МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ОДЕСЬКИЙ ДЕРЖАВНИЙ ЕКОЛОГІЧНИЙ УНІВЕРСИТЕТ

Methodical instructions for practical work, test performance, distance learning of PhD students in the discipline "QUANTUM GEOMETRY AND SPECTROSCOPY AND DYNAMICS OF RESONANCES", Part 2. (Training of PhD students of the specialty: 104 – "Physics and Astronomy" and others)

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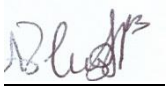 проф. Глушков О.В.

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на засіданні кафедри математики та квантової механіки
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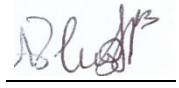 проф. Глушков О.В.

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Methodical instructions for practical work, test performance, distance learning of PhD students in the discipline “Quantum Geometry, Spectroscopy and Dynamics of Resonances”, Part 2. (Training specialty: 104 - “Physics and Astronomy” and others)

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PREFACE

Discipline "Quantum geometry, spectroscopy and dynamics of resonances" is an elective discipline in the cycle of professional training of postgraduate or PhD students (third level of education) in the specialty 104-Physics and Astronomy.

It is aimed at Acquisition (providing) of a number of competencies, in particular, the achievement of relevant knowledge, understanding and the ability to use the methods of quantum geometry and dynamics of resonances, the ability to develop new and improve existing mathematical methods of analysis, modeling and forecasting based on fractal geometry and elements of the chaos theory of regular and chaotic dynamics (evolution) of complex systems, the ability to develop fundamentally new and improve existing modern computational methods and algorithms of quantum mechanics, geometry and electrodynamics for analysis, modeling and prediction of the properties of classical and quantum systems with pronounced resonant behavior. mastering the modern apparatus of fractal geometry and chaos theory.

Competencies that must be acquired or developed include; i) K11 Ability to analyze and identify a complex of major problems in a certain field of modern physics and, in particular, optics and spectroscopy of atoms, multi-charged ions, molecular, quantum, laser systems, solid bodies, as well as the atmosphere and ocean; Ability to develop new and improve existing methods of describing optical and spectroscopic properties of solids based on methods of quantum mechanics, quantum chemistry of solids, as well as methods of relativistic quantum theory; ii) K12 The ability to create physical, mathematical and computer models in optics and spectroscopy of physical systems with the implementation of effective algorithms and specialized software; Ability to acquire new fundamental knowledge in optics and spectroscopy of atoms, molecules, solids, laser systems, as well as geophysical systems (atmosphere and ocean).These methodical instructions are for self-studying work of the second-year PhD students and tests performance in the discipline “Quantum Geometry and Dynamics of Resonances”.

The main topic is a Resonance dynamics for quantum systems in an electromagnetic field. Methods for calculating energies and widths of Stark resonances (Look Syllabus of the discipline, edition 2023)

I. Topic: The theory of resonances for quantum systems in the electromagnetic field. Theory of the Stark effect

Topic: Теорія резонансів для квантових систем в електромагнітному полі. Теорія ефекта Штарка, (Л 2.3; 2.5а)

1 Introduction

An external electric field shifts and broadens the bound state atomic levels. The standard quantum -mechanical approach relates complex eigenenergies (EE) $E = E_r + i\Gamma/2$ and complex eigenfunctions (EF) to the shape resonances [1-51]. The field effects drastically increase upon going from one excited level to another. The highest levels overlap forming a "new continuum" with lowered boundary. The calculation difficulties inherent to the standard quantum mechanical approach are well known. Here one should mention the well-known Dyson phenomenon. The Wentzel-Kramers-Brillouin (WKB) approximation overcomes these difficulties for the states lying far from the " new continuum" boundary. Some modifications of the WKB method [1] are introduced in Stebbings and Dunning (1983), Kondratovich and Ostrovsky (1982, 1984), Popov et al (1988, 1990) and Glushkov, Ivanov and Letokhov (1975, 1992), where the first theoretical estimation of the effectiviness of the selective ionization of the Rydberg atom using electric and laser fields has been fulfilled. The usual WKB approximation applicability is substantiated in the case of a relatively weak electric field [23]. One can show that the standard form of the WKB method applicability condition can be reformulated as the requirement that the examined resonances be well separated one from other. The same is so regarding the widespread asymptotic phase method (Damburg and Kolosov 1976), based on the Breit-Wigner parameterization for the asymptotic phase shift dependence on scattering energy and the method by Luc-Koenig and Bachelier, who have used a normalization constant [18]. Different calculational procedures are used in the Pade and then Borel summation of the divergent Rayleigh-Schrödinger perturbation theory (PT) series (Franceschini et al 1985, Popov et al 1990) and in the sufficiently exact numerical solution of the difference equations following from expansion of the wave function over finite basis (Benassi ans Grecchi 1980, Maquet et al 1983, Kolosov 1987, Telnov 1989, Anokhin-Ivanov 1994), complex-scaling method [1-25]. It should be

noted that the latter has been extensively used to describe the resonance behavior in different atomic and even molecular systems. Its mathematical foundation is linked with the theory of dilatation analyticity. Surely, though the Hamiltonian of an atom in a DC electric field is not a dilatational analytic operator, Reinhardt has performed the numerical experiments on the diagonalization of the complex-scaled Stark Hamiltonian for a hydrogen with a real L basis set. The same method has been used by Cerjan et al. to get new data on the ground and low-excited states of a hydrogen atom in a DC and AC fields. Farrelly and Reinhardt have used the complex coordinate rotation method in combination with numerical integration of the separated equation. Ivanov-Ho have applied the method for the Dirac Hamiltonian. Different applications are reviewed in Refs. [1-15].

Rao, Liu and Li (1994) have studied theoretically the DC strong-field Stark resonances by a complex-scaling plus B-spline approach and shown that the high accuracy is attributed to the good stationarity behavior of eight trajectories with a well-adjusted 8-spline basis. Rao and Li (1995) have also studied the behavior of the resonances of a hydrogen atom in parallel magnetic and electric fields with a complex scaling plus B-spline method too and received a consistent data on the corresponding resonance parameters in dependence upon the ratio of the magnetic-field strength to the electric-field strength. It is worth to remind that the similar approaches have been developed to describe the Zeemane resonances. Namely, for hydrogen atoms in pure magnetic fields, the properties of resonant states were calculated by the complex scaling, the R matrix, the operator PT (OPT) and other methods (look, for example, [4-7]. The generalization of methods to account for the resonance interference, non-H and relativistic effects is still an important problem, though here a definite progress has been reached too. One should mention such approaches as a model potential method, quantum defect approximation, the OPT, complex scaling plus B-spline method etc [1-19, 24-28]. Regarding the quantum chaos phenomenon in atoms in electromagnetic fields (note that this topic should not be considered here. Let us only note that the approach presented below together with the various methods of the theory of chaos in options has been effectively used to describe the chaotic behavior of the hydrogen and non-H atoms in the magnetic and microwave fields.

Here a consistent uniform quantum-mechanical approach to the solution of the non-stationary state problems including the DC strong-field Stark effect and

also scattering problem is presented. It allows calculation of complex EE and especially is destined for investigation of the spectral region near the new continuum boundary. The essence of the method is the inclusion of the well known “distorted waves approximation” method in the frame of the formally exact PT. The zero-order Hamiltonian H_0 of this PT possesses only stationary bound and scattering states. To overcome formal difficulties, we define the zero-order Hamiltonian by the set of orthogonal eigenfunctions (EF) and EE without specifying the explicit form of the corresponding zeroth-order potential. To ensure rapid PT convergence, a physically reasonable spectrum (EE and EF) must be chosen as the zero order, similar to the “distorted waves” method [3]. In a case of the optimal zeroth-order spectrum, the PT smallness parameter is of the order of Γ/E, where Γ and E are the field width and bound energy of the state level examined. The successive PT corrections can be expressed through the matrix elements of the total Hamiltonian calculated between the zeroth-order basis functions. This method is called the OPT. We will define H_0 so that it coincides with the total Hamiltonian H at $\varepsilon \Rightarrow 0$ (ε is the electric field strength.) Let us emphasize that perturbation in our theory does not coincide with the electric field potential though they disappear simultaneously. We also present a generalization of the OPT for calculation of the DC strong field Stark effect in the non-H atoms in an electric field [3]. The difference between the atomic and Coulomb field is taken into account by introducing the quantum defects on a parabolic basis. The results of calculation of the Stark resonance energies and widths for the H and sodium atoms are listed and compared with other theoretical and experimental data.

2 Complex resonance energy. Methods for calculating energies and widths of Stark resonances Operator perturbation theory for Stark effect

2.1 *DC strong –field Stark effect for the hydrogen atom*

The Schrödinger equation for the electron function taking into account the uniform electric field and field of the nucleus (Coulomb units are used: for length, 1 unit is h^2/Ze^2m; for energy 1 unit is mZ^2e^4/h^2) is [3]:

$$[-(1 - N/Z) / r + V_m(r) + \varepsilon z - 1/2\Delta - E\ ]\ \psi = 0, \qquad (1)$$

where E is the electron energy, Z is the nucleus charge, N is the number of electrons in the atomic core (for the hydrogen atom: Z=1, N=0), V_m is an model potential (for the hydrogen atom V_m=0). Firstly, we only deal with the Coulomb part of the electron- atomic residue interaction. The non-Coulomb part, as well as relativistic effects, can be approximately accounted for next step. The separation of variables in the parabolic coordinates ($\xi = r + z,\ \eta = r - z,\ \varphi = \tan^{-1}(y/x)$) :

$$\psi(\zeta, \eta, \varphi) = f(\zeta)\, g(\eta)(\zeta \cdot \eta)^{|m|/2} \exp(im\varphi)/(2\pi)^{1/2} \qquad (2)$$

transforms it to the system of two equations for the functions f, g:

$$f'' + \frac{|m|+1}{t} f' + [1/2E + (\beta_1 - N/Z) / t - 1/4\varepsilon(t)\, t\ ]\, f = 0, \qquad (3)$$

$$g'' + \frac{|m|+1}{t} g' + [1/2E + \beta_2 / t + 1/4\varepsilon(t)\ t\ ]\, g = 0, \qquad (4)$$

coupled through the constraint on the separation constants:

$$\beta_1 + \beta_2 = 1 \qquad (5)$$

For the uniform electric field

$$\varepsilon(t) = \varepsilon\,.$$

In principle, the more realistic models can be considered in the framework of our approach. Potential energy in equation (4) has the barrier. Two turning points for the classical motion along the η axis, t_1 and t_2, at a given energy E are the solutions of the quadratic equation ($\beta = \beta_1,\ E = E_0$):

$$t_2 = \{[\ E_0^2 - 4\varepsilon(1-\beta)]^{1/2} - E_0\ \}/\varepsilon, \qquad (6)$$

$$t_1 = \{-[E_0^2 - 4\varepsilon(1-\beta)]^{1/2} - E_0\ \}/\varepsilon, \quad t_1 < t_2 \qquad (7)$$

Here and below t denotes the argument common for the whole equation system. To simplify the calculational procedure, the uniform electric field ε in (3) and (4) should be substituted by the function [23]:

$$\varepsilon(\mathrm{t}) = \frac{1}{t}\,\varepsilon\left[(t-\tau)\frac{\tau^4}{\tau^4+t^4}+\tau\right] \tag{8}$$

with sufficiently large τ (τ=1.5t_2).

The function $\varepsilon(t)$ practically coincides with the constant ε in the inner barrier motion region ($t<t_2$) and disappears at $t>>t_2$.

The minimal acceptable value of τ introduced in the spatial dependence of the electric field, which does not influence the final results, can be established experimentally.

Thus, the final results do not depend on the parameter τ (the further calculation has entirely confirmed this fact). Besides the pure technical convenience, the case of an asymptotically disappearing electric field is more realistic from the physical point of view. Now we deal with the asymptotically free (without electric field) motion of the ejected electron along the η-axis. The corresponding effective wavenumber is:

$$k = (E/2 + \varepsilon\tau/4)^{1/2}. \tag{9}$$

The scattering states energy spectrum now spreads over the range $(-\varepsilon\tau/2,+\infty)$, compared with $(-\infty,+\infty)$ in the uniform field. In contrast to the case of a free atom in scattering states in the presence of the uniform electric field remain quantified at any energy E, i.e. only definite values of β_1 are possible.

The latter are determined by the confinement condition for the motion along the η-axis. The same is true in our case, but only for

$$E \subset \left(-\frac{1}{2}\varepsilon\tau,+\frac{1}{2}\varepsilon\tau\right).$$

The motion with larger E is non-quantified, similar to the free atom case.

2.2 Energy and width of the Stark resonance

The total Hamiltonian $H\ (\varsigma,\ \nu,\ \varphi)$ does not possess the bound stationary states. According to OPT [6, 56-58]), one has to define the zero order Hamiltonian H_0, so that its spectrum reproduces qualitatively that of the initial one. In contrast to H, it must have only stationary states. To calculate the width Γ of the concrete quasistationary state in the lowest PT order one needs only two zeroth–order EF of H_0: bound state function $\Psi_{Eb}(\varepsilon,\ \eta,\ \varphi)$ and scattering state function $\Psi_{Es}(\varepsilon,\ \eta,\ \varphi)$ with the same EE. We solve a more general problem: a construction of the bound state function along with its complete orthogonal complementary of scattering functions Ψ_E with $\mathrm{E} \subset \left(-\frac{1}{2}\varepsilon\tau, +\infty\right)$. First, one has to define the EE of the expected bound state. It is the well known problem of states quantification in the case of the penetrable barrier [65,66]. Following [57], we solve the system (3) and (4) with the total Hamiltonian H under the conditions:

$$f(t) \to 0 \text{ at } t \Rightarrow \infty\,, \qquad (10a)$$

$$\partial x(\beta, E) / \partial E = 0 \qquad (10b)$$

with $$x(\beta, E) = \lim_{t \Rightarrow \infty} [\ g^2(t) + \{g'(t) / \mathrm{k}\}^2\ ]\ t^{|m|+1}. \qquad (11)$$

The first condition ensures the finiteness of motion along the ς-axis, the second condition minimizes the asymptotic oscillation amplitude for the function describing the motion along the η-axis. These two conditions quantify the bound energy E and separation constant β_1. We elaborated a special numerical procedure for this two-dimensional eigenvalue problem. Our procedure deals repeatedly with the solving of the system of the ordinary differential equations (3) and (4) with probe pairs of $E,\ \beta_1$. The corresponding EF:

$$\psi_{Eb}(\zeta,\ \eta,\ \varphi) = f_{Eb}(\zeta)\ g_{Eb}(\eta)(\zeta\,\eta)^{|m|/2}\ exp\ (im\varphi)(2\pi)^{-1/2}\,. \qquad (12)$$

Here $f_{Eb}(t)$ is the solution of (3) (with the just determined E, β_1) at $t \subset (0,\infty)$ and $g_{Eb}(t)$ is the solution of (4) (with the same E, β_1) at $t < t_2$ (inside barrier) and $g(t) = 0$ otherwise. These bound state EE, eigenvalue β_1 and EF for the zero-order

Hamiltonian H_0 coincide with those for the total Hamiltonian H at $\varepsilon \Rightarrow 0$, where all the states can be classified due to the quantum numbers $n,\ n_1,\ n_2, m$ (principal, parabolic, azimuthal) connected with E, β_1, m by the well known expressions. We preserve the $n,\ n_1, m$ states classification in the non-zero ε case. The scattering state functions:

$$\psi_{E's}\ (\zeta,\ \eta,\ \varphi) = f_{E's}(\zeta)\ g_{E''s}(\eta)\ (\zeta\ \eta)^{|m|/2}\ \exp\ (im\varphi)(2\pi)^{-1/2} \qquad (13)$$

must be orthogonal to the above defined bound state function and to each other. In addition, these functions must describe the motion of the ejected electron, i.e. $g_{E's}$ must satisfy the equation (4) asymptotically. Following the OPT ideology [23], we choose the next form of $g_{E's}$:

$$g_{E's}(t) = g_1\ (t) - z_2'\, g_2(t) \qquad (14)$$

with $f_{E's}$ and $g_1(t)$ satisfying the differential equations (3) and (4). The function $g_2(t)$ satisfies the non-homogeneous differential equation, which differs from (4) only by the right-hand term, disappearing at $t \Rightarrow \infty$. The total equation system, determining the scattering function, reads

$$\begin{aligned} f''_{E's} + \frac{|m|+1}{t} f'_{E's} + [1/2E' + (\beta_1' - N/Z) / t - 1/4\ \varepsilon(t)t]\ f_{E's} = 0, \\ g_1'' + \frac{|m|+1}{t} g_1' + + [1/2E' + \beta_2' / t + \ 1/4\varepsilon(t)t]\ g_1 = 0, \\ g_2'' + \frac{|m|+1}{t} g_2' + + [1/2E + \beta_2' / t + \ 1/4\varepsilon(t)t]\ g_2 = 2g_{Eb}, \end{aligned} \qquad (15)$$

($\beta_1' + \beta_2' = 1$). As mentioned above there remains motion quantification for $E' \subset \left(-\frac{1}{2}\varepsilon\tau, +\frac{1}{2}\varepsilon\tau\right)$. At the given E', the only quantum parameter β_1' is determined by the natural boundary condition: $f_{E's} \Rightarrow 0$ at $t \Rightarrow \infty$.

Of course: $\beta_1' = \beta_1$, $f_{E's} = f_{Eb}$ at $E' = E$; only this case is needed in the particular problem we deal with here. The coefficient z_2' ensures the orthogonality condition $\langle \Psi_{Eb} | \Psi_{E's} \rangle = 0$:

$$z_2' = \{ \iint d\zeta d\eta\, (\zeta+\eta)\, f^2_{Eb}(\zeta) g_{Eb}(\eta) g_1(\eta)\} /$$

$$/\{ \iint d\zeta d\eta\, (\zeta+\eta)\, f^2_{Eb}(\zeta) g_{Eb}(\eta) g_2(\eta)\}. \qquad (16)$$

One can check that

$$\langle \psi_{Es'} | \psi_{E''s} \rangle = 0 \quad \text{for} \quad E' \neq E''$$

The imaginary part of state energy in the lowest PT order is

$$\mathrm{Im}E = \Gamma/2 = \pi | < \Psi_{Eb} | H | \Psi_{Es} > |^2 \qquad (17)$$

with the total Hamiltonian H. The state functions Ψ_{Eb} and Ψ_{Es} are assumed to be normalized to 1 and by the $\delta(k-k')$ condition, accordingly. The action of H on Ψ_{Eb} is defined unambiguously by (15):

$$(H-E')\psi_s = 2|m|(\zeta\cdot\eta^2)\cdot f_{E's}(\zeta)\, g_{Eb}(\eta) z_2' \exp(im\varphi)/[(2\pi)^{1/2}(\zeta+\eta)],$$

$$\langle \psi_{Eb} | H | \psi_{E's} \rangle = \iint d\zeta d\eta\, (\zeta\eta)^{|m|} \eta\, f^2_{Eb}(\zeta) f^2_{E's}(\zeta) g_{Eb}(\eta)\, z_2'. \qquad (18)$$

The matrix elements $\langle \Psi_{Eb} | H | \Psi_{E's} \rangle$ entering the high- order PT corrections can be determined in the same way. All the two-dimensional integrals in (16)-(18) and the normalization coefficients can be expressed through the next set of one-dimensional integrals:

$$\begin{aligned}
I_1 &= \int dt\, f_b^2(t) t^{|m|}, & I_2 &= \int dt\, f_b^2(t) t^{|m|+1}, \\
I_3 &= \int dt\, g_b(t)\, g_1(t)\, t^{|m|}, & I_4 &= \int dt\, g_b(t) g_1(t) t^{|m|+1}, \\
I_5 &= \int dt\, g_b(t)\, g_2(t)\, t^{|m|}, & I_6 &= \int dt\, g_b(t)\, g_2(t) t^{|m|+1}, \\
I_7 &= \int dt\, g_b{}^2(t)\, t^{|m|}, & I_8 &= \int dt\, g_b^2(t) t^{|m|+1}, \qquad (19)
\end{aligned}$$

calculated with the arbitrary normalized functions f_{Eb}, g_{Eb}, f_2, g_2, and $f_1 = f_{Eb}$, $g_1 = g_{Eb}$. In this notation

$$\Gamma = 32\pi z_2^2 N_s^2 I_1^2 I_8^2 / [I_2 I_7 + I_1 I_8],$$

$$z_2 = [I_1 I_4 + I_2 I_3]/[I_1 I_6 + I_2 I_5] \qquad (20)$$

with

$$N^2_s = \lim_{t \Rightarrow \infty} X(t)/\{2\pi\eta^{2|m|+1}[\, g^2_s(\eta) X^2(t) + g'^2_s(\eta)]\}\ ,$$

$$X(t) = \{E/2 + (\beta - N/Z)/t - E\, t/4\}^{1/2} \qquad (21)$$

Remember that arbitrary normalized state functions are assumed in (20) and (21). The whole calculational procedure at known resonance energy E and separation parameter β_1 has been reduced to the solution of one system of the ordinary differential equations. This master system includes the differential equations for the state functions f_{Eb}, g_{Eb}, f_{Es}, g_{Es}, as well as the equations for the integrals $I_1 - I_8$. Thus, our calculational procedure is one-dimensional. The procedure is sufficiently simple and realized as the numerical code with using the fourth-order Runge–Kutta method of solving the differential equations (the atomic code “Superatom-ISAN-Stark”).

II. Task options for self-sufficient work

Task Option 1.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength,

Explain all definitions in theory of Stark resonances for atomic systems in DC electric field on the example of the hydrogen, helium and any alkali atom , preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.

2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **Li.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

Task Option 2.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength; Explain all definitions in theory of Stark resonances for atomic systems in DC electric field on the example of the hydrogen, helium and any alkali atom , preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.
2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **Na.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

Task Option 3.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength; Explain all definitions in theory of Stark resonances for atomic systems in DC electric field on the example of the

hydrogen, helium and any alkali atom, preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.
2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **K.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

Task Option 4.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength; Explain all definitions in theory of Stark resonances for atomic systems in DC electric field on the example of the hydrogen, helium and any alkali atom , preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.
2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **Rb.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

Task Option 5.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength; Explain all definitions in theory of Stark

resonances for atomic systems in DC electric field on the example of the hydrogen, helium and any alkali atom, preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.
2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **Cs.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

Task Option 6.

1). Give the key definitions of a theoretical approach to definition of the energy and spectral characteristics of the Stark resonances using the standard quantum-mechanical amplitude approach and new formalism of operator perturbation theory by Glushkov-Ivanov: i) mathematical and physical essense of quantum-mechanical amplitude approach , ii) mathematical and physical essense of operator perturbation theory by Glushkov-Ivanov: iii) calculation of the Stark resonances energies and widths s, iv) calculation of the ionization cross section in a presence of DC electric field, v) analysis of the role of correlation effects and value of the field strength; Explain all definitions in theory of Stark resonances for atomic systems in DC electric field on the example of the hydrogen, helium and any alkali atom, preliminarily describing the corresponding spectrum of a free system , i.e. without an external electric field.
2).To apply the operator perturbation theory by Glushkov-Ivanov for computing the Stark resonances energies and widths of any alkali atom, say **Fr.** To perform its pracrical realization (using Fortran Power Station , Version 4.0; PC Code: "Superatom-Stark" for quantum system from the first task of the option (all necessary numerical parameters should be self-taken).

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