Modeling chaotic dynamics of complex systems with using chaos theory, geometric attractors, and quantum neural networks

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Abstract Nonlinear simulation and forecasting chaotic evolutionary dynamics of complex systems has been effectively performed using the concept of compact geometric attractors. We present an advanced approach to analyze complex system dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation.

Keywords Geometric attractor conception, quantum neural networks, chaotic dynamics

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1. Introduction

In this work nonlinear simulation and forecasting chaotic evolutionary dynamics of complex systems are carried out using the concept of compact geometric attractors . We are developing a new approach to analyze complex system dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation. This work develops our studies, presented in [1-12].

The basic idea of the construction of our approach to prediction of chaotic processes in complex systems is in the use of the traditional concept of a compact geometric attractor in which evolves the measurement data, plus the implementation of neural network algorithms. The existing so far in the theory of chaos prediction models are based on the concept of an attractor, and are described in a number of papers (e.g. [1,13-20]). From a mathematical point of view, it is a fact that in the phase space of the system an orbit continuously rolled on itself due to the action of dissipative forces and the nonlinear part of the dynamics, so it is possible to stay in the neighborhood of any point of the orbit *y (n)* other points of the orbit $y^r(n), r = 1, 2, ..., N_B$, which come in the neighborhood *y (n)* in a completely different times than *n*. Of course, then one could try to build different types of interpolation functions that take into account all the neighborhoods of the phase space and at the same time explain how the neighborhood evolve from *y (n)* to a whole family of points about *y (n+1)*. Use of the information about the phase space in the simulation of the evolution of some physical (geophysical etc.) process in time can be regarded as a fundamental element in the simulation of random processes.

In terms of the modern theory of neural systems, and neuro-informatics (e.g. [1]), the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations). Imitating the further evolution of a complex system as the evolution of a neural network with the corresponding elements of the self-study, self- adaptation, etc., it becomes possible to significantly improve the prediction of evolutionary dynamics of a chaotic system. Considering the neural network with a certain number of neurons, as usual, we can introduce the operators S_{ij} synaptic neuron to neuron u_i u_j, while the corresponding synaptic matrix is reduced to a numerical matrix strength of synaptic connections: $W = w_{ij}$. The operator is described by the standard activation neuro-equation determining the evolution of a neural network in time:

$$s_i^{'} = sign(\sum_{j=1}^{N} w_{ij}s_j - \theta_i), \tag{1}$$

where $1 < i < N$.

From the point of view of the theory of chaotic dynamical systems, the state of the neuron (the chaos-geometric interpretation of the forces of synaptic interactions, etc.) can be represented by currents in the phase space of the system and its the topological structure is obviously determined by the number and position of attractors. To determine the asymptotic behavior of the system it becomes crucial a information aspect of the problem, namely, the fact of being the initial state to the basin of attraction of a particular attractor.

Modelling each physical attractor by a record in memory, the process of the evolution of neural network, transition from the initial state to the (following) the final state is a model for the reconstruction of the full record of distorted information, or an associative model of pattern recognition is implemented. The domain of attraction of attractors are separated by separatrices or certain surfaces in the phase space. Their structure, of course, is quite complex, but mimics the chaotic properties of the studied object. Then, as usual, the next step is a natural construction parameterized nonlinear function $\mathbf{F}$ $(\mathbf{x}, \mathbf{a})$, which transforms: $\mathbf{y}(n) \to \ \mathbf{y}(n+1) = \mathbf{F}(\mathbf{y}(n, a))$,and then to use the different (including neural network) criteria for determining the parameters a (see below). The easiest way to implement this program is in considering the original local neighborhood, enter the model(s) of the process occurring in the neighborhood, at the neighborhood and by combining together these local models, designing on a global nonlinear model. The latter describes most of the structure of the attractor.

Although, according to a classical theorem by Kolmogorov-Arnold -Moser, the dynamics evolves in a multidimensional space, the size and the structure of which is predetermined by the initial conditions, this, however, does not indicate a functional choice of model elements in full compliance with the source of random data. One of the most common forms of the local model is the model of the Schreiber type [14] (see also [1,15-19]).

Nonlinear modelling of chaotic processes can be based on the concept of a compact geometric attractor, which evolve with measurements. Since the orbit is continually folded back on itself by the dissipative forces and the non-linear part of the dynamics, some orbit points $\mathbf{y}^r(k), r = 1, 2, \ldots, N_B$ can be found in the neighbourhood of any orbit point $\mathbf{y}(k)$, at that the points $\mathbf{y}^r(k)$ arrive in the neighbourhood of $\mathbf{y}(k)$ at quite different times than k. Then one could build the different types of interpolation functions that take into account all the neighborhoods of the phase space, and explain how these neighborhoods evolve from $y(n)$ to a whole family of points about $y(n+1)$. Use of the information about the phase space in modelling the evolution of the physical process in time can be regarded as a major innovation in the modelling of chaotic processes.

This concept can be achieved by constructing a parameterized nonlinear function $F(x, a)$, which transform $y(n)$ to $y(n+1) = F(y(n), a)$, and then using different criteria for determining the parameters a. Further, since there is the notion of local neighborhoods, one could create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global nonlinear model that describes most of the structure of the attractor.

As shown Schreiber [14], the most common form of the local model is very simple:

$$s(n+\Delta n) = a_0^{(n)} + \sum_{j=1}^{d_A} a_j^{(n)} s(n-(j-1)\tau) \tag{2}$$

where Δn - the time period for which a forecast .

The coefficients $a_j^{(k)}$, may be determined by a least-squares procedure, involving only points $s(k)$ within a small neighbourhood around the reference point. Thus, the coefficients will vary throughout phase space. The fit procedure amounts to solving $(d_A + 1)$ linear equations for the $(d_A + 1)$ unknowns. When fitting the parameters a, several problems are encountered that seem purely technical in the first place but are related to the nonlinear properties of the system. If the system is low-dimensional, the data that can be used for fitting will locally not span all the available dimensions but only a subspace, typically. Therefore, the linear system of equations to be solved for the fit will be ill conditioned. However, in the presence of noise the equations are not formally ill-conditioned but still the part of the solution that relates the noise directions to the future point is meaningless .Other modelling techniques are described, for example, in [3,10, 17-20].

Assume the functional form of the display is selected, wherein the polynomials used or other basic functions. Now, we define a characteristic which is a measure of the quality of the curve fit to the data and determines how accurately match $y(k+1)$ with $F(y(k), a)$, calling it by a local deterministic error:

$$\epsilon_D(k) = \mathbf{y}(k+1) - \mathbf{F}(\mathbf{y}(k), \mathbf{a}).$$

The cost function for this error is called $W(\epsilon)$. If the mapping $F(y, a)$, constructed by us, is local, then one has for each adjacent to $y(k)$ point, $y^{(r)}(k)(r = 1, 2, ..., N_B)$,

$$\varepsilon_D^{(r)}(k) = \mathbf{y}(r, k+1) - \mathbf{F}(\mathbf{y}^r(k), \mathbf{a}),$$

where $y(r, k+1)$ - a point in the phase space which evolves $y(r, k)$. To measure the quality of the curve fit to the data, the local cost function is given by

$$W(\varepsilon, k) = \frac{\sum_{r=1}^{N_B} \left|\varepsilon_D^{(r)}(k)\right|^2}{\sum_{r=1}^{N_B} \left[y(k) - \langle y(r, k)\rangle\right]^2} \tag{3}$$

and the parameters identified by minimizing $W(\epsilon, k)$, will depend on a.

Furthermore, formally the neural network algorithm is launched, in particular, in order to make training the neural network system equivalent to the reconstruction and interim forecast the state of the neural network (respectively, adjusting the values of the coefficients). The starting point is a formal knowledge of the time series of the main dynamic parameters of a chaotic system, and then to identify the state vector of the matrix of the synaptic interactions w_{ij} etc. Of course, the main difficulty here lies in the implementation of the process of learning neural network to simulate the complete process of change in the topological structure of the phase space of the system and use the output results of the neural network to adjust the coefficients of the function display.

Further we consider implementation of the quantum neural networks algorithm into general scheme of studying chaotic dynamics. The basic aspects of theory of the photon echo based neural networks are stated previously (see, for example, [21]). So here we mention only the essential elements. Photon echo is a nonlinear optical effect, in fact this is the phenomenon of the four wave interaction in a nonlinear medium with a time delay between the laser pulses. We have used a software package for numerical modeling of the dynamics of the photon echo neural network, which imitates evolutionary dynamics of the complex system. It has the following key features: multi-layering, possibility of introducing training, feedback and controlled noise. There are possible the different variants of the connections matrix determination and binary or continuous sigmoid response (and so on) of the model neurons. In order to imitate a tuition process we have carried out numerical simulation of the neural networks for recognizing a series of patterns (number of layers N=5, number of images $CЪ$ = 640; the error function:

$$SSE = \sum_{p=1}^{p_{\max}} \{.. \sum_{k=}^{k_{\max}} [t(p,k) - O(p,k)]^2\}, \qquad (4)$$

where $O(p,k)-$ neural networks output *k* for image *p* and *t(p,k)* is the trained image *CЪ* for output *Pe*; *SSE* is determined from a procedure of minimization; the output error is $RMS = sqrt(SSE/P_{max})$; As neuronal function there is used function of the form: $f(x) = 1/[1 + \exp(-\delta x)]$. In our calculation there is tested the function $f(x,T) = exp[(xT)^4]$ too.

The result of the PC simulation (with using our neural networks package NNW-13-2003 [21]) of dynamics of the quantum multilayer neural networks with the input rectangular and soliton-like pulses is listed in fig.1 and fig 2. The same results for sinusoidal and noisy input sequence are listed in [21]. Analysis of the

PC experiment results allows to make conclusion about sufficiently high-quality processing the input signals of very different shapes and complexity by a photon echo based neural network.

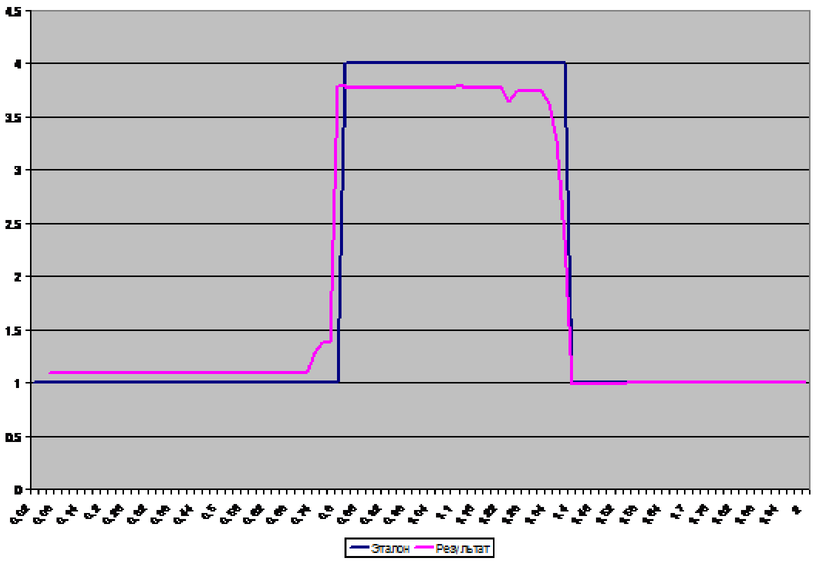

Fig. 1. The results of modeling the dynamics of multilayer neural networks with rectangular input pulse

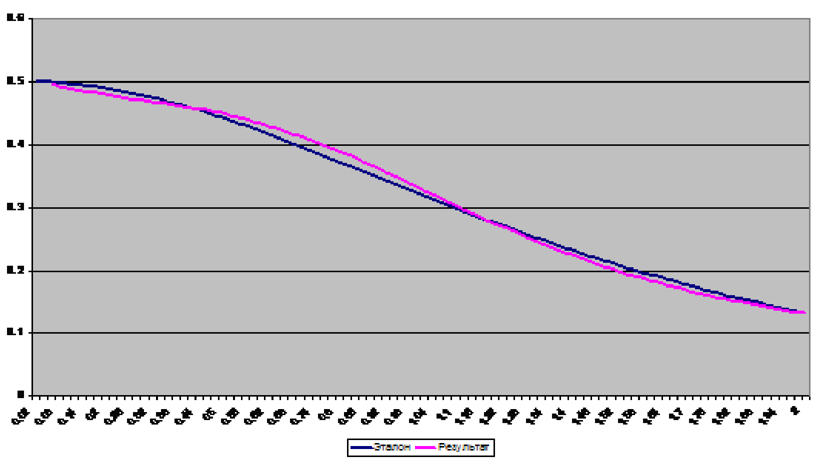

Fig. 2. The results of modeling the dynamics of multilayer neural networks with soliton-like input pulse

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