E4 transition and with the low-lying level  $p_{3/2}$  by the E2 transition. The levels  $p_{3/2}$  and  $f_{7/2}$  are connected by the E2 transition. One could consider also magnetic transitions between these levels. The life-time for the isolated nucleus in the excited state is of order  $10^{-11}$  s.

There are possible the different approaches to modelling the proton motion in a field of a nuclear core [17-20]. The first model corresponds to the well-known Dirac-Wood-Saxon model [12]. Another approach uses the Bloumkvist-Wahlborn potential [11] (see also Refs. [4,6,20,21]). The electric core potential is given by the Gauss model potential [22]. In the relativistic Dirac-Woods-Saxon model the wave functions are determined by solution of the Dirac equations with the Woods-Saxon potential, where the parameters  $V_0$ ,  $a_0$ ,  $R_0$  are fitting using the levels energies. Namely, the parameters are calculated from the fitting condition for the experimental and theoretical energies of the ground and first excited states. For the values  $V_0=-47.6$ ,  $R_1=2$ ,  $R_2=7.65$ ,  $R=4.75$  it has been obtained for the proton states:  $E(f_{7/2})=-9.62$ ,  $E(p_{3/2})=-6.53$ ,  $E(p_{1/2})=-5.24$  and for the muon states:  $E(1s)=-1.05$ ,  $E(2s)=-0.272$ ,  $E(2p)=-0.281$  (the units of energy 1 MeV and length units  $10^{-13}$  cm are used). It should be noted that in the last years the relativistic mean field model with using the Dirac-Woods-Saxon orbital set is developed too.



**Figure 2** Schemes of energy levels of proton (the left part of the figure) and muon (the right part of the figure) in  ${}^{49}_{21}\text{Sc}_{28}$ . Transitions of proton and muon on the first and second stages are noted by the solid and dotted lines.

We present numerical data (calculation with two nuclear potentials) for the scandium nucleus. The probabilities of the muon-atomic decay (in  $\text{s}^{-1}$ ) for a most interesting nuclear transitions are:

i). for the Bloumkvist-Wahlborn potential [11]:

$$P_2(p_{1/2}-p_{3/2})=3,93 \cdot 10^{15}, P_2(p_{1/2}-f_{7/2})=3,15 \cdot 10^{12}, P_2(p_{3/2}-f_{7/2})=8,83 \cdot 10^{14},$$

ii). for the Woods-Saxon potential:

$$P_2(p_{1/2}-p_{3/2})=3,87 \cdot 10^{15}, P_2(p_{1/2}-f_{7/2})=3,09 \cdot 10^{12}, P_2(p_{3/2}-f_{7/2})=8,75 \cdot 10^{14}.$$

For both potentials the presented values are significantly higher than the corresponding non-relativistic estimates of Refs. [4,6]. For example, according to [4]:  $P_2(p_{1/2}-p_{3/2})=3.3 \cdot 10^{15}$ . If a muon-atomic system is in the initial state  $p_{1/2}$ , then the cascade discharge occurs with an ejection of the muon on the first stage and the gamma quantum emission on the second stage. To consider a case when the second channel is closed and the third channel is opened, let us suppose that  $E^p(p_{1/2})-E^p(p_{3/2})=0.92$  MeV (figure 2). The nuclear transition energy is not sufficient to provide transition of the muon to the continuum state. However, it is sufficient for the excitation to the  $2p$  state. It is important to note here that this energy is not lying in the resonant range. The diagram  $C_1$  (see figure 1) describes the proton transition  $p_{1/2}-p_{3/2}$  with the virtual excitation of muon to states of the series  $nd$  and with emitting gamma quantum of the following energy:

$$h\nu=E^p(p_{1/2})+E^\mu(1s)-E^p(p_{3/2})-E^\mu(2p).$$

Further the dipole transition  $2p-1s$  can occur. The calculated value for the probability of this transition is  $P_3=1.9 \cdot 10^{13}$ . It should be noted that the value  $P_3$  is more than the probability value for the radiation transition  $p_{1/2}-p_{3/2}$  and the probability value for the transition  $p_{1/2}-f_{7/2}$ . The transition  $p_{3/2}-f_{7/2}$  occurs during  $\sim 10^{-15}$  s without emission, but with the ejection of a muon.

The experimental possibilities of search of the metastable nucleus discharge effect have been discussed in Refs. [4,6,12] and require a choice of the special type nuclei (a target). Probability of the  $\bar{\mu}$  capture by the excited nucleus must be comparable or being more than a probability of the capture by other (non-excited) nuclei of a target. As result, the target must be prepared as the excited nuclei concentrate with the minimal size of order or more than the free  $\bar{\mu}$  running length  $l$  in relation to a capture by a nucleus. The condition for fewest excited nuclei in a target is  $N_{\min}>l^3 n_0$ , where  $n_0$  is a

density of the target atoms. For initial  $\mu$  slowing to energies of 0.1-0.3MeV, the free running length is  $\sim 0,1$  cm. The required number of metastable nuclei is  $N_{\min} > 10^{19}$  for the density  $n_0 = 10^{22} \text{ cm}^{-3}$ . The radioactivity of such a target is  $R = N_{\min}/T$ , where  $T$  is the decay time. For example, for  $T = 10^2$  days one can get the estimate  $R \sim 10^3 \text{ Ci}$  ( $1 \text{ Ci} = 3.7 \cdot 10^{10}$  decays per s). In conclusion, let us note that a development of electron- $\mu$ -nuclear spectroscopy of atoms (nuclei) is of a great theoretical and practical interest. The development of adequate approaches to studying the cooperative e-, $\mu$ - $\gamma$ -nuclear processes promises the rapid progress in our understanding of a nuclear decay. Such an approach is useful, providing perspective for developing advanced nuclear models, search for new cooperative effects on the boundary of atomic and nuclear physics, carrying out new methods for treating (muonic chemistry tools) the spatial structure of molecular orbitals, studying the chemical bond nature and checking different models in quantum chemistry, atomic and nuclear-laser spectroscopy [4-12].

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