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ANALYSIS OF THE FRACTAL STRUCTURES IN CHAOTIC PROCESSES: TIME SERIES OF THE DANUBE RIVER'S DAYLY RUNOFF AND THE EXTREMAL HYDROLOGICAL EVENTS

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This paper goes on our investigations of the fractal structures in the chaotic and turbulent processes and connected with a great importance the experimental and theoretical studying of the non-linear dynamical systems with aim to discover the fractal features and elements of dynamical chaos. In this paper on the basis of wavelet analysis and multifractal formalism it is carried out an analysis of fractal structures in the chaotic processes (the time series of the daily runoffs for the Danube river, 1989-1998 years) and the spectrum of the fractal dimensions has been computed. It is carried out numerical modelling and fulfilled a comparison of theoretical data on runs with observed ones on the basis of the new approach to modeling the extremal hydrological events (flood etc.). The latter is based on the multi-factor systems formalism, in particular, system model with many inputs and one output.

Key words: hydrological systems, fractals structures, chaotic processes, the extremal hydrological events, fractal dimensions

1. INTRODUCTION

This paper goes on our investigations of the fractal structures in the chaotic and turbulent processes [1,2]. Let us remind that in last years it is of a great importance the experimental and theoretical studying of the non-linear dynamical systems with aim to discover the fractal features and elements of dynamical chaos (e.g. [3-23]). One of the effective approaches to solving such a problem is the multifractal and wavelet analyses. The foundations and application information on the continuous wavelet transform-based method of multifractal analysis are presented in Ref. [3]. An extension of the concept of multifractals to irregular functions through the use of wavelet transform modulus maxima and potential and limitations of the multifractal formalism in the study of non-stationary processes and short signals are in details considered in these references. Especial attention is turned to the multifractality loss effects in the dynamics of different types of systems. A review of fundamental results on the manifestation of fractal structure in wave (turbulent) processes is presented in [3].

As it is indicated in many references (e.g. [3]) the most natural and effective illustration of the chaos effect can be observed in turbulent flows. In papers by Zaslavsky et al (e.g. [5]) the fractal properties of the sea surface have been considered on the scales which are more than the distortion correlation radius.

In this paper On the basis of wavelet analysis and multifractal formalism it is carried out an analysis of fractal structures in the chaotic processes (the time series of the daily runoffs for the Danube river, 1989-1998 years) and the spectrum of the fractal dimensions has been computed. It is carried out numerical modelling and fulfilled a comparison of theoretical data on runs with observed ones on the basis of the new approach to modeling the extremal hydrological events (flood etc.).

2. METHOD

2.1 Wavelet expansions and multifractals

Let us further consider the utilized version of the wavelet analysis and multi-fractal formalism. Note that a;; details of the method have been in details presented in the earlier papers [1,2], so here we are limited only by the key aspects. The theoretical tool is in fact based on the wavelet decomposition for analyzing various signals. At present, the family of analyzing function dubbed wavelets is being increasingly used in problems of pattern recognition; in processing and synthesizing various signals; in analysis of images of any kind (X-ray picture of a kidney, an image of mineral, etc.); for study of turbulent fields, for contraction (compression) of large volumes of information, and in many other cases. Wavelets are fundamental building block functions, analogous to the sine and cosine functions. Fourier transform extracts details from the signal frequency, but all information about the location of a particular frequency within the signal is lost. At the expense of their locality the wavelets have advantages over Fourier transform when non-stationary signals are analyzed. Here, we use non-decimated wavelet transform that has temporal resolution at coarser scales [1,2].

The dilation and translation of the mother wavelet ψ(t) generates the wavelet as follows: ψj,k(t) = 2j/2ψ(2jt – k). The dilation parameter j controls how large the wavelet is, and the translation parameter k controls how the wavelet is shifted along the t-axis. For a suitably chosen mother wavelet ψ(t), the set {ψj,k}j,k provides an orthogonal basis, and the function f which is defined on the whole real line can be expanded as

$$f(t) = \sum_{k=-\infty}^{\infty} c_{0k}\varphi_{0,k}(t) + \sum_{j=1}^{J}\sum_{k=-\infty}^{\infty} d_{jk}\psi_{j,k}(t) , \quad (1)$$

where the maximum scale J is determined by the number of data, the coefficients c0k represent the lowest frequency smooth components, and the coefficients djk deliver information about the behavior of the function f concentrating on effects of scale around 2–j near time k × 2–j. This wavelet expansion of a function is closely related to the discrete wavelet transform (DWT) of a signal observed at discrete points in time. In practice, the length of the signal, say n, is finite and, for our study, the data are available monthly, i.e. the function f(t) in Eq. (1) is now a vector f = (f(t1),…, f(tn)) with ti = i/n and i = 1,…,n. With these notations, the DWT of a vector f is simply a matrix product d = Wf, where d is an n × 1 vector of discrete wavelet coefficients indexed by 2 integers, djk, and W is an orthogonal n × n matrix associated with the wavelet basis. For computational reasons, it is simpler to perform the wavelet transform on time series of dyadic (power of 2) length. One particular problem with DWT is that, unlike the discrete Fourier transform, it is not translation invariant. This can lead to Gibbs-type phenomena and other artefacts in the reconstruction of a function. The non-decimated wavelet transform (NWT) of the data (f(t1), …, f(tn)) at equally spaced points ti = i/n is defined as the set of all DWT's formed from the n possible shifts of the data by amounts i/n; i = 1, …, n.

Thus, unlike the DWT, there are 2j coefficients on the jth resolution level, there are n equally spaced wavelet coefficients in the NWT

$$d_{jk} = n^{-1}\sum_{i=1}^{n} 2^{j/2}\,\psi\left[2^{j}\left(i/n - k/n\right)\right]y_i ,$$

k = 0, …, n–1,

on each resolution level j. This results in log2(n) coefficients at each location. As an immediate consequence, the NWT becomes translation invariant. Due to its structure, the NWT implies a finer sampling rate at all levels and thus provides a better exploratory tool for analyzing changes in the scale (frequency) behavior of the underlying signal in time. These advantages of the NWT over the DWT in time series analysis are demonstrated in Nason et al (e.g.[12]). As in the Fourier domain, it is important to assess the power of a signal at a given resolution. An evolutionary wavelet spectrum (EWS) quantifies the contribution to process variance at the scale j and time k. Another way of viewing the result of a NWT is to represent the temporal evolution of the data at a given scale. This type of representation is very useful to compare the temporal variation between different time series at given scale. To obtain the results, smooth signal S0 and the detail signals Dj (j =1, …, J) are:

$$S_0(t) = \sum_{k=-\infty}^{\infty} c_{0k}\,\varphi_{0,k}(t)$$

and

$$D_j(t) = \sum_{k=-\infty}^{\infty} d_{jk}\,\Psi_{j,k}(t) . \qquad (2)$$

The fine scale features (high frequency oscillations) are captured mainly by the fine scale detail components DJ and DJ–1. The coarse scale components S0, D1, and D2 correspond to lower frequency oscillations of the signal. Note that each band is equivalent to a band-pass filter. Further we use the Daubechies wavelet as mother wavelet [11]. This wavelet is bi-orthogonal and supports discrete wavelet transform. Using a link between wavelets and fractals, one could make calculating the multi-fractal spectrum. As usually, the homogeneous fractals are described by single fractal dimension D(0). Non-homogeneous or multifractal objects are described by spectrum D(q) of fractal dimensions or multifractal spectrum A problem of its calculation reduces to definition of singular spectrum f(α) of measure μ. It associates Haussdorff dimension and singular indicator α, that allows calculating a degree of singularity: Nα(ε)=ε-f(α). Below we use a formalism, which allows defining spectra of singularity and fractal dimension without using standard Legandre transformations. Wavelet transformation of some real function F can be also defined as [1]

$$\mathrm{W}_{\Psi}[F](b,a) = (1/\alpha)\int_{-\infty}^{+\infty} F(x)\Psi\left(\frac{x-b}{a}\right)dx \qquad (3)$$

where parameter b denotes a shift in space (a space scale). The analyzing splash Ψ has to be localized as in space as on frequency characteristics. The most correct way of estimate of the function D(h), f(α) is in analysis of changing a dependence of the distribution function Z(q,a) on modules of maximums of the splash-transfers under scale changes

$$Z = \sum_{i=1}^{N(a)} (\omega_i(a))^q \qquad (4)$$

where I=1,…,N(a); N(a) is a number of localized maximums of transformation WΨ[F](b,a) for each scale a; function ω(a) can be defined in terms of coefficients of the splash-transformations as

$$\omega_i(a) = \max_{\substack{(x,a')\in L \\ a'<a}} |W_{\Psi}[F](x,a')|, \qquad (5)$$

where li∈L(a); L(a) is a set of such lines, which make coupling the splash-transformation coefficient maximums (they reach or make cross-section of a level, which is corresponding to scale a). In the limit a→0+ the distribution function Z(q,a) manifests the behaviour, which is corresponding to a degree law:

Z(q,a)~aτ(q) .

To calculate a singularity spectrum, the standard canonical approach can be used. It is based on using such functions:

$$h(a,q) = \frac{1}{Z(a,q)} \frac{\partial Z(a,q)}{\partial q}, \qquad (6)$$

$$\frac{\partial Z}{\partial q} = \sum_{i=l}^{N(a)} \omega_i(a)^q \ln \omega_i(a), \qquad (7)$$

$$D(a,q) = qh(a,q) - \ln Z(a,q). \qquad (8)$$

The spectra D(q) and h(q) are defined by standard way as follows:

$$D(q) = \lim_{a \to 0} \frac{D(a,q)}{\ln a}, \qquad (9)$$

$$h(q) = \lim_{a \to 0} \frac{h(a,q)}{\ln a}. \qquad (10)$$

Other details can be found in Refs. [11,15-18].

2.2 Multisystem multifactor response model

Let us briefly describe the key points of the method of extreme hydrological events, following [21] .. The systemic approach output function of a nonlinear system is determined by the sum of the component of the nonlinear-determined instant-ing retarded and response system and linear components associated with the linear response of the system we [7,9]. The master equation for the output

$$Q_t = \sum_{j=1}^{J} \sum_{i=1}^{n(j)} \sum_{k=i}^{n(j)} U_{i,k}^{(j)} P_{t-i+1}^{(j)} P_{t-k+1}^{(j)} + \sum_{j=1}^{J} \sum_{i=1}^{k(j)} U_{i+n}^{(j)} P_{t-(i+n)+1}^{(j)} \qquad (11)$$

where j = 1,2, ..., J - the number of independent inputs (including due to rainfall), J - the number of small catchments (total floortion giving catchment); n is the number of time slots corresponding. rainfall, which contributes to the prompt and delayed flow components (non-linear part of the "memory" of the catchment), l - number of similar time intervals (the linear part of the "memory"); (n + l) - total length of the "memory" of the model; P - matrix precipitation j input series corresponding guide-j-th mini-catchment; $U_{i,k}$ – it refers to a series of discrete ordinates of the nonlinear response functions, which are summarized below, say, in the runoff coefficient; U_i – the same for the linear part. The important advantages of the model are adequate consideration essentially nonlinear response of the system, the possibility of introducing reverse governing relations [4], quite effective procedure for numerical programming. The model is calibrated further according to the number series of separate data on rainfall and runoff co-responsible. Уравнение (11) с учетом p (p=1, NN) числа серии данных записывается в следующем виде Equation (11)with accounting for p (p=1, NN) number of a series of data is written as follows

$$Q_t^p = \sum_{j=1}^{J} \sum_{i=1}^{n(j)} \sum_{k=1}^{n(j)} U_{i,k}^{(j)} P_{t-i+1}^{(j),p} P_{t-k+1}^{(j),p} + \sum_{j=1}^{J} \sum_{i=1}^{l(j)} U_{i+n}^{(j)} P_{t-(i+n)+1}^{(j),p} \qquad (12)$$

The solution of equation (12) for the calibration the series of the N runoffs Q1,Q2,..,QN can be represented in the vector-matrix form

$$\mathbf{Q} = P^{(1)} U^{(1)} + P^{(2)} U^{(2)} + \ldots + P^{(J)} U^{(J)}$$

Equation (11) can also be written as

$$Q = PU, \qquad (13)$$

where *P* - matrix of size

(N,M): $\mathbf{P} = \left[P^{(1)} P^{(2)}, \ldots, P^{(J)} \right]$

and

$$M = \sum_{j=1}^{J} \mathrm{mm}(j).$$

As a result, {PTP} is a square (MxM) symmetric matrix and U – (Mx1) vector (column). Then a solution of (13) by standard numerical methods. Other details can be found in Refs. [11,15-18].

3. RESULTS AND CONCLUSIONS

Using the above described formalism, we have carried out a multifractal analysis of spatial spectrum and time series of the daily runoffs for the Danube river (1989-1998 years), The corresponding detailed data on runoffs, particularly in the seasonal distribution for the years indicated are taken from the UNESCO report (data Slovakian and Romanian research groups) [21,22]. For example, Figure 1 shows the average daily cost to the district. Danube in the period 1989-1998 [18] .

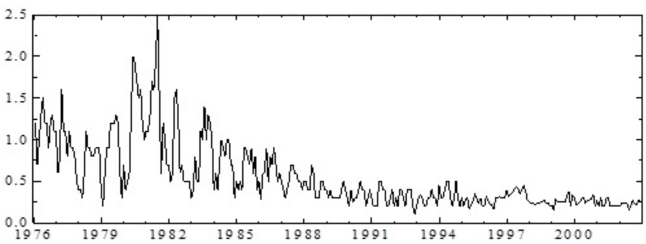

Fig. 1 - The average daily runoff for the Danube river in the period 1989-1998 (see text).

The process is analyzed on the time intervals which are more than the correlation scale, i.e., as one could wait for here, a intermittency has a multi-fractal nature. Using the PC complex "Geomath" (c.f.[15]) we have performed the numerical calculations of the fractal spectrum. Our numerical estimates have shown that the fractals dimensions are lying in the interval [2.4-2.7].

These data are satisfactory agreed with the preliminary estimates within the simple standard multifractal definition modelling.

Table 1 - The calculated runoffs (m^3/ s), corresponding to extreme floods for the Danube river (see text)

Date	Cunovo	Komarno	Sturovo
1.8.3:00	3770	3770	3771
1.8.6:00	3709	3767	3767
1.8.9:00	3650	3763	3763
1.8. 12:00	3608	3756	3761
1.8. 15:00	3583	3745	3758
1.8. 18:00	3565	3736	3755
1.8.21:00	3539	3725	3746
2.8. 0:00	3513	3710	3735
2.8. 3:00	3441	3692	3724
2.8. 6:00	3386	3671	3710
2.8. 9:00	3333	3652	3692
2.8. 12:00	3297	3623	3676
2.8. 15:00	3272	3595	3652
2.8. 18:00	3258	3562	3630
2.8.21:00	3250	3540	3601
3.8. 0:00	3245	3510	3575
3.8.3:00	3240	3485	3545
3.8. 6:00	3255	3460	3524
3.8. 9:00	3288	3435	3496
3.8. 12:00	3320	3416	3470
3.8. 15:00	3690	3411	3451
3.8. 18:00	4180	3430	3442
3.8.21:00	4727	3475	3437
4.8. 0:00	5136	3556	3472
4.8. 3:00	5475	3657	3520
4.8. 6:00	5713	3778	3586
4.8. 9:00	5929	3905	3674
4.8. 12:00	6039	4050	3775
4.8. 15:00	6182	4195	3890
4.8. 18:00	6304	4340	4010
4.8.21:00	6395	4485	4139
5.8. 0:00	6474	4628	4269
5.8.3:00	6618	4770	4398
5.8. 6:00	6715	4905	4538
5.8.9:00	6793	5040	4670
5.8. 12:00	6810	5171	4798
5.8. 15:00	6800	5294	4926
5.8. 18:00	6747	5410	5049
5.8.21:00	6680	5517	5167
6.8. 0:00	6550	5615	5280

In [21-23] as an application of the multisystem, multi-factor method have been performed evaluating flood discharges and compared with those observed ones for the river Danube from the station Devin (Bratislava) to the station Nagymaros [18,19].All empirical data needed for implementation of the model have been taken from the UNESKO reports [18,19] (and references cited therein). The model was calibrated according to the 1991, 1992 years. Table 1 shows the results of numerical modeling the characteristics of extremely high flood (flood scenario; see [18]). Detailed analysis and comparison of the results and possibilities of our method shows that application of the system model allows you to track quantitatively acceptable runoffs in a case of the extreme events. Of course, the critical point of the model is its proper calibration and adjustment. Resolving this issue could lead to further improvements in the model options. Therefore, our analysis confirms the universal conclusion regarding availability of the multifractal features for the daily runoff series for the Danube river.

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24.

АНАЛИЗ ФРАКТАЛЬНЫХ СТРУТУР В ХАОТИЧЕСКИХ ПРОЦЕССАХ: ВРЕМЕННЫЕ РЯДЫ СУТОЧНЫХ РАСХОДОВ ДЛЯ РЕКИ ДУНАЙ И ЭКСТРЕМАЛЬНЫЕ ГИДРОЛОГИЧЕСКИЕ ЯВЛЕНИЯ

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Данная работа продолжает наши исследования фрактальных структур в хаотических и турбулентных процессах и связана с большой актуальностью и важностью экспериментального и теоретического изучения нелинейных хаотических динамических систем с целью обнаружения фрактальных структур и свойств и элементов динамического хаоса. На основе вейвлет-анализа и мультифрактального формализма осуществляется анализ фрактальных структур в хаотических процессов (временные ряды суточных стоков для реки Дунай, 1989-1998 годы) и вычислено выдповыдний спектр фрактальных размерностей. На основе нового метода описания экстремальных гидрологических явлений, в частности, паводков, базирующегося на многофакторном системном моделировании и системной модели с «множеством входов» и «одним выходом» проведен численный расчет и приведены результаты характеристик экстремально высоких паводков (на примере р.Дунай).

Ключевые слова: гидрологические системы, фрактальные структуры, хаотические процессы, экстремальные гидрологические явления, фрактальные размерности

АНАЛІЗ ФРАКТАЛЬНИХ СТРУТУР У ХАОТИЧНИХ ПРОЦЕСАХ: ЧАСОВІ РЯДИ ДОБОВИХ ВИТРАТ ДЛЯ РІЧКИ ДУНАЙ І ЕКСТРЕМАЛЬНІ ГІДРОЛОГІЧНІ ЯВИЩА

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Дана робота продовжує наші дослідження фрактальних структур в хаотичних і турбулентних процесах і пов'язана з великою актуальністю і важливістю експериментального і теоретичного вивчення нелінійних хаотичних динамічних систем з метою виявлення фрактальних структур і властивостей та елементів динамічного хаосу. На основі вейвлет-аналізу та мультіфрактального формалізму здійснюється аналіз фрактальних структур в хаотичних процесів (часові ряди добових стоків для річки Дунай, 1989-1998 роки) і обчислено відповідний спектр фрактальних розмірностей. На основі нового методу опису екстремальних гідрологічних явищ, зокрема, паводків, що базується на багатофакторному системному моделюванні та системної моделі з «безліччю входів» і «одним виходом» проведено чисельний розрахунок та наведено результати характеристик екстремально високих паводків (на прикладі р.Дунай).

Ключові слова: гідрологічні системи, фрактальні структури, хаотичні процеси, екстремальні гідрологічні явища, фрактальні розмірності

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