УДК 551.509.328 + 517.938

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RENORM-GROUP AND FRACTAL APPROACH TO TURBULENCE SPECTRUM IN PLANETARY ATMOSPHERE SYSTEM, "COSMIC PLASMA – GALACTIC COSMIC RAYS"

Renorm-group and fractal approach is used to study a turbulence spectrum in a general dynamics of atmosphere, turbulent pulsation in planetary atmosphere – cosmic plasma and galactic cosmic rays system.
Keywords: *renorm-group approach, turbulence, dynamics of atmosphere, cosmic rays*

Introduction. It is well known that the equations of hydrodynamics are relatively well adjusted to the high-frequency processes in the atmosphere such as the evolution of the cyclonic formation in the period up to two days, but it is not able to well describe the low-frequency processes such as changing forms of circulation [1-6]. The equation of macroturbulent regime of atmosphere are low-frequency on its basis. Surely, there is some experience in their solving on the basis of a number of techniques, such as spectral, etc. The method for solving these equations in the low frequency range is used for modeling of the changing forms of circulation and, accordingly, for the mathematical parameterization homologues circulation [3-6]. In principle, an alternative approach to the study of the spectrum of turbulence in a general geodynamics, apparently, should be considered as methods of the renormalization group (RG). The RG methods originally developed in quantum field theory are also used to describe fully developed turbulence. The fact the RG approach is essentially a method of describing multimode systems with a large range of specific size and strong mode coupling [7-14]. According to the Kuzmin-Patashinskii's hypothesis, such systems tend to localize interactions in wavenumber space and cascade mechanism of interaction modes with significantly different scales. The first attempt to apply the RG approach to modeling a power-law behavior of the statistical moments of turbulent fluctuations of the velocity field are based on the Kadanov's iterative procedure of partial averaging and the RG field formulation. Here we outline the ways of applying the RG analysis to studying a spectrum of large-scale turbulence in a general dynamics of atmosphere with confirming a renormalization and scaling, as well as consider the turbulent fluctuations of in "planetary atmosphere - cosmic plasma (CP)- galactic cosmic rays (GCR)" system. Some applications were earlier considered (look, eg., [14-21]).

Initial equations and renormalization procedure. We consider a standard atmospheric system of the Navier-Stokes equations with adding an external force. The latter can be a Gaussian random process such as "white noise." The hydrodynamic field-pressure and velocity vector components v_i in point $1=(r_1,t_1)$ should be considered in a space of d measurements as the components of $(d+1)$- dimensional vector [8]

$$\psi_\alpha(1)=\{\psi_0(1),\psi_i(1)\}=\{p(r_1,t_1),v_i(r_1,t_1)\},\alpha=0,1,...d,i=1,2,...d. \tag{1}$$

In a formalism of "doubling the fields" (see [8-10,14]), the system is given by the action:

$$\begin{aligned} &S[\psi,\hat{\psi}]=S_0[\psi,\hat{\psi}]+\lambda_0 S_1[\psi,\hat{\psi}]:\\ &S_0[\psi,\hat{\psi}]=-\hat{\psi}_\alpha(1)L_{\alpha\beta}(12)\psi_\beta(2)+(i/2)\hat{\psi}_\alpha(1)D_{\alpha\beta}(12)\hat{\psi}_\beta(2),\\ &S_1[\psi,\hat{\psi}]=-1/2\,\hat{\psi}_\alpha(1)V_{\alpha\beta\gamma}(123)\psi_\beta(2)\psi_\gamma(3). \end{aligned} \tag{2}$$

where the linear part of the Navier-Stokes equations $L_{\alpha\beta}$, the correlation function of random external forces $D_{\alpha\beta}(12)$ and the coefficient $V_{\alpha\beta\gamma}(123)$ determined by the relations:

$$L_{\alpha\beta}(12)=\begin{bmatrix}0 & \partial_j^{(0)}\\ \partial_i^{(0)} & (\partial_t^{(1)}-v_0\Delta\delta_{ij})\end{bmatrix}\delta(1-2)$$

$$D_{ij}(12)=\delta_{ij}D(|r_1-r_2|)\delta(t_1-t_2), \qquad (3)$$

$$V_{ijk}(123)=-[\delta_{ij}\partial_k^{(2)}+\delta_{ik}\partial_j^{(3)}]\delta(1-2)\delta(1-3),$$

where v_0 - the coefficient of molecular viscosity, λ_0 - a formal expansion parameter (the end result is usually set equal to unity). Further consideration of the objects of interest-averaged velocity field of the linear response to an external action, the Green's function $G_{ij}(12)=i<\psi_i(1)\hat{\psi}_j(2)>$ and correlation function $C_{ij}(12)=<\psi_1(1)\psi_2(2)>$. The representation of the characteristic functional of system in the standard classical perturbation theory

$$\psi[\eta,\hat{\eta}]=\int d[\psi]d[\hat{\psi}]\exp\{i(S_0[\psi,\hat{\psi}]+\lambda_0 S_1[\psi,\hat{\psi}]+\eta\psi+\hat{\eta}\hat{\psi})\}$$

is presented as an expansion in powers $\lambda_0 S_1$. As usual, the partition of the action on the unperturbed part and a perturbation is performed taking into account the finite renormalization of the field amplitudes with the addition of a compensating counter-terms. The non-dependence of the results on the choice of the renormalization constants corresponds to the requirement of invariance with respect to the renormalization of the perturbation. The renormalization of the field amplitude $\hat{\psi}$ and the viscosity in the system (2), which describes the dynamics of the perturbed atmosphere, is easily introduced as [9,10]:

$$\hat{\psi}\to\hat{\psi}^R=z\hat{\psi},\quad \lambda_0\to\lambda=z^{-1}\lambda_0,\quad D\to D^R=z^{-2}D,\quad v_0\to v \qquad (4)$$

and adding the counter-terms to S_1 of the following form

$$\delta S_1=-(z_{-1}-1)\hat{\psi}_i\partial_t\psi_i+(v_0z^{-1}-\hat{v})\psi_i\Delta\psi_i. \qquad (5)$$

Renormalization of the parameters are chosen in such a way that a total Green's function in the renormalized theory has the form, well known in quantum field theory :

$$G_{ij}^R(k,w)=P_{ij}(k)[-iw+vk^2-\sum{}^R(k^2,w)]^{-1},\quad (P_{ij}(k)=\delta_{ij}-k_ik_jk^{-2}) \qquad (6)$$

and coincides with a free Green's function at the point of the renormalization $w=0,\quad k^2=\mu^2$, of course, imposed a condition of the relations:

$$\sum{}^R(\mu^2,0)=0,\quad \frac{\partial}{\partial w}\sum{}^R(\mu^2,w)|_{w=0}=0. \qquad (7)$$

In fact, the desired approach in a different representation is used in a number of problems in quantum field theory, quantum geometry (see, eg., [22]).

The RG analysis in general geodynamics. Generally the function $D(k)$ is as

$$D(k)=D_0(k^2)^{-d/2+2-\varepsilon}. \qquad (8)$$

The dimension of the parameter D_0 coincides with the dimension of the energy dissipation rate in the Kolmogorov theory for $\varepsilon=2$ [7]. There is a logarithmic divergence of the operator's self-energy for $\varepsilon=0$. The real expansion parameter in a perturbation theory is proportional to ε and procedure ε - expansion is reduced to the analytic continuation on ε from the logarithmic theory ($\varepsilon=0$) to the Kolmogorov ($\varepsilon=2$).Indeed, a real situation (in geodynamics) is characterized by a non-integer value $\varepsilon=\varepsilon_{real}$. Naturally, the processes of momentum transfer in a real turbulent atmosphere are carried out both molecular and turbulent vortex motion. The effective viscosity $\tilde{v}$ is determined by the usual expression [10]

$$G_{ij}^{-1}(k,w) = [-iw + \tilde{v}(k^2,w)k^2]\delta_{ij}. \quad (9)$$

By the invariance with respect to the renormalization, the renormalized Green's function (for 2 different points of normalization μ, μ_1) are related by of the following type

$$z^{-1}(\mu^2)G^R(k,w;\mu^2) = z^{-1}(\mu_1^2)G^R(k,w;\mu_1^2). \quad (10)$$

The condition of changing a normalization under passing from one point to another one

$$Z(v,\lambda,D;\mu^2 \mid v_1,\lambda_1,D_1;\mu_1^2) = z(v_1,\lambda_1,D_1;\mu_1^2)z^{-1}(v,\lambda,D;\mu^2) \quad (11)$$

dimensional considerations provide that the normalized Z is the function of the parameter $h = \lambda^2 Dv^{-3}(\mu^2)^{-\varepsilon}$, relation μ_1^2/μ^2 and corresponds to the group composition rule

$$Z(\frac{\mu_1^2}{\mu^2},h_1) = Z(\frac{\mu_1^2}{\mu_1^2},h_2)Z^{-1}(\frac{\mu_1^2}{\mu_2^2},h_2). \quad (12)$$

Here the function $\tilde{h}$ is in fact analog of the invariant charge in quantum theory of field or topological charge of a vector soliton. One could write the corresponding equation:

$$\{-x\frac{\partial}{\partial x} + \beta(h)\frac{\partial}{\partial h}\}\tilde{h}(x,h) = 0, \beta(h) = \frac{\partial \tilde{h}(x,h)}{\partial x}|_{x=1}. \quad (13)$$

For the class of problems we are interested in the problem boils down to the definition of the self-energy operator in the lowest approximation of the renormalized perturbation theory. As usual, the required operator can be written as

$$\sum\nolimits_{ij}^{R}(k,w) = \sum\nolimits_{ij}(k,w) + iw(z^{-1}-1)\delta_{ij} - k^2(v_0 z^{-1} - v)\delta_{ij}, \quad (14)$$

where the first term is determined as

$$\sum\nolimits_{ij}^{R}(k,w) = \lambda^2 V_{imn}(k)\int \frac{dq}{(2\pi)^d}\frac{d\Omega}{2\pi}G_{mm'}(q,\Omega)C_{nn'}(k-q,w-\Omega)V_{m'n'j}(q), \quad (15)$$

2nd and 3rd terms are the counter-terms. A link between a renormalization constant and self-energy operator $\sum$ is as $z^{-1} = 1 + i\partial\sum(\mu^2,w)/\partial w|_{w=0}$. Detailed description of this procedure for the finite Fermi systems is given in [22]. With this in mind, for the effective viscosity of the lowest order perturbation theory calculation $\sum$ one could get the known expression, but with the principal difference. Instead of the ideal value $\varepsilon = 2$ one should take the real ε_{real}. Some concrete applications have been in details presented in [13,16-21].

Turbulent pulsation in "Planetary atmosphere - CP- GCR" system. The known experiments of Pudovkin and Raspopov (1992) have detected that processes in the atmosphere at heights 10–20 km are extremely influenced by the GCR with the protons' energies of $10^{11} \div 10^{15}$ eV. Strong variations of these rays (a few tens of percents) coincide with the solar activity cycles and atmospheric perturbation variations induced by the separate flares on the Sun. Pudovkin and Raspopov (1992) have also shown that both the value of incoming energy from the GCR spectrum in the magnetosphere and the magnitude of consequent processes in the magnetosphere-ionosphere coincide with values of actual energy for atmospheric processes (~$10^{19} \div 10^{20}$ J day^{-1}). The GCR spectrum is striking stable and, in the limits of $10^{11} \div 10^{15}$ eV, is defined by the law:

$$\frac{dN}{dE} = CE^{-\nu}, \quad \nu \approx 2.4 \div 2.74. \quad (16)$$

We aim to define of the spectrum of the turbulent GCR-induced pulsations in the atmosphere and to determine possible manifestation of genesis of fractal dimensions in the system of "spectrum of the CP turbulent pulsations – GCR spectrum – spectrum of atmospheric turbulent pulsations". According to Eq. (16) the integral GCR spectrum in the limits of $10^{11}\div10^{15}$ eV is $N \sim E^{\mu}$, $\mu = 1.7$. Assume that the GCR energy is absolutely absorbed by the atmosphere; then the average energy, E_g, transferred to the atmospheric gas is evaluated as $E_g \sim NE \sim E^{1-\mu}$. Let's consider that each cosmic particle induces the initiation of an eddy with a size of λ in the moving gaseous medium. This size is inversely proportional to the energy of particle, E, i.e.$E \sim \lambda^{-1}$. According to [8], we introduce the appropriate spatial "wave numbers" of pulsations (eddies) as $k \sim 1/\lambda$ instead of scales λ. Then the integral spectrum of eddies, E_g, looks like $E_g \sim k^{1-\mu}$ and the appropriate spectral density of turbulence is $E_g(k) \sim k^{-\mu}$, where $E_g(k)$ is the kinetic energy of gaseous eddy with the spatial wave number k. Since $\mu \approx 5/3$, it is obvious that this is the well-known Kolmogorov-Obukhov spectrum describing the dynamics of high-frequency perturbations or, in other words, the structure of small-scale turbulized medium as a skeleton of eddy cluster with the fractal dimension $D = 5/3$ [15]. The scaling laws, scale ratios, and spectral dynamics, in particular within the inertial interval theory that results in the energy spectrum of the Kolmogorov-Obukhov eddies, are conventionally applied at the atmospheric turbulence modelling but for planetary boundary layer only, i.e. for the air layer, in which the interaction of atmosphere with the underlying surface is directly appeared [3]. This implies that the detected GCR-induced Kolmogorov-Obukhov spectrum differs not only by the cause, but by the principal location of appearance: homosphere – upper atmosphere. There is natural question: what consequences should be experimentally observed in this case? Such a time-stable and large-value increment (e.g. as the Joulean heat into upper atmosphere) should essentially revise the "centre of gravity" in the Earth's energy balance by accounting for a turbulent heat flux G generated by the variations of the galactic and solar cosmic rays:

$$\frac{\partial U}{\partial t} = S[1-\alpha(T)] - I_a(T) - Q(T) + G(T), \qquad (17)$$

where $\partial U/\partial t$ is the rate of heat generation in the Earth's climate system, T is the temperature, S is the solar irradiance onto top of the atmosphere, α is the albedo of the atmosphere-Earth system, I_a is the intensity of outgoing long-wave atmospheric radiation, Q is the heat quantity leaving the considered volume of the climate system due to the horizontal transport of sensible, Q_1, and latent, Q_2, heat. Represent I_a, Q, G, and α as the functions of temperature. First energy term I_a is responsible for the long-wave radiation of the Earth with the mean temperature T; with approximation sufficient for our model it is equal to $I_a = \gamma_a \sigma T^4$, where σ is the Stephan-Boltzmann constant, γ_a is the coefficient allowing for the area of atmospheric external boundary parallel with the Earth's surface. The heat flux Q is formulated as follows:

$$Q = Q_1 + Q_1 = \gamma_{adv}\mu_{adv}T + \gamma_{adv}m_{wv}c(T_{wv} - T), \qquad (18)$$

where μ_{adv} is the advection coefficient, γ_{adv} is the coefficient allowing for the total area of the lateral sides of the Earth's climate system, m_{wv} is the weight velocity of the condensation of water vapour molecules, c is the specific heat. The dependence of the effective value of albedo for Earth-atmosphere system from temperature is taken as the continuous Fegr's parameterization: $\alpha(T) = 0.486 - \eta_\alpha(T - 273)$, where $\eta_\alpha = 0.0092$ K^{-1}. Regarding the heat transport G note. in the turbulent mode. The universal behaviours obtained within the inertial interval theory or, in other words, the Kolmogorov-Obukhov scaling laws were developed to describe the statistical structure of temperature turbulent pulsations when they not essentially affect the structure of flow [7]. . It has been also shown that the structure of temperature field for the turbulent mode is defined not only by the dissipation rate of turbulent kinetic energy per mass unit ε, but also by the dissipation rate of intensity of temperature fluctuations, N_T, which is equal in order of magnitude to $N_T \cong (\Delta T)^2 \Delta u L^{-1}$, where Δu and L are the typical size for the velocity and length of main energy-bearing eddies, ΔT is the typical temperature variation in the flow at its external scale L. It is also easily to show that the integral spectrum of eddies E_g looks like: $E_T = C_{1t}(\Delta T)^2$. Assuming $\Delta T \sim \beta T$, where $\beta < 1$, this dependence can be expressed as $G \sim g(\Delta T)^2 \sim (g\beta)T^2$, where g is the dimensional coefficient, W K^{-2}. So, we get

$$\frac{1}{4\gamma_a\sigma}\left(S - \frac{\partial U}{\partial t}\right) = F(T,a,b) = \frac{1}{4}T^4 + \frac{1}{2}aT^2 + bT + \text{free term}, \qquad (19)$$

where $a = -\frac{g\beta}{2\gamma_a\sigma}$, $b = \frac{\gamma_{adv}\mu_{adv} + \eta_a S - \gamma_{adv} m_{wv} c}{4\gamma_a\sigma}$. Here, we assume that the power $F(T, a, b)$ is not time-dependent, which seems as physically lawful. So, there is the family of functions $F(T, a, b)$ depending on the two control parameters, a and b. In our opinion, it is interesting to model the long-period behaviour of mean value, $\langle T\rangle$, and dispersion, $\langle \Delta T^2\rangle$, of temperature for the manifold of assembly-type catastrophe. To reveal the nontrivial capabilities of the proposed method, let us consider two cases of cyclic path. The first case is modelled under the condition that $a = -1$ and $b(\mathrm{t}) = b\cos\omega t$; $T_1^{-1} >> \omega$. The time evolutions of $\langle T\rangle$ and $\langle \Delta T^2\rangle$ are shown in Fig. 1a (the symmetric cyclic path C). The second case, which corresponds to the asymmetric (with regard to coordinate axis a) cyclic path A in plane (a – b), is modelled under following conditions: $a = -0.5$, $b(t) = [-b\cos\omega t + \Delta]$, where $\Delta = (b - 2|b_c|)/4$, and b_c is determined by the equation of semi-cubical parabola describing the bifurcation set of assembly-type catastrophe (Fig. 1b). It is noteworthy that the analysis of well-known experimental data from the Antarctic station Vostok, which are related to the temperature variations during the last 420 ky, confirms the existence of period ~120 ky [15,16]. The analysis demonstrates that the reason of such an periodical behaviour of control parameter b is the periodical variations of Earth's orbit geometry (eccentricity) initiating the variations of solar radiation or, in other words, the physical mechanism for "controlling" of global climate, which was long time ago concerned in the well known Milankovitch's theory of ice age rhythms (look, e.g. [14,15).

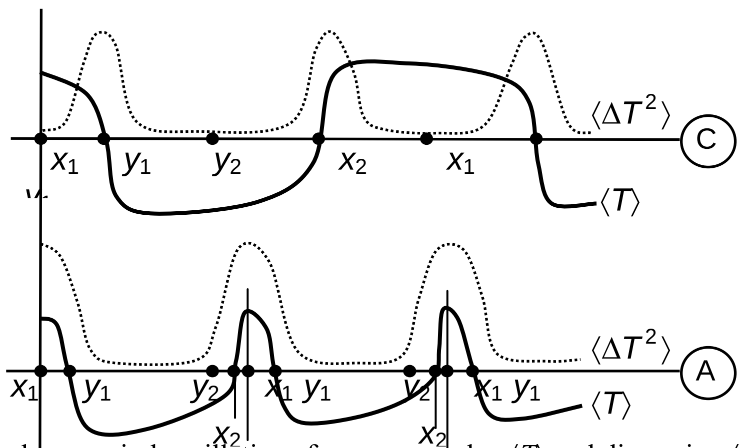

Figure 1 - Modelling long-period oscillations for a mean value $\langle T\rangle$ and dispersion $\langle \Delta T^2\rangle$ of temperature corresponding to symmetric (a) and asymmetric (b) paths in space of controlling parameters (*a, b*).

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Ренорм-груповий та фрак тильний підхід до опису спектру турбулентності у системі «планетарна атмосфера – космічна плазма- галактичні космічні промені. Глушков О.В.

У роботі вперше ренорм-груповий аналіз у загальному формулюванні застосовано до аналізу спектру турбулентності в системі «планетарна атмосфера –космічна плазма-галактичні космічні промені».

***Ключові слова**: ренорм-груповий аналіз, турбулентність, динаміка атмосфери, космічні промені*

Ренорм-групповой и фрактальный подход к описанию спектра турбулентности в системе “планетарная атмосфера – космическая плазма - галактические космические лучи". Глушков А.В.

Изложен новый ренорм-групповой и фрактальный подход к описанию спектра турбулентности в системе “планетарная атмосфера – космическая плазма - галактические космические лучи".

***Ключевые слова**: ренорм-групповой подход, турбулентность, динамика атмосферы, космические лучи*